Canadian Institute of
Guided Ground Transport
Queen's University
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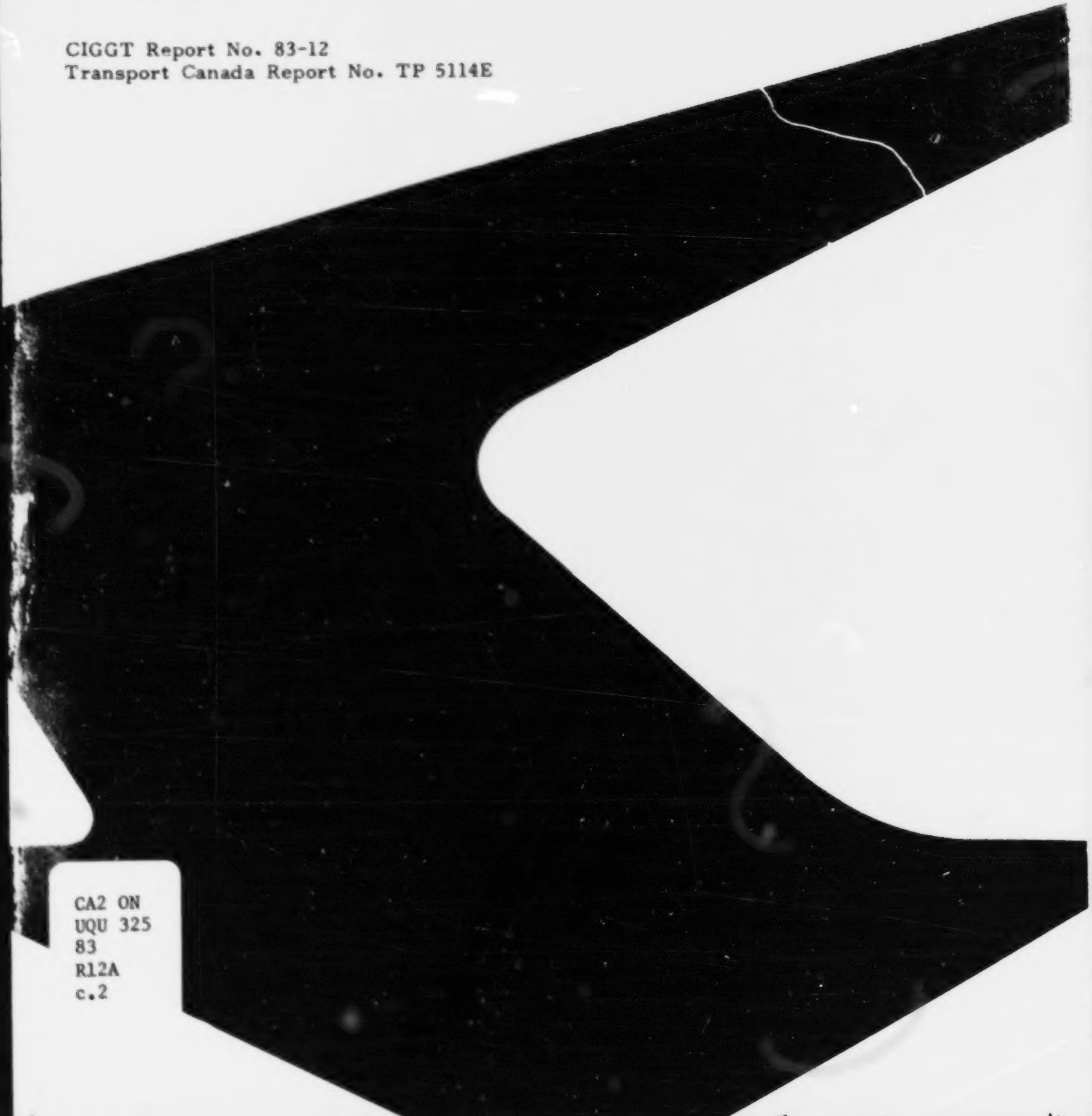

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EVALUATION OF DESIGN ALTERNATIVES FOR A FREIGHT TRAIN CONTROLLER

Appendix A

Automatic Control of Freight Trains

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EVALUATION OF DESIGN ALTERNATIVES FOR A FREIGHT TRAIN CONTROLLER

Appendix A

AUTOMATIC CONTROL OF FREIGHT TRAINS

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This project on development and optimization of control schemes for freight trains was motivated by the need to find optimal or suboptimal closed-loop control schemes for long freight trains. Several on-line and off-line freight train control strategies have been studied and reported. The relation between these control strategies is discussed in this report. The structure of an overall train operation scheme which consists of an off-line energy saving strategy and an on-line control strategy to maintain the velocity profile specified by the off-line scheme is proposed. There are five separately-bound appendices: Appendix A: AUTOMATIC CONTROL OF FREIGHT TRAINS; Appendix B: A REDUCED-ORDER MODEL FOR LONG FREIGHT TRAINS; Appendix C: A COMPARATIVE STUDY OF SELF-TUNING CONTROLLERS FOR FREIGHT TRAINS; Appendix D: MINIMUM ENERGY CONSUMPTION CONTROL OF PASSENGER/FREIGHT TRAINS WITH FIXED TERMINAL TIME; Appendix E: OPTIMAL CONTROL STRATEGY FOR MINIMUM ENERGY AND MINIMUM FUEL CONSUMPTION OF FREIGHT TRAINS

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17. Résumé

L'étude dont ce rapport rend compte a trait aux programmes visant l'automatisation et l'optimalisation de la conduite des longs convois de marchandises, étude motivée par le besoin pressant de trouver des techniques en boucle fermée qui soient optimales, ou tout proche. Le rapport présente un certain nombre de programmes de marche prédétermines et de programmes d'optimalisation de la conduite; de plus, il traite de l'interaction entre eux. Il préconise un programme du premier type axé sur l'économie d'énergie, secondé par un programme du deuxième type assurant à la fois le respect des consignes imposées et une consommation minimale d'énergie. Le rapport contient cinq annexes distinctes, intitulées comme suit: A: Automatisation de la conduite des convois de marchandises; B: Logiciel en réduction approprié aux longs convois de marchandises; C: Etude comparative des logiciels tenant compte des variations dans les paramètres de conduite; D: Consommation minimale des trains de voyageurs et de marchandises à horaires fixes; E: Stratégie d'optimalisation de la consommation d'énergie et de carburant dans le cas des trains de marchandises.

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1. INTRODUCTION

This report presents a multivariable proportional and integral (PI) controller for automatic operation of long freight trains. The controller achieves asymptotic tracking of a given velocity profile, and minimizes a quadratic performance index subject to physical and practical constraints. The performance index accounts for the energy consumption, velocity deviation, and intercar coupling forces. The new controller is also capable of accommodating various disturbances such as changes in grade profile and aerodynamic resistance, small variations in train parameters, and inaccuracies arising from linearization and simplification of the train model. A technique is also developed to combine the scheduled (offset) brakes and throttle with the controller operation to ensure better regulation of train dynamics. A description is given of a train simulation program developed to test the effectiveness of the controller. Finally, simulation results to demonstrate the performance of the controller under various operating conditions are presented.

The existing train operation consists of pneumatic brakes on cars, the 'Locotrol'[1] radio communication system between locomotives, and an open-loop manual control which results in relatively slow train operation. In recent years, control methods for the brakeless operation of long freight trains were suggested and discussed in several reports.[2-5] Barry and Davis[4,5] designed an optimal controller for brakeless operation using a distributed parameter model of the train. The implementation of the controller, however, still poses some problems. The controller design for trains with brake operation was studied by Gruber and Bayoumi,[6,7] and by Wong, Bayoumi and Gruber.[8] Throttle and brake coordination is achieved by switching between two (or more) controllers, depending on whether brake service is needed or not. However, some simulation results show that undesirable transient fluctuations may occur at the switching moments.

This study introduces the PI controller to overcome this difficulty and to achieve better velocity tracking in addition to minimizing the coupler forces, velocity deviations, and energy consumption.

2. TRAIN MODEL

The train model makes use of the studies of McLane et al,[3] Gruber and Bayoumi,[6] and the Freight Train Simulator (FTS).[15] The simulation of train dynamics is performed using a full nonlinear model of the train,[15] while the controller design is based on a simplified linear model. In the nonlinear model, each train member is represented as shown in Figure 1.

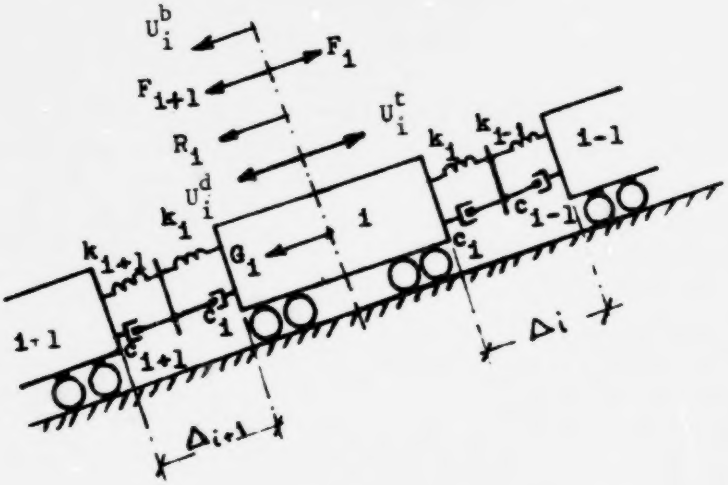


Figure 1 Model of Train Member i

The draw bar forces may be computed using the following formula:

$$\dot{v}_i = \frac{1}{m_i}\left[F_i(t) - F_{i+1}(t) - R_i(t) - G_i(t) + u_i^t - U_i^d - U_i^b\right] \quad (1)$$

$$\dot{\Delta}_i = v_{i-1} - v_i$$

where: m_i = mass of the ith train member

v_i = velocity of the ith train member

Δ_i = dynamic displacement between train members #i and #i-1

F_i = draft gear forces between train member #i-1 and member #i

R_i = total rolling and aerodynamic resistance acting on train member #i

G_i = component of gravitational force against direction of motion acting on train member #i

U^t = throttling force acting on train member #i

U_i^d = dynamic braking force on train member #i

U_i^b = electropneumatic braking force on train member #i

Each coupler of the i^th train member is represented by a spring having a spring constant, k_i, a damping coefficient, c_i, and a dead zone (stack), DZ. Thus, the draft gear force, F_i, is given by:

$$\begin{aligned} F_i &= {}_i\, k_i + c_i\, \dot{\delta}_i & \text{if } |\Delta_i| \geq \mathrm{DZ} \\ &= 0.0 & \text{if } |\Delta_i| < \mathrm{DZ} \end{aligned} \qquad (2)$$

where δ_i is the elongation (or compression) of the front coupler of the i^{th} car. δ_i in turn is computed via the following nonlinear differential equation:

$$\dot{\delta}_i = \frac{1}{c_i + c_{i-1}} [- (k_i + k_{i-1})\, \delta_i + d_1 (\Delta_i, k_{i-1}, \mathrm{DZ})$$

$$+ d_2 (\dot{\Delta}_i, c_{i-1}, \mathrm{DZ}, {}_i)]$$

$$\text{where: } \begin{aligned} d_1 (x,y,z) &= xy & \text{if } |x| \geq Z \\ &= 0.0 & \text{if } |x| < Z \\ d_2 (x,y,z,w) &= xy & \text{if } |w| \geq Z \\ &= 0.0 & \text{if } |w| < Z \end{aligned}$$

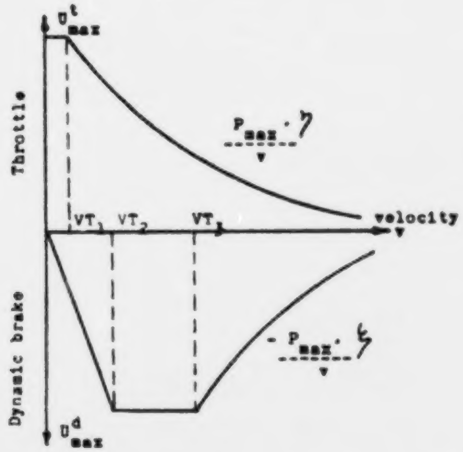
Note that Δ_i, δ_i and δ_{i-1} are related by $\Delta_i = \delta_{i-1} + \delta_i$.

The rolling resistance and aerodynamic drag force are nonlinear functions of speed and are given by:

$$R_i = r_{0i} + r_{1i} v_i + r_{2i} v_i^2 \tag{3}$$

where r_{0i}, r_{1i} and r_{2i} are constants which depend on the mass, number of axles of the train member under consideration, its cross-sectional area, and location in the train if train member i is a locomotive.

The locomotives utilized in this study are diesel-electric units in which the maximum allowable throttle/dynamic braking force obeys approximately the relation shown in Figure 2.



P_{max} : maximum power
η : transmission efficiency
ξ : locomotive constant

Figure 2
Typical Tractive
Force Constraints

In the dynamic braking state, the traction motors act as generators which absorb the kinetic energy of the train and feed it to a dummy electric load. The throttle/dynamic braking range is quantized according to the following approximate levels:

$$U^t = \frac{n}{8} \frac{P_{max} \times \eta}{v} \tag{4}$$

$$U^d = -\frac{n}{8} \frac{P_{max} \times \xi}{v} \tag{5}$$

where: v = velocity of the locomotive

n = 0, 1, 2, ... 8, the power level.

It is assumed here that the quantization levels are uniformly distributed among the eight quantization positions.

If hydraulic braking is used in the train, for each braking input signal, U_i, there is a different delay time before the braking force is actually activated. If electropneumatic braking is used, the delay time for activating the brakes is negligible, but for both cases there will be an additional time for the pressure in the brake cylinder to build up or release. The asymmetrical behaviour of the brake cylinder pressure is approximated by linear segments during build-up and exponential segments for release. For the controller design, a series of simplifications has been made as follows:

- the model is linearized to a time invariant system for a certain speed range
- power constraints on the throttle and dynamic brake are not considered, but the one-sided constraints are taken into account
- build-up and release time constants of the brake cylinder pressure are set to zero
- coupler slack is neglected.

The linearized state variable form can now be written as:

$$\delta \underline{\dot{x}} = A\ \delta \underline{x} + B\ \delta \underline{u} + \delta \underline{d} \qquad (6)$$

where $\delta \underline{x}$, $\delta \underline{u}$ and $\delta \underline{d}$ are the changes from the nominal values of the states, control inputs, and disturbances.

The train configuration of interest consists of n train members; specifically n_L locomotives and n_c loaded gondola cars. m_1 of the locomotives are located at the front and the rest of the locomotives, m_2, are placed after about two-thirds of the train. The train is divided into two sections for electropneumatic brake application. Thus, two brake control inputs are utilized, one for each section. The application is performed in parallel to all members of the section. This train configuration was recommended by the Transportation Development Centre of Transport Canada. In this case, the states and input vectors are given by:

$$\delta\underline{x}^{T} = [\delta v_1, \Delta_1, \delta v_2, \ldots\ldots, \Delta_{n-1}, \delta v_n] \qquad (7)$$

$$\delta\underline{u}^{T} = [\delta u_1, \delta u_2, \delta u_3, \delta u_4, \delta u_5, \delta u_6] \qquad (8)$$

where: δu_1, δu_2 = the throttle of each locomotive in front, and the middle group of locomotives, respectively

δu_3, δu_4 = the dynamic brake of each locomotive in front, and the middle group of locomotives, respectively.

δu_5, δu_6 = brake control inputs of the two blocks of cars described previously

The entries a_{jk} of matrix A in the linearized model (6) are given by (i = integer; $2 \leq i < 2n-1$).

$$a_{2i-1,\, 2i-3} = \frac{\tilde{C}_{i-1}}{m_i}$$

$$a_{2i-1,\, 2i-2} = \frac{\tilde{k}_{i-1}}{m_i}$$

$$a_{2i-1,\, 2i-1} = -(\tilde{C}_{i-1} + \tilde{C}_i + r_i)/m_i$$

$$a_{2i-1,\, 2i} = -\tilde{k}_i/m_i$$

$$a_{2i-1,\, 2i+1} = \tilde{C}_i/m_i$$

$$a_{2i-2,\, 2i-3} = 1$$

$$a_{2i-2,\, 2i-1} = -1$$

$$a_{2,1} = 1, \quad a_{2,3} = -1$$

$$a_{1,1} = (r_1 + \tilde{C}_1)/m_1$$

$$a_{1,3} = \tilde{C}_i/m_1, \quad a_{1,2} = -\tilde{k}_1/m_1$$

$$a_{2n-1,\,2n-3} = \tilde{C}_{n-1}/m_n$$

$$a_{2n-1,\,2n-2} = \tilde{k}_{n-1}$$

$$a_{2n-1,\,2n-1} = -(\tilde{C}_{n-1} + r_n)/m_n$$

all other entries of matrix A are zero.

where $\tilde{k}_i$ and $\tilde{C}_i$ are the equivalent spring constant and damping constant of the draft gear between cars #i and i+1. r_i is the average rolling and aerodynamic resistance. r_i is obtained by performing least-squares curve-fitting on relation (3) over the speed range, 2.5 metres/second to 28 metres /second. The entries b_{ij} of matrix B in the linearized model (6) are given by (i = integer; $1 \leq i \leq n$).

$$\left.\begin{array}{ll} b_{2i-1,\,1} & = \dfrac{1}{m_i} \\[2ex] b_{2n-1,3} & = -\dfrac{1}{m_i} \end{array}\right\} \text{if i corresponds to one of the front locomotives}$$

$$b_{2i-1,1},\ b_{2i-1,3} = 0.0 \text{ otherwise}$$

$$\left.\begin{array}{ll} b_{2i-1,2} & = \dfrac{1}{m_i} \\[2ex] b_{2i-1,4} & = -\dfrac{1}{m_i} \end{array}\right\} \text{if i corresponds to one of the middle locomotives}$$

$$b_{2i-1,2},\ b_{2i-1,4} = 0.0 \text{ otherwise}$$

$$
\begin{aligned}
b_{2i-1,5} &= -\frac{1}{m_i} \quad \text{if train member i belongs to the front train brake section} \\
&= 0.0 \quad \text{otherwise} \\
b_{2i-1,6} &= -\frac{1}{m_i} \quad \text{if train member i belongs to the second train brake section} \\
&= 0.0 \quad \text{otherwise}
\end{aligned}
$$

all other entries of the B matrix are zero

A computer program to build this linearized model for any arbitrary train configuration is given in Annex C.

3. THE MULTIVARIABLE PI CONTROLLER

There has been a great deal of interest in the design of multivariable servomechanism systems in recent years.[9-12] In this type of problem, the mathematical model of the plant to be controlled is known and it is desired to design a controller so that the outputs of the system asymptotically track a specific reference signal, independent of any disturbances occurring in the system. The design should be such that closed-loop stability is maintained and other constraints on system performance are satisfied, i.e. tolerances to variations in plant parameters and inaccuracies arising from linearization and simplification of the plant model. The multivariable proportional and integral feedback controller proposed by Smith and Davison[11] will be adopted here, since it is relatively easy to build using the standard linear quadratic regulator theory. The control objective is to keep the coupler forces below 11,400 kg, and the relative velocities between cars should not exceed one metre per second. It is also desirable to minimize velocity deviations and energy consumption, and achieve asymptotic velocity tracking. The controller design proceeds from the following augmented system:

$$
\begin{bmatrix} \delta \underline{\dot{x}} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} \delta \underline{x} \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \delta \underline{u} + \begin{bmatrix} \delta \underline{d} \\ 0 \end{bmatrix} \qquad (9a)
$$

$$\delta y = \begin{bmatrix} C \mid 0 \end{bmatrix} \begin{bmatrix} \delta \underline{x} \\ \underline{z} \end{bmatrix} \tag{9b}$$

$$\dot{z} = \delta y = y - y_{ref} \tag{9c}$$

The output, y, is taken to be the velocity of the first locomotive, and y_{ref} is the reference speed of the train. The augmented system may then be written as:

$$\dot{\underline{x}}_a = A_a \underline{x}_a + B_a U_a + \underline{d}_a \tag{10}$$

$$y = C_a \underline{x}_a$$

$$\underline{x}^T_a = [\ \delta \underline{x}^T\ z]$$

$$C_a = [C \mid 0],\ B_a = [\ {B \atop 0}\ ]$$

$$\underline{U}_a = \delta \underline{u}$$

$$A_a = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \text{ and } d\underline{a}^T = (\delta \underline{d}^T,\ 0)$$

The performance index is taken to be:

$$J = \int_0^\infty \left\{ [\ \underline{x}^T,\ z] \begin{bmatrix} Q_1 & 0 \\ 0 & q_2 \end{bmatrix} \begin{bmatrix} \delta \underline{x} \\ z \end{bmatrix} + 2\ \underline{U}_a^T\ E\ \ \underline{x}_a + \underline{U}_a^T\ R\ \underline{U}_a \right\} dt \tag{11}$$

Matrix Q_1 penalizes the deviation of the state variables from their nominal values. q_2 penalizes the integral of the velocity tracking error. Matrix R accounts for the control amplitude. Matrices Q_1 and R are chosen using an argument similar to that used in the ordinary LQR approach.[6] However, extensive simulation has shown that q_2 should carry considerably greater weight than Q_1 for the integral part of control to give the desired effect on train performance. The higher the weight of the integral part, the better the velocity tracking action; however, this is achieved at the cost of higher energy consumption and fast fluctuations in control inputs. Matrix E accounts for energy consumption. Since energy consumption is determined soley by the locomotives' throttle and velocity, the energy weighting can be expressed as:

$$\underline{U}_a^T E \underline{x}_a = \alpha_1 \delta u_1 v_{\ell 1} + \alpha_2 \delta u_2 v_{\ell 2}$$

where $v_{\ell 1}$ and $v_{\ell 2}$ are the velocities of the two groups of locomotives. If the costs of running the locomotives are the same, we may set:

$$\alpha_1 = \alpha_2 = \alpha, \alpha \geq 0$$

The scalar is bounded from above by the condition:[16]

$$Q_a - E^T R^{-1} E \geq 0 \qquad (12)$$

As the weight of energy increases, i.e. the closer α is to this bound, an undesirable slowdown in train dynamics occurs. Moreover, the Riccati equation, which solves this minimization problem, tends to be ill-conditioned and numerical difficulties are encountered. The machinery of the calculus of variation[17] is applied to minimize (11) subject to the dynamic constraint (10) as well as the following physical constraints:

$$\left.\begin{array}{ll} U_j = \delta u_j + U_{0j} \geq 0 , & j = 1, \dots 6 \\ U_i U_j = 0 & , i = 1,2 , j = 3, 4, 5, 6 \end{array}\right\} \qquad (13)$$

The last constraint implies that no simultaneous braking and throttling are allowed. The nonlinear relation between the traction force and velocity, and the quantized nature of the locomotives' power are not included at this stage. However, the target throttle/dynamic braking reflected from the feedback law is then adjusted to the nearest effective value, in accordance with Figure 2 and relations (4) and (5).

4. DERIVATION OF THE CONTROL LAW

It should be mentioned at this stage that for the control problem defined by equations (10) and (11) to have a solution, it is necessary that:[11]

I - (A,B) be controllable

$$\text{II} \quad - \quad \text{rank of} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = 2\,n$$

$$\text{III} \quad - \quad \begin{bmatrix} Q_a & E \\ E^T & R \end{bmatrix} \geq 0 \quad \text{and} \quad R > 0$$

Fortunately, the first two conditions are satisfied, and condition 3 can alway be satisfied by a proper choice of Q_a, E and R.

Let us now introduce a new variable, $\underline{\eta}$, such that:

$$u_j = \eta_j^2$$

The Hamiltonian function is given by:[17]

$$H = \underline{x}_a^T Q_a \underline{x}_a + 2\,\underline{U}_a^T E\,\underline{x}_a + \underline{U}_a^T R\,\underline{U}_a + 2\,\underline{\lambda}_1^T (A_a \underline{x}_a + B_a \underline{U}_a)$$

$$+ \sum_{i=1}^{2} \sum_{j=3}^{6} 2\,\beta_{ij}\, u_i\, u_j + \sum_{\ell=1}^{6} 2\,\gamma_\ell\,(u_\ell - \eta_\ell^2) \qquad (14)$$

where $\underline{\lambda}_1$, $\{\beta_{ij}\}$ and $\underline{\gamma}$ are Lagrange multipliers.

Now we must have:

$$\frac{\partial H}{\partial \underline{x}_a} - \frac{d}{dt}\,\frac{\partial H}{\partial \dot{\underline{x}}_a} = 0 \qquad (15)$$

$$\frac{\partial H}{\partial \underline{U}} = 0 \qquad (16)$$

and

$$\frac{\partial H}{\partial \underline{\eta}} = 0 \qquad (17)$$

Equations (15) through (17) become, respectively,

$$Q_a\,\underline{x}_a + E^T\,\underline{U}_a + A_a{}^T\,\underline{\lambda}_1 = -\,\dot{\underline{\lambda}}_1 \qquad (18)$$

$$E\,\underline{x}_a + R\,\underline{U}_a + B_a{}^T\,\underline{\lambda}_1 + \left[\begin{array}{c|c} 0 & \beta \\ \hline \beta^T & 0 \end{array}\right] \underline{U} + \underline{\gamma} = 0 \qquad (19)$$

$$\gamma_j\,\eta_i = 0 \qquad (20)$$

where β is a 2 x 4 matrix whose elements are the Lagrange multipliers $\{\beta_{ij}\}$.

Equation 20 implies that $\gamma_j = 0$ if $U_j > 0$, and γ_j is arbitrary if U_j fails to take positive values.

To simplify the equations, we split the vector, $\underline{U}$, as

$$\underline{U} = \left[\begin{array}{c} \underline{U}^t \\ \hline \underline{U}^b \end{array}\right]$$

such that the vector, $\underline{U}^t$ contains the throttle inputs and $\underline{U}^b$ contains the dynamic and electropneumatic braking inputs. Without loss of generality, we shall assume R to be a diagonal matrix. We may then normalize $\underline{\gamma}$ to $R\underline{\gamma_1}$, (by setting $\underline{\gamma} = R\underline{\gamma_1}$), then (19) can be written as:

$$-\left(E\,\underline{x}_a + B_a^{T}\,\underline{\lambda}_1\right) = R\,\delta\underline{U}_a + \left[\begin{array}{c|c} 0 & \beta \\ \hline \beta^{T} & 0 \end{array}\right] \left[\begin{array}{c} \underline{U}^{t} \\ \hline \underline{U}^{b} \end{array}\right] + R\,\underline{\gamma}_1 \quad (21)$$

If we set $R = \left[\begin{array}{c|c} R_1 & 0 \\ \hline 0 & R_2 \end{array}\right]$, we can then split (21) as follows:

$$-\vartheta_1\,(\underline{x}_u,\,\underline{\lambda}_1) = \delta\underline{U}_a^{\,t} + R_1^{-1}\,\beta\,\underline{U}^{b} + \underline{\gamma}_1^{\,t} \quad (22)$$

$$-\vartheta_2\,(\underline{x}_a,\,\underline{\lambda}_1) = \delta\underline{U}_a^{\,b} + R_2^{-1}\,\beta^{T}\,\underline{U}^{t} + \underline{\gamma}_1^{\,b} \quad (23)$$

where: $R^{-1}\left(E\,\underline{x}_a + \beta_a^{T}\,\underline{\lambda}_1\right) = \begin{bmatrix} \vartheta_1\,(\underline{x}_a,\,\underline{\lambda}_1) \\ \vartheta_2\,(\underline{x}_a,\,\underline{\lambda}_1) \end{bmatrix}$

$$\delta\underline{U}_a = \begin{bmatrix} \delta\underline{U}_a^{\,t} \\ \delta\underline{U}_a^{\,b} \end{bmatrix}$$

$$\gamma_1 = \begin{bmatrix} \underline{\gamma}_1^{\,t} \\ \underline{\gamma}_1^{\,b} \end{bmatrix}$$

Clearly, if $\underline{U}^{t} > 0$, then by condition (13), $\underline{U}^{b}$ must be zero; also from (20) $\underline{\gamma}_1^{\,t}$ must be zero. Hence, (22) reduces to:

$$\delta\underline{U}_a^{\,t} = -\underline{\vartheta}_1\,(\underline{x}_a,\,\underline{\lambda}_1) \quad (24)$$

Similarly, if $\underline{U}^{b} > 0$, then by condition (13), $\underline{U}^{t}$ must be zero, and from (20) $\underline{\gamma}_1^{\,b} = 0$. Thus, equation (23) becomes:

$$\underline{\delta}\,\underline{U}_a^{\,b} = -\,\underline{\vartheta}_2\,(\underline{x}_a, \underline{\lambda}_1) \tag{25}$$

From (24) and (25), we come up with the following decision rule. First we find the vectors $\underline{\vartheta}_1$ and $\underline{\vartheta}_2$ from the relation:

$$\begin{bmatrix} \underline{\vartheta}_1 \\ \underline{\vartheta}_2 \end{bmatrix} = -\,R^{-1}\,[E\,\underline{x}_a + B_a^{\,T}\,\underline{\lambda}_1] \tag{26}$$

Then we set:

$$\underline{U}_a = \begin{bmatrix} -\,\underline{\vartheta}_1 \\ 0 \end{bmatrix} \text{ if } \underline{U}^t \geq 0$$

or

$$\underline{U}^b = \begin{bmatrix} 0 \\ -\,\underline{\vartheta}_2 \end{bmatrix} \text{ if } \underline{U}^b \geq 0$$

To find the feedback law, let $\underline{\lambda}_1 = P\,\underline{x}_a$ and substitute:

$$\underline{U}_a = -\,R^{-1}\,[E + B_a^{\,T}\,P]\,\underline{x}_a \tag{27}$$

into equation (18). This gives:

$$[Q_a - [E^T + P\,B_a]\,R^{-1}\,[E + B_a^{\,T}\,P] + A_a^{\,T}\,P + P\,A_a]\,\underline{x}_a = 0$$

For the above equation to hold for all $\underline{x}_a$, P must satisfy the following algebraic Riccati equation (A.R.E.):

$$A_a^{\,T}\,P + P\,A_a - [E^T + P\,B_a]\,R^{-1}\,[E + B_a^{\,T}\,P] + Q_a = 0 \tag{28a}$$

or

$$(A_a - B_a\,R^{-1}\,E)^T\,P + P\,(A_a - B_a\,R^{-1}\,E) - P\,B_a\,R^{-1}\,B_a^{\,T}\,P$$

$$+\,(Q_a - E^T\,R^{-1}\,E) = 0 \tag{28b}$$

Clearly, we should have $Q - E^T R^{-1} E \geq 0$ for (28) to yield a positive solution, P, as mentioned earlier.

We can now summarize the solution procedure of this control problem as follows (also see Figure 3):

Step 1:

Find the positive matrix solution, P, of the A.R.E. (28b).

Step 2:

Compute the vector:

$$\underline{\upsilon} = - R^{-1} [E + B_a^T P] \underline{x}_a$$

$$= - K_1 \delta\underline{x} - K_2 \int_0^t (v_{\ell 1} - v_{ref})\, dt$$

Step 3:

Compute the target control input according to the following nonlinear decision rule:

I - If any control inupt, U_j, $j=1,2,\ldots,6$, takes a negative value, it is set to zero.

II - If the throttle has been in effect, set the braking forces to zero, i.e. the control vector will have the form:

$$\underline{U}_a^T = [\upsilon_1, \upsilon_2, 0, 0, 0, 0]$$

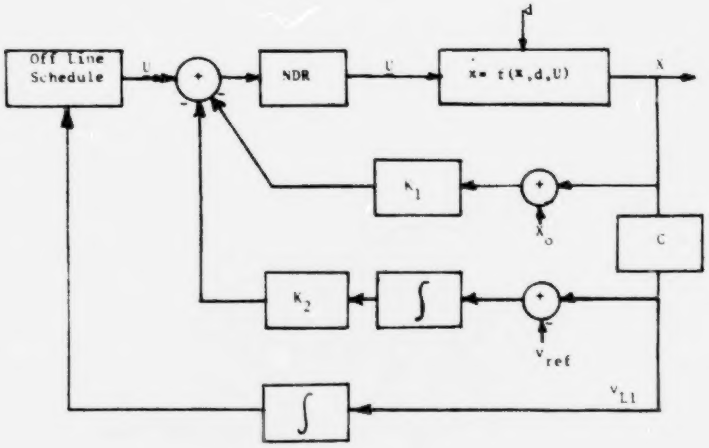
This should be in effect until U_1 and U_2 become zero. If brake application is then requested, i.e. one or more of $(U_{0j} + U_j)$, $j = 3,4,5,6$ becomes positive, set the thrust to zero and the braking forces should be in effect. Hence, the control vector will have the form:

$$\underline{U}_a^T = [0, 0, U_3, U_4, U_5, U_6]$$

The braking forces remain in effect until at least one U_j, $j=3,4,5,6$ becomes zero, and throttle is requested.

Step 4:

Adjust the throttle/dynamic brake to the nearest permissible level according to the nonlinear relation depicted in Figure 2, and relations (4) and (5).



$\underline{U}_o$: scheduled control inputs
NDR : nonlinear decision rule
$\underline{U}$: feedback control inputs
d : disturbance
K_1 : proportional feedback gain matrix
$\underline{x}_o$: nominal value of the states
C : output matrix
K_2 : integral feedback gain matrix
v_{ref} : reference speed
VL1 : velocity of front locomotive

Figure 3 The Closed-Loop Control System

During the course of tuning the cost function to obtain an appropriate performance, the following physical constraints must be taken into consideration:

- coupler forces must be kept below the breakdown level of 114,000 kg
- relative velocity between cars must not exceed the dangerous impact level of 1 metre per second
- velocity regulation within five per cent is required
- abrupt fluctuations in braking or between throttle and dynamic braking are not allowed
- throttle/dynamic braking should be increased incrementally from a throttle/dynamic braking level to the next one, and should stay at any level not less than 2.5 seconds.

The controller has been implemented for a test case of thirty loaded gondola cars and two U25 locomotives. The first locomotive is placed at the front, the second is located after the first twenty cars. The train data are given in Table 1. The number of states of the augmented system in this case is 64. Sixty-three of them are contributed by the train model. The extra state comes from the PI controller. The process of tuning the cost function is computationally very expensive, since every parameter change requires solving the A.R.E. and possibly some simulation runs. The cost function for the best performance we could achieve with our limited resources is as follows:

- Q_1 is a unit matrix
- $q_2 = 800$
- $\alpha = 1$
- $R = \text{diag.}\ (2\text{x}10^{-4},\ 2\text{x}10^{-4},\ 1.2\text{x}10^{-3},\ 1.2\text{x}10^{-3},\ 1\text{x}10^{-2},\ 1\text{x}10^{-2})$

The most sensitive parameter of the cost function is matrix R. The values shown above give the best balance and coordination of the throttle, dynamic brake, and electropneumatic brake. q_2 is weighted considerably greater than the train states for the integral part of the control to give the desired performance. Values of q_2 higher than 100 give adequate performance. However, when q_2 took values between 100 and 800, there was little change. The rate of the integral part can be adjusted more effectively by multiplying the velocity error $(v_{\ell 1} - v_{ref})$ by some scalar value, α_I where $0.5 \leq \alpha_I \leq 2$.

TABLE 1

DATA OF THE TWO-LOCOMOTIVE, THIRTY-CAR TRAIN

	Locomotives	Cars
Type	U25	gondola
Weight	139 tons	102 tons
Length	18.3 m	19.7 m
Maximum Braking Force	4×10^5 N	1.47×10^5 N
Maximum Power	2,000 hp	--
Locomotive Transition Velocities	8, 13, 26 km/h	--
Coupler Spring Constant	70×10^5 N/m	70×10^5 N/m
Coupler Damping Constant	2.04×10^5 N.sec/m	2.04×10^5 N.sec/m

5. THE OFFLINE SCHEDULE

The known data of the train and terrain sections are used in an offline schedule to provide steady-state braking and throttle on each terrain zone. The offline schedule can be chosen to minimize travel time subject to velocity limits and other route constraints, such as distance, curvature, grades, and available traction power, using a simple model for the train. A technique is also developed to superimpose the scheduled braking and throttle on the feedback controller operation to ensure better regulation of train dynamics. The proposed technique is a modified version of the WGB method prepared by Wong, Gruber and Bayoumi in [8]. The basic differences may be stated as follows:

(1) In [8] the throttle or brakes are applied on each train section separately when a section of the train crosses the boundary of a terrain zone. However, this technique may result in large, abrupt swings in throttle or braking when the train enters a steep uphill or downhill zone. In our proposed technique, the throttle as well as the brakes are applied on the whole train gradually in proportion to the length of the train that has passed the boundary of the new terrain zone.

(2) In [8], to perform a speed reduction (or speed increase), the brakes are applied only on sections of the train which have crossed the speed limit zone. Such method of brake application may not be practical because it could cause serious impacts between cars and excessive compression of the train. In the proposed method, the brakes are applied all over the train simultaneously and increased gradually to the required level. According to this method, if the train is going downhill, the brakes are applied when the locomotive is at a distance, s, before the new speed zone, where s is given by $l = (50 + 12 v_{ref})$ metres (v_{ref} being the current reference speed in metres/second).

In the next section we shall provide simulation results which demonstrate the performance of both schemes under various operating conditions. The WGB technique is applied here at four increments, as follows:

- when the front end of the train crosses the boundary of a terrain zone, half of the full service is applied on the first section of the train
- when the centre of mass of the first section passes the terrain zone, full service is applied on the first section
- half of the full service is applied on the second train section upon arrival at the boundary of the new zone
- full service is applied on the second train section when its centre of mass crosses the boundary of the new zone.

6. SIMULATION RESULTS

6.1 Speed Reduction Tests

These simulations, were performed to test the ability of the controller to track rapidly decelerating reference speeds on level ground. The performance criterion here is the velocity error, and the coordination of the throttle, dynamic brakes and electropneumatic brakes. Figures 4(a) through 4(d) illustrate the performance of the PI controller without any offline scheme. Table 2 shows the test conditions.

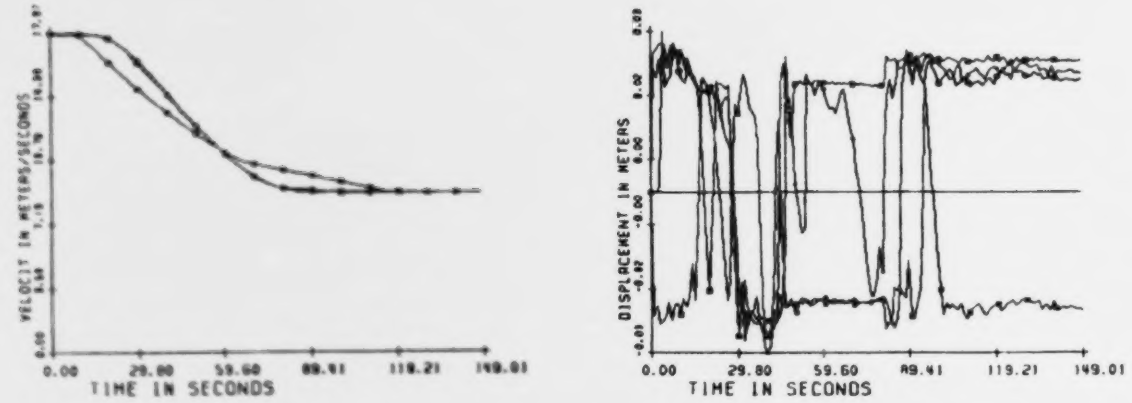
TABLE 2

TEST CONDITIONS FOR SPEED REDUCTION
USING PI CONTROLLER WITHOUT OFFLINE SCHEDULE

	Terrain Zone			
	1	2	3	4
Length of Zone (m)	885	724	241	5,630
Grade (per cent)	0.0	0.0	0.0	0.0
Target Speed at the Start of Zone (km/h)	64.34	64.34	38.6	32.2
Resistance and Acceleration Force (N)	7.71×10^4	-4.83×10^5	-1.81×10^5	5.42×10^4
Gravitational Force (N)	0.0	0.0	0.0	0.0

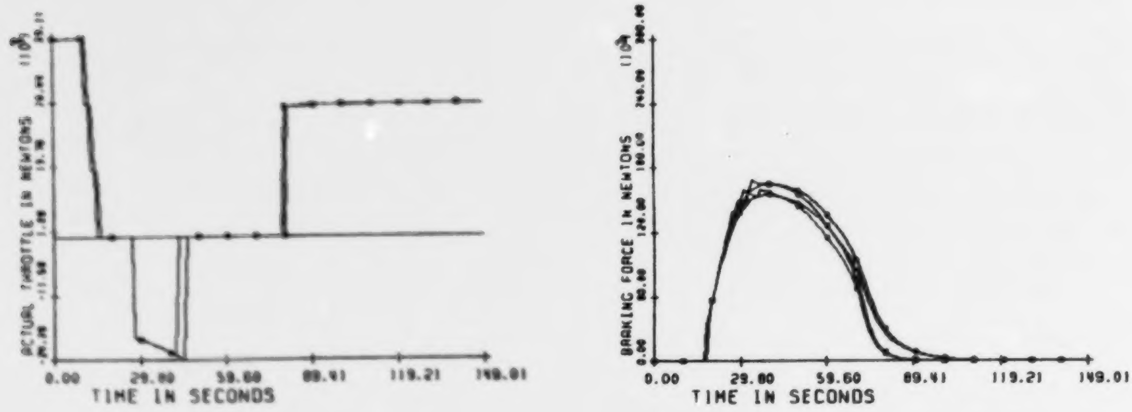
Although the controller provides zero steady-state velocity error, the five per cent objective velocity regulation has not been met. Apart from this drawback, the controller shows excellent coordination of throttle and brakes. The control law is modified by the preposed offline schedule and by the WGB technique. The simulations for both cases are given in Figures 5(a) through 5(d) and 6(a) through 6(d), respectively. The test conditions here, shown in Table 3, are basically the same as those in Table 1 except that the original speed has been increased to about 72 km/h (45 mph).

Figure	Description	Graphic Symbol □	○	✱	⌗	⧖
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity

(b) Intercar Displacement

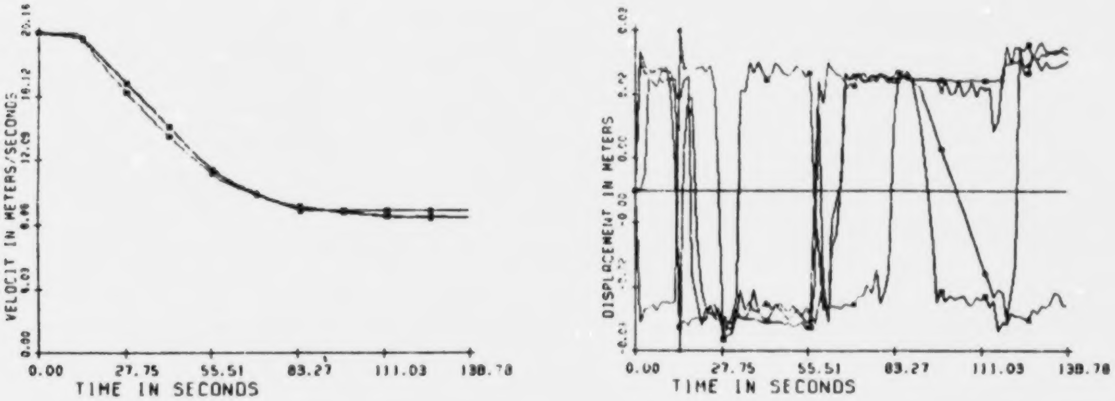


(c) Throttle/Dynamic Braking

(d) Brake Force

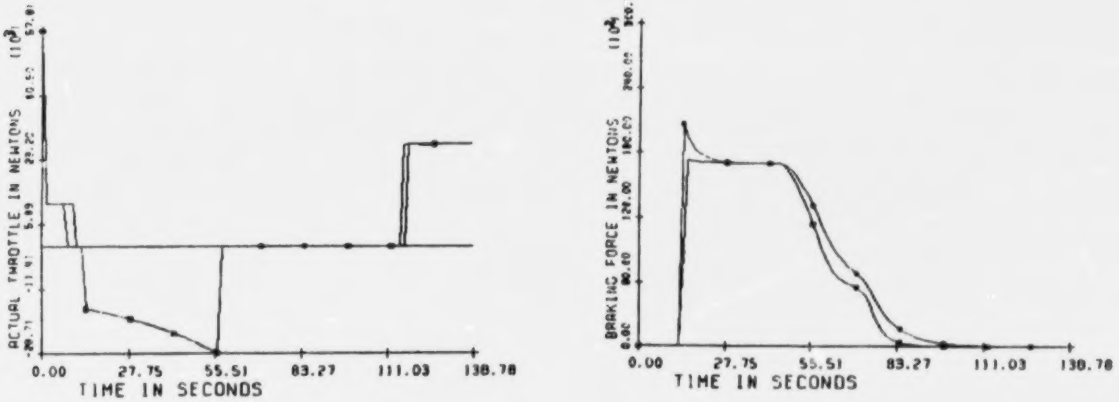
Figure 4 Speed Reduction Test Using the PI Controller Only, Without Offline Schedule

Figure	Description	Graphic Symbol □	○	✱	♯	⧗
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity

(b) Intercar Displacement

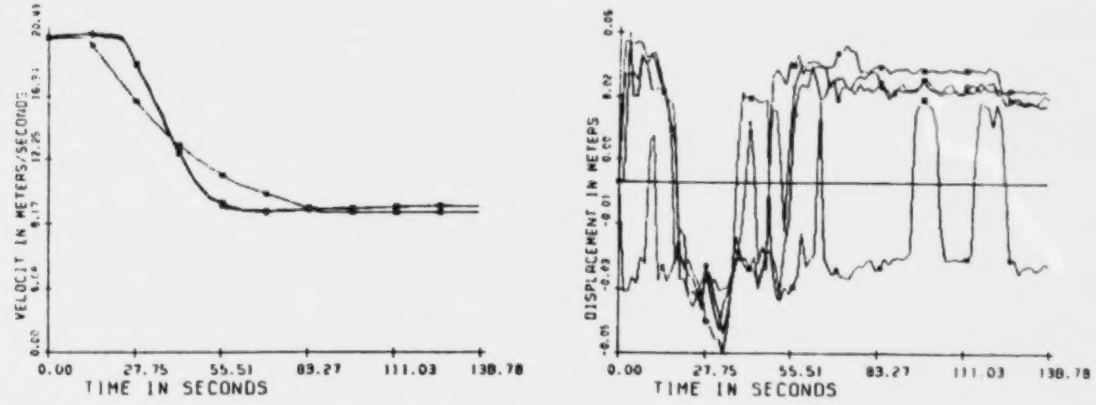


(c) Throttle/Dynamic Braking

(d) Brake Force

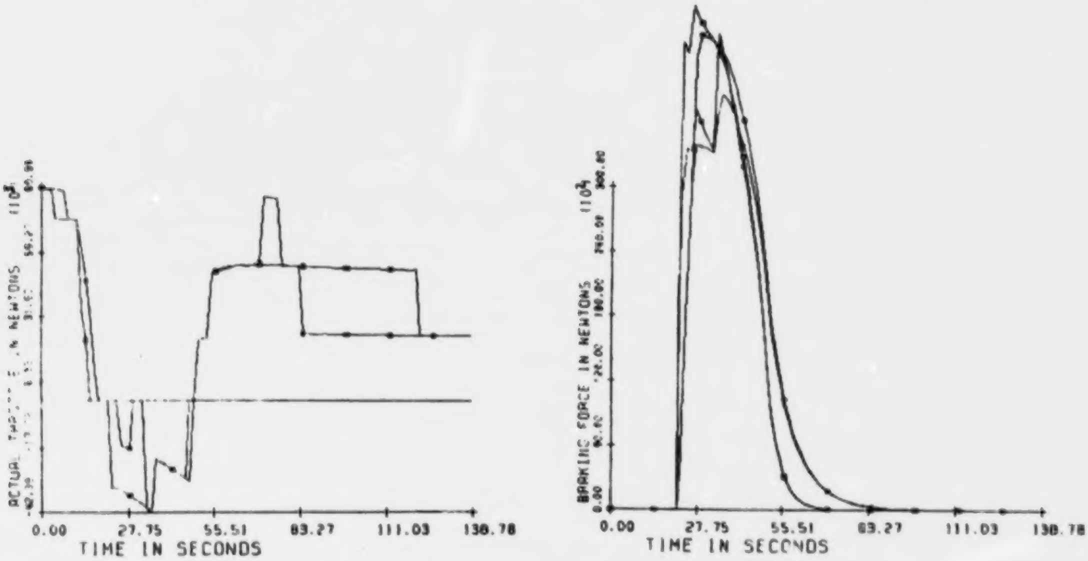
Figure 5 Speed Reduction Test Using a PI Controller and New Offline Schedule

Figure	Description	Graphic Symbol □	○	*	#	⧖
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity

(b) Intercar Displacement



(c) Throttle/Dynamic Braking

(d) Brake Force

Figure 6 Speed Reduction Test Using a PI Controller and the WGB Schedule

TABLE 3

TEST CONDITIONS FOR SPEED REDUCTION USING A PI CONTROLLER AND OFFLINE SCHEDULE

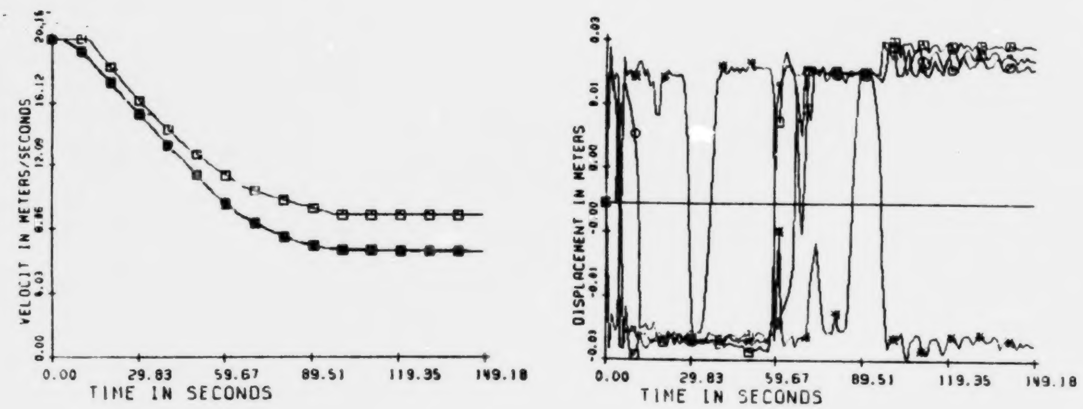
	Terrain Zone			
	1	2	3	4
Length of Zone (m)	885	724	241	5,630
Grade (per cent)	0.0	0.0	0.0	0.0
Target Speed at the Start of Section (km/h)	72.4	72.4	38.62	32.18
Resistance and Accelerating Force (N)	$8.39\text{x}10^4$	$-5.825\text{x}10^5$	$-1.82\text{x}10^5$	$5.42\text{x}10^4$
Gravitational Force (N)	0.0	0.0	0.0	0.0

It should be noted that the WGB scheme fails to meet the five per cent velocity regulation. The test also demonstrates the general performance of the two offline schemes. In particular, it is clear that the proposed offline schedule has less coupler forces as well as smoother brake application.

The small velocity error at the end of this simulation test should eventually disappear. However, as the feedback action is proportional to the integral of the velocity error, smaller velocity errors may need longer time to be completely eliminated.

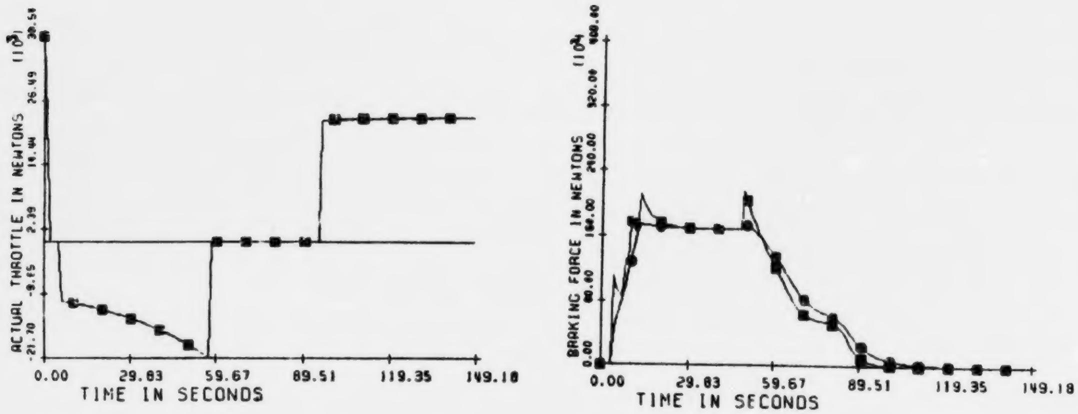
Finally, Figures 7(a) and 7(d) illustrate the operation of the train in an open-loop manner under the new offline schedule alone, i.e. the feedback controller was disconnected. The simulation shows the significant deterioration of train performance without the PI controller.

Figure	Description	Graphic Symbol □	○	✱	#	⧖
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity

(b) Intercar Displacement



(c) Throttle/Dynamic Braking

(d) Brake Force

Figure 7 Speed Reduction Test Using the New Offline Schedule Only

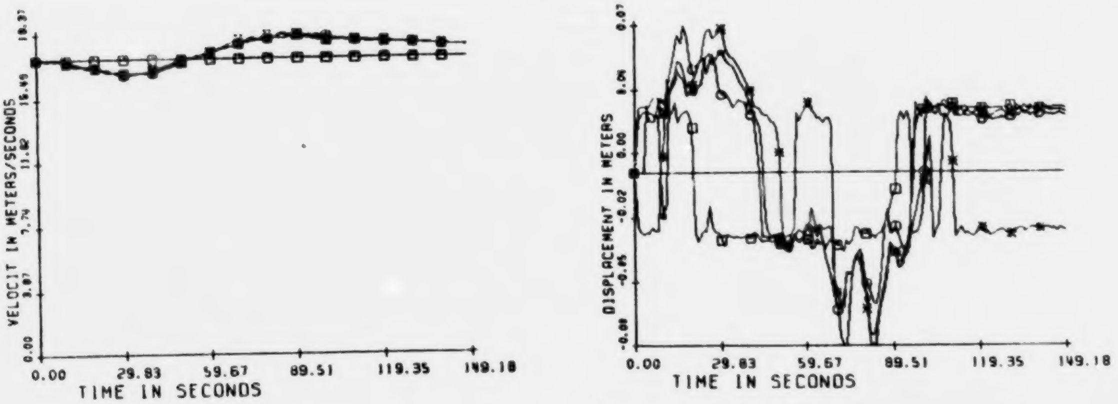
6.2 Steep Downhill Zone Test

In this section, we provide two illustrative examples of train handling techniques over a severe downhill zone. In the first example, the train runs under manual operation in accordance with AAR recommendations[18] to avoid impacts between cars on entering the downhill zone. The air brakes are applied upon arrival at the downhill zone, and the throttle is decreased to idle. The dynamic brake plays a minor role here. It is used only to assist the air brake. However, this technique has two serious drawbacks for long, steep, downhill zones. In the firt place, it is a costly method. The application of brakes for long durations requires frequent replacement of the brake shoes. Second, as the temperature of the brake shoes increases, the effectiveness of the brakes decreases, and hence the speed of the train starts to drift, causing more heating and further deterioration of the brakes. In the meantime, the use of dynamic braking on steep descents requires high skill to estimate the proper amount of dynamic brakes and the proper rate of application to avoid impacts between cars and possible derailment of the train. The multivariable PI controller supported by the new offline schedule is tested on this case. The control law is designed in such a way that the role of the dynamic brake is emphasized. The inherent features of this type of controller provide the proper corrective action in case of any decrease in the effectiveness of one of the braking methods and eliminates the drift in train speed. The simulation tests for both cases are shown in Figures 8(a) through 8(d) and 9(a) through 9(d), respectively. The test conditions are given in Table 4.

6.3 Uphill Tests

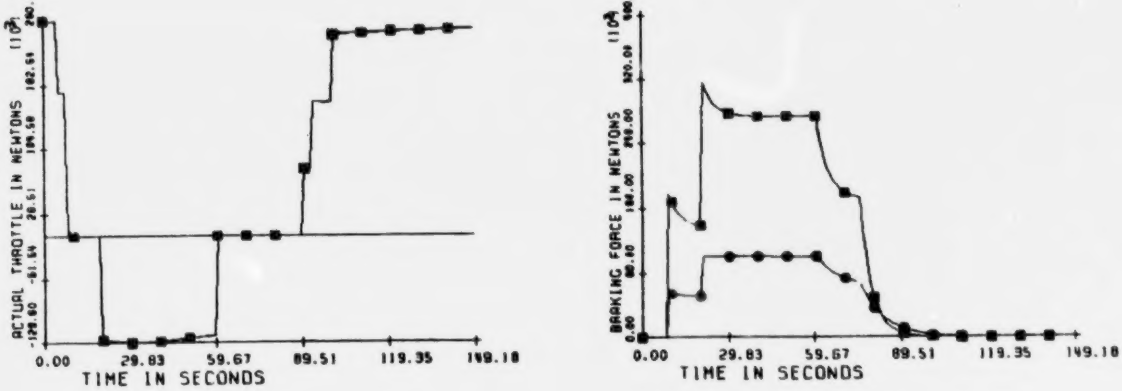
In this simulation, the performance of the controller has been investigated on a long, steep, uphill zone. The grade of this zone is taken to be two per cent, which represents the severest ascending zone on Canadian railways. The train configuration of thirty loaded cars and two small locomotives may not have enough power to climb this uphill grade at the reference speed. This situation has been studied in Figures 10, 11 and 12 with the feedback controller only, with the controller plus the WGB offline scheme, and with the feedback controller and the new offline scheme, respectively. The little velocity undershoot in Figure 10(a) is due to the fact that the feedback

Figure	Description	Graphic Symbol □	○	✱	#	⧖
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity

(b) Intercar Displacement

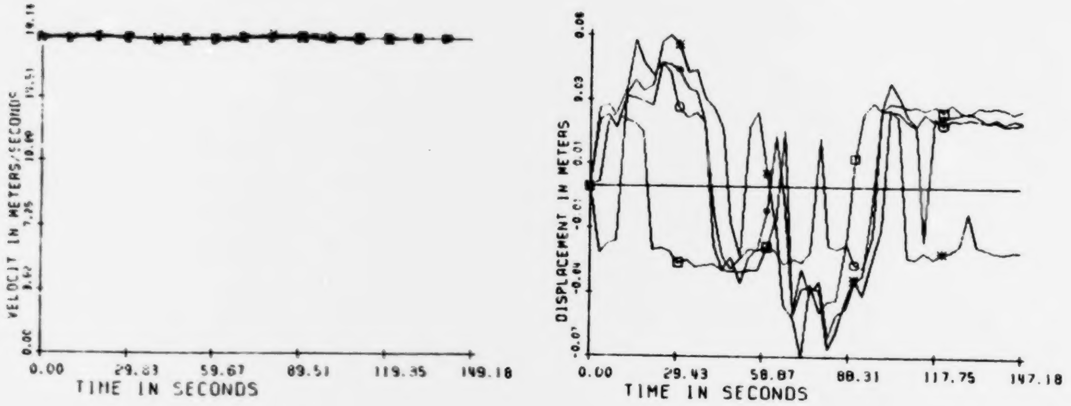


(c) Throttle/Dynamic Braking

(d) Brake Force

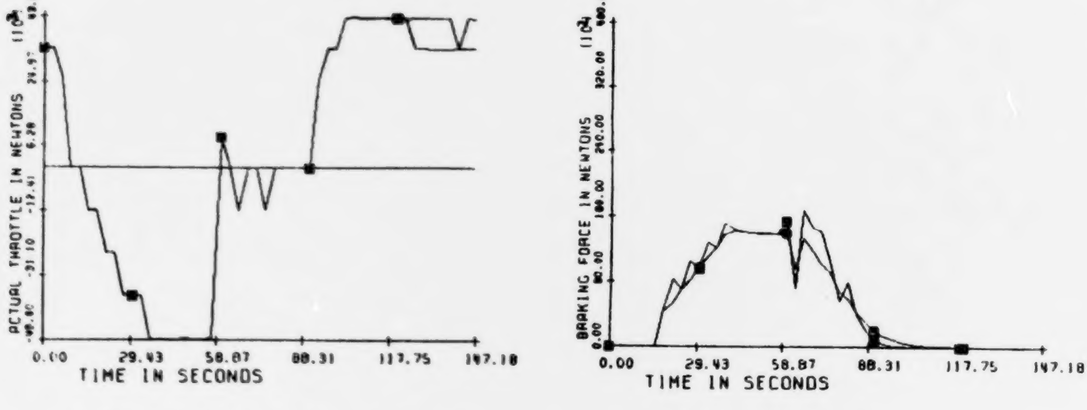
Figure 8 Downhill Test Using Manual Operation

Figure	Description	Graphic Symbol				
		□	○	*	#	⧖
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity

(b) Intercar Displacement



(c) Throttle/Dynamic Braking

(d) Brake Force

Figure 9 Downhill Test Using a PI Controller and the New Offline Schedule

TABLE 4

CONDITIONS FOR THE DOWNHILL TEST

	Terrain Section		
	1	2	3
Length of Terrain Section (m)	724	965	5,632
Grade (per cent slope)	0.0	-2.0	0.0
Target Velocity (km/h)	64.36	64.36	64.36
Total Rolling and Air Resistance (N)	$7.72\text{x}10^4$	$7.72\text{x}10^4$	$7.72\text{x}10^4$
Total Gravitational Forces (N)	0.0	$-6.4\text{x}10^5$	0.0

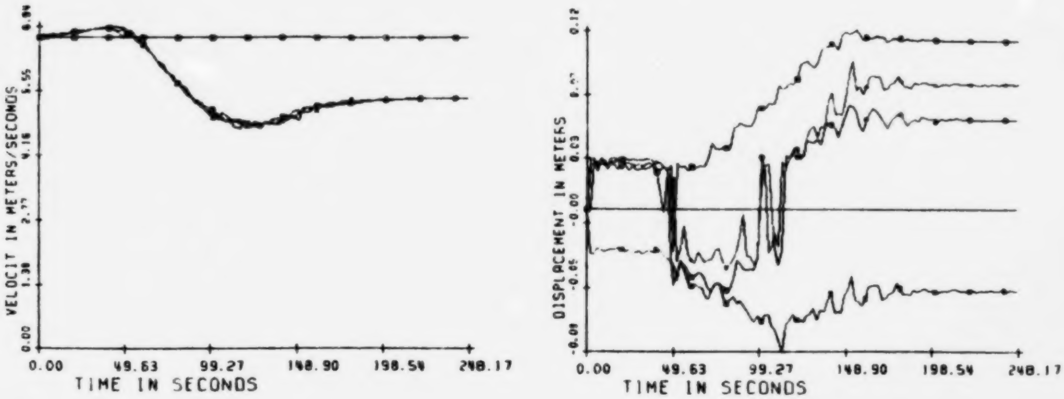
controller alone has no ability to predict the coming uphill zone. Corrective actions start only after a velocity error has developed and is integrated. In contrast to this situation, when the controller is supported by the WGB offline scheme, Figure 11, a speed increase occurs before the uphill grade. This feature may be desirable for short uphill sections to compensate for the speed loss over this zone.

In Figure 12(a) no velocity overshoot or undershoot occurs because the new offline schedule increases the traction force in propoortion to the rate of increase of the gravitational force until the maximum power is reached. The speed of the train then decreases until the traction forces balance the sum of the gravitational force and the aerodynamic rolling resistance.

A new train configuration has been examined consisting of thirty loaded gondola cars plus six U25 locomotives. Three locomotives were located at the front and three after the first twenty cars. This configuration has about forty per cent more traction power than the average power needed to drive the train at the given target speed over this uphill grade. The simulation results are shown in Figures 13, 14 and 15.

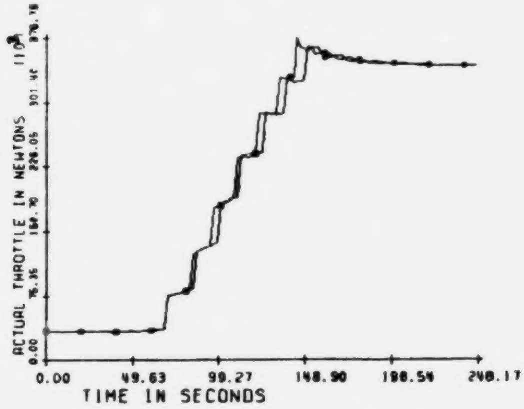
The test conditions for the two configurations are given in Tables 5 and 6, respectively.

Figure	Description	Graphic Symbol □	 ○	 ✱	 #	 ⧖
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity

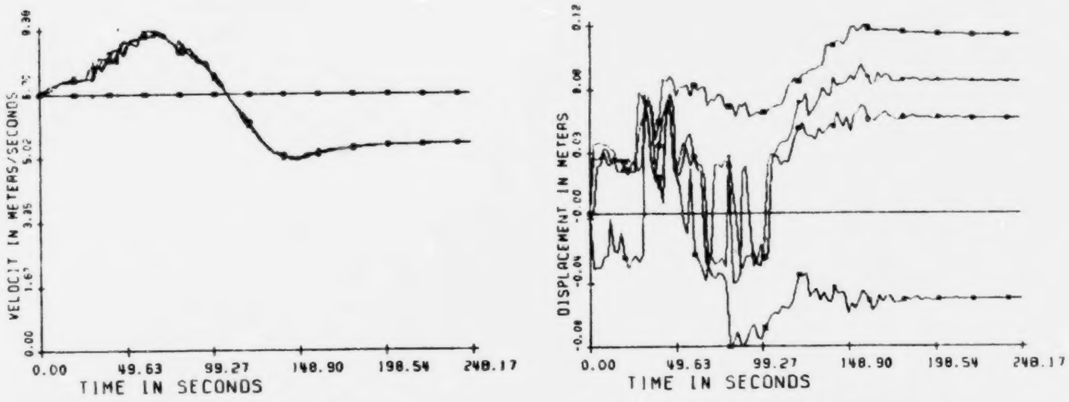
(b) Intercar Displacement



(c) Throttle/Dynamic Braking

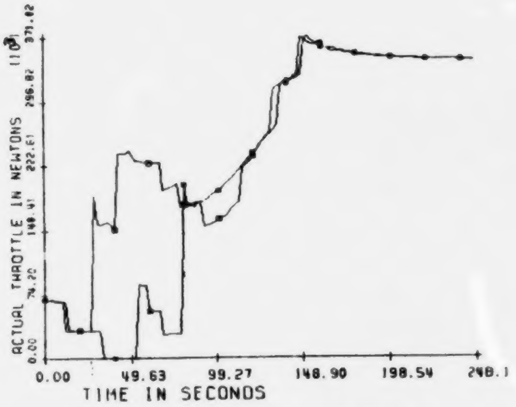
Figure 10 Uphill Test of an Underpowered Train Using a PI Controller Without Offline Schedule

Figure	Description	Graphic Symbol ☐	 ○	 ✱	 #	 ⧖
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity

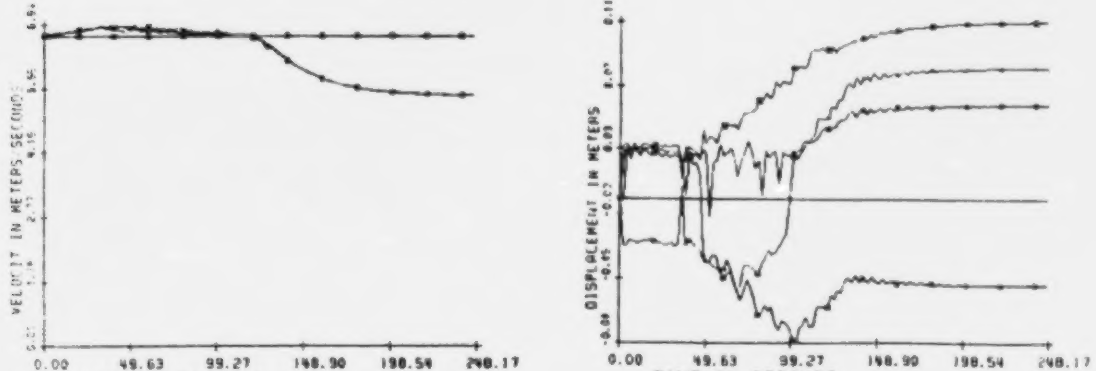
(b) Intercar Displacement



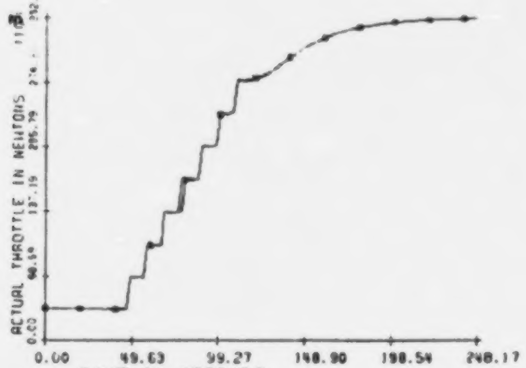
(c) Throttle/Dynamic Braking

Figure 11 Uphill Test of an Underpowered Train Using a PI Controller and the WGB Schedule

Figure	Description	Graphic Symbol □	○	✱	#	⧖
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity (b) Intercar Displacement



(c) Throttle/Dynamic Braking

Figure 12 Uphill Test of an Underpowered Train Using a PI Controller and the fnw Offline Schedule

TABLE 5

TEST CONDITIONS FOR FIGURES 10, 11 AND 12

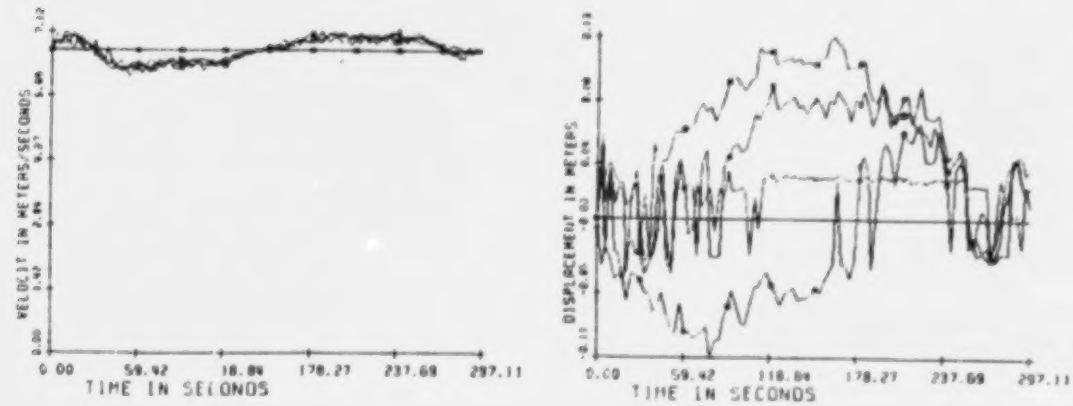
	Terrain Zone			
	1	2	3	4
Length of Zone (m)	724	8,280	--	--
Grade (per cent)	0.0	2.0	--	--
Target Speed at Start of Section (km/h)	24.14	24.14	24.14	--
Resistance and Acceleration Force (N)	4.94×10^4	4.94×10^4	--	--
Gravitational Force (N)	0.0	6.4×10^5	--	--

TABLE 6

TEST CONDITIONS FOR FIGURES 13, 14 AND 15

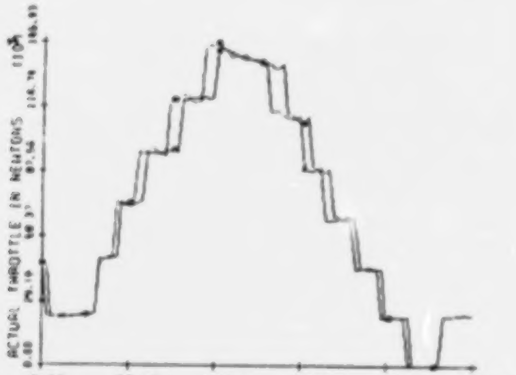
	Terrain Zone			
	1	2	3	4
Length of Zone (m)	724	965	8,280	--
Grade (per cent)	0.0	2.0	0.0	--
Target Speed at Start of Section (km/h)	24.14	24.14	24.14	24.14
Resistance and Acceleration Force (N)	5.6×10^4	5.6×10^4	5.6×10^4	--
Gravitational Force (N)	0.0	7.47×10^5	0.0	--

Figure	Description	Graphic Symbol □	○	✱	#	⧖
(a)	Velocity	v_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity

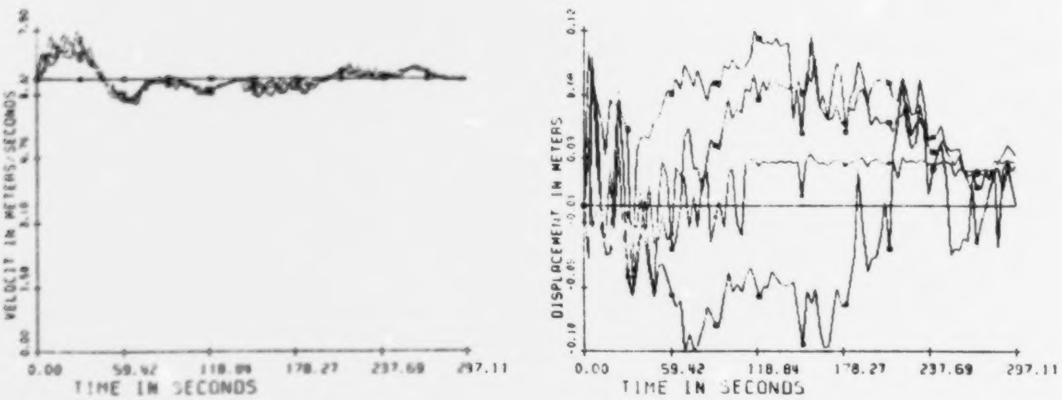
(b) Intercar Displacement



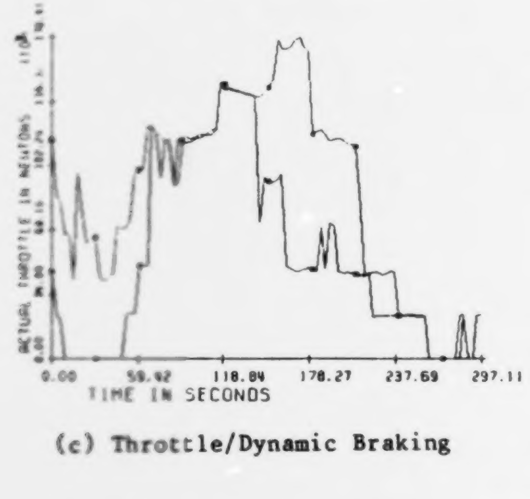
(c) Throttle/Dynamic Braking

Figure 13 Uphill Test of a Thirty-Car, Six-Locomotive Train Using a PI Controller Without Offline Schedule

Figure	Description	Graphic Symbol				
		□	○	✳	#	⧖
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



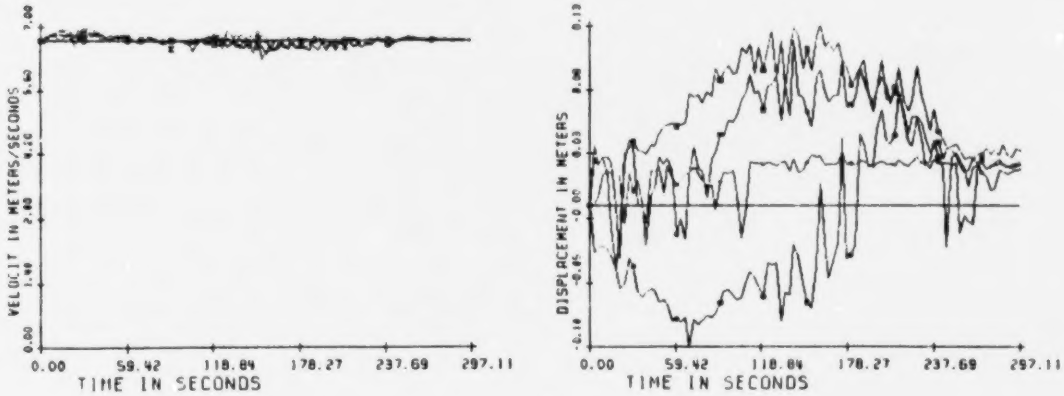
(a) Velocity (b) Intercar Displacement



(c) Throttle/Dynamic Braking

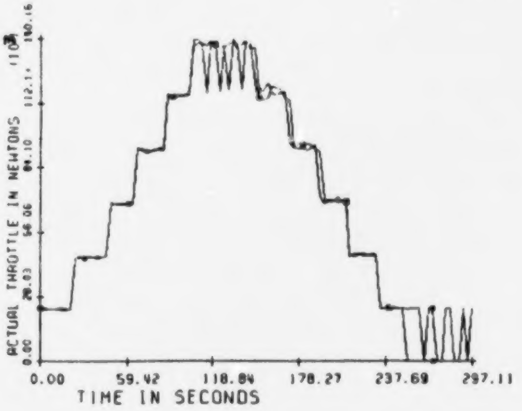
Figure 14 Uphill Test of a Thirty-Car, Six-Locomotive Train Using a PI Controller and the WGB Schedule

Figure	Description	Graphic Symbol □	 ○	 ✱	 #	 ⧖
(a)	Velocity	V_{ref}	V1	V10	V22	V32
(b)	Coupler displacement between train members	1,2	10,11	21,22	22,23	--
(c)	Trottle	1	1	22	--	--
(d)	Brake force	1	10	22	32	--



(a) Velocity

(b) Intercar Displacement



(c) Throttle/Dynamic Braking

Figure 15 Uphill Test of a Thirty-Car, Six-Locomotive Train Using a PI Controller and the New Offline Schedule

More tests have been performed using a thirty-car, two-locomotive configuration, and using hypothetical power rating for the two locomotives. The study shows that the controller and the new offline schedule require about 25 per cent more power than the average required to drive the train steady at the target speed up the hill. This extra power capacity is essential to perform the transient throttle manipulations at the boundary of the uphill zone.

7. CONCLUSION

A multivariable PI controller is designed to achieve regulation of train dynamics and asymptotic tracking of a given target speed profile in the presence of disturbances and uncertainties in train parameters. Extensive simulation tests under various conditions illustrate the potential advantages of this type of controller. The proposed controller can easily be modified to achieve tracking of a target velocity and of a target acceleration profile simultaneously. Further studies are needed to simplify the controller structure and to minimize the number of measurements using the recent model reduction techniques.

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ANNEX A

PROGRAM LISTING

```
FILE: LINMDL   FORTRAN  A1                 VM/SP CONVERSATIONAL MONITOR SYSTEM

C*****************************************************************C
C     PROGRAM LINMDL
C    FUNCTION      COMPUTES THE MATRICES A AND B OF THE LINEAR MODEL
C                  DXDT=A*X + B*U
C    PARAMETERS
C    MM    - NUMBER OF TRAIN MEMBERS . MAXIMUM 103
C    M     - NUMBER OF STATES =MM*2-1
C    NS    - NUMBER OF TRAIN SECTIONS FOR BRAKE APPLICATION.
C    NSC   - VECTOR OF LENGTH NS CONTAINING THE NUMBER OF VEHICLES IN
C            EACH SECTION .
C    MKS   - MKS=1 (MKS=0)IF DATA IN THE MKS SYSTEM( ENGLISH SYSTEM).
C    W     - VECTOR OF LENGTH MM OF THE VEHICLES WEIGHT IN TONS .
C    NLOC  - NUMBER OF THE LOCOMOTIVES .
C    ML1   - VECTOR OF THE LOCOMOTIVES POSITION .
C
C    SUBROUTINE NEEDED    RLL
C    INPUT      FILE TRN DATA
C    OUTPUT     FILE TRN1 MODEL
C
C    WRITTEN BY MOUSTAFA ELSHAFEI AHMED
C    DATE  18-04-1982
C ****************************************************************C
      DIMENSION A(206,206),B(206,10),W(103),SPC(103),SPD(103),R(103),
     +          NSC(10),ML1(10),ML2(10)
C
C
      INTEGER  MM,ML,MKS,NI,NS,ML2,M
C
C     READ TRAIN DATA FROM FILE TRN DATA
C
 5    FORMAT(6I5)
10    FORMAT(6F12.4)
      READ(5,5) MM,NLOC,NS,MKS
      READ(5,5) (ML1(I),I=1,NLOC)
      READ(5,10)(W(I),I=1,MM)
      READ(5,10) SPRGL,SPRGC,CLOC,CCAR
      READ(5,5)(NSC(I),I=1,NS)
      READ(5,5)NAXC,NAXL
C
C
      NI= NS+NLOC
      M=2*MM
      DO 2  I=1,NLOC
 2    ML2(I)=ML1(I)*2
      MMM1=MM-1
      M1=M-1
      IF(MKS.EQ.1) GO TO 30
C
C     CONVERT TO THE MKS SYSTEM
C
      DO 20 I=1,MM
      W(I)=W(I)*0.9072
 20   CONTINUE
         SP=         0.4536*3.2808*9.81
      SPRGL=SPRGL*SP
      SPRGC=SPRGC*SP
      CCAR=CCAR*SP
      CLOC=CLOC*SP
30    AC= 90.*0.0929
      AL=  120.*.0929
C
C
      DO 40 I=1,MM
      W(I)=W(I)*1000.
      SPC(I)=SPRGC
      SPD(I)=CCAR
      DO 40 J=1,M1
40    A(I,J)=0.0
C
      DO 44 I=1,NLOC
      I1=ML1(I)
      SPD(I1)=CLOC
      SPC(I1)=SPRGL
44    CONTINUE
C
      WRITE(6,110)
110   FORMAT(/,20X,'FREIGHT TRAIN LINEARIZED MODEL '/)
      MM3=MM-NLOC
      WRITE(6,115)NLOC,MM3
115   FORMAT(10X,'THIS IS A LINEAR MODEL OF',I3,' LOCOMOTIVES AND ',
     +    I5,' CARS')
      WRITE(6,120)(W(ML1(I)),I=1,NLOC)
120   FORMAT(6X,' THE LOCOMOTIVES WEIGHTS IN KGM :  '/,3(10X,F8.1))
      WRITE(6,130)SPRGL,SPRGC,CLOC,CCAR
130   FORMAT('  THE COUPLER PARAMETERS IN MKS UNITS : '/,' KL= ',E12.3,
     +  ', KC= ',E12.3,/,  '  CL= ',E12.3,' ,  CC= ',E12.3)
      WW  =0.0
      DO 140 I=1,MM
      WW  =WW  +W(I)
140   CONTINUE
      DO 145 I=1,NLOC
145   WW=WW-W(ML1(I))
C
C
```

```
         WW   =WW/MM3
        WRITE(6,150) WW
 150    FORMAT(6X,' THE AVERAGE CAR WEIGHT IN KGM = ',F8.1)
 C
 C
             CALL  RLL(MM,NLOC,ML1,ML2,W,R,AC,AL,NAXL,NAXC)
 C
        DO 50 I=2,MMM1
        I3=2*I-1
        I2=2*I
        SPD1=SPD(I)*SPD(I-1)/(SPD(I)+SPD(I-1))
        SPD2=SPD(I)*SPD(I+1)/(SPD(I)+SPD(I+1))
        SPC1=SPC(I)*SPC(I-1)/(SPC(I)+SPC(I-1))
        SPC2=SPC(I)*SPC(I+1)/(SPC(I)+SPC(I+1))

        A(I3,I2-2)=SPC1

        A(I3,I2-3)=   SPD1
        A(I3,I3)=-(SPD1      +SPD2   +R(I))
        A(I3,I2)   =-SPC2
        A(I3,I2+1)=SPD2
        A(I2-2,I2-3)=1.
        A(I2-2,I3)=-1.
 50     CONTINUE
        A(M-2,M-3)=1
        A(M-2,M-1)=-1
 C
        A(2,1)=1
        A(2,3)=-1.
        SPD2=SPD(1)*SPD(2)   /(SPD(1)+SPD(2))
        SPC2=SPC(1)*SPC(2)   /(SPC(1)+SPC(2))
        A(1,1)=-(R(1)   +SPD2 )
        A(1,2)=-SPC2
        A(1,3)=SPD2
 C
        SPC1=SPC(MM)*SPC(MMM1)/(SPC(MM)+SPC(MMM1))
        SPD1=SPD(MM)*SPD(MMM1)/(SPD(MM)+ SPD(MMM1))
        A(M-1,M-3)=SPD1
        A(M-1,M-2)=SPC1
        A(M-1,M-1)=-(SPD1      +R(MM))
 C
        NSUM=0
        DO 60 II=1,NS
  60    NSUM=NSUM+NSC(II)
        IF(NSUM.EQ.MM) GO TO 70
        WRITE(6,1000)
 1000   FORMAT(/,'ERROR      ',/,5X,'SUM OF CAR SECTIONS NOT EQUAL MM')
 C
  70    CONTINUE
        DO 80 I=1,MM
        DO 80 J=1,NI
  80    B(I,J)=0.0
        NSUM= NSC(1)
        J=1
        DO 90 I=1,MM
        I3=2*I-1
        IF(I.LE.NSUM) GO TO 85
        J=J+1
        NSUM=NSUM+NSC(J)
 85     J3=J+NLOC
        B(I3,J3)=-1
 90     CONTINUE
        DO 99 I=1,NLOC
        I2=ML2(I)-1
 99     B(I2,I)=1
        DO 101 I=1,MM
        I3=2*I-1
        DO 100 J=1,M1
 100    A(I3,J)=A(I3,J)/W(I)
 101    CONTINUE
        DO 105 I=1,MM
        I3=2*I-1

        DO 105 J=1,NI
  105   B(I3,J)=B(I3,J)/W(I)
 C      NOW WRITE THE RESULTS IN THE FILE " TRN1 MODEL "
 C
 C
        WRITE(6,155)
 155    FORMAT(/,20X,' THE MATRIX A ',/)
        DO 170 I=1,M1
        WRITE(6,160) (A(I,J),J=1,M1)
  160   FORMAT(/,5(1PE14.7,2X))
 170    CONTINUE
 C
        WRITE(6,180)
 180    FORMAT(//,20X,' THE MATRIX B ',/)
        DO 200 I=1,M1
        WRITE(6,160)(B(I,J),J=1,NI)
 200    CONTINUE
        STOP
        END
 C
 C
 C
 C
 C
 C
         SUBROUTINE RLL(MM,NLOC,ML1,ML2,W,R,AC,AL,NAXL,NAXC)
 C
 C       PERFORMS LEAST SQUARES CURVE FITTING FOR THE ROLLING RESISTANCE
 C       IN THE RANGE OF VELOCITY
 C       VMIN=2.25 M/SEC (5 MILES/HR)
 C       VMAX=28 METERS/SEC. (60 MILES/HR)
 C      THE LEAD LOCO
 C
```

```
      DIMENSION R(MM),W(MM),ML1(NLOC),ML2(NLOC)
C
      RO=8.165*NAXL+.75*W(1)/1000.
      R1=.335612*W(1)/10000.
      R2=.58657*AL/10.
      X2=RO*28.+R1*28.*28./2.+R2*28.*28.*28./3.
      X1=RO*2.25+R1*2.25*2.25/2.+R2*2.25*2.25*2.25/3.
      R(1)=2.*(X2-X1)/(28.*28.-2.25*2.25)
C
C     ALL  CARS
C
      DO 10 K=2,MM
      RO=13.1543*NAXC+.65*W(K)/1000.
      R1=5.03412*W(K)/100000.
      R2=.1222059*AC/10.
      X2=RO*28.+R1*28.*28./2.+R2*28.*28.*28./3.
      X1=RO*2.25+R1*2.25*2.25/2.+R2*2.25*2.25*2.25/3.
      R(K)=2.*(X2-X1)/(28.*28.-2.25*2.25)
10    CONTINUE
C
C    THE MIDDLE LOCOMOTIVES
C
      DO 12 I=1,NLOC
      I1=ML1(I)
      RO=8.165*NAXL+.75*W(I1)/1000.
      R1=.335612*W(I1)/10000.
      R2=.83100*AL/100.
      X2=RO*28.+R1*28.*28./2.+R2*28.*28.*28./3.
      X1=RO*2.25+R1*2.25*2.25/2.+R2*2.25*2.25*2.25/3.
      R(I1)=2.*(X2-X1)/(28.*28.-2.25*2.25)
12    CONTINUE
C
C
C     CONVERT THE ROLLING RESISTANCE TO NEWTONS
C
       DO 30 J=1,MM
       R(J)=R(J)*9.81
C
30    CONTINUE
      RETURN
      END
```

ANNEX B

THE FREIGHT TRAIN SIMULATOR USER'S MANUAL

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B.1 INTRODUCTION

This manual explains the technical characteristics of the Freight Train Simulator (FTS) program. Description of the mathematical model and subroutines, as well as some illustrative examples, are included. The FTS is designed to simulate the dynamics of trains composed of several diesel-electric locomotives and freight cars.[1] However, it is a special-purpose package which models in some detail the longitudinal drawbar forces, draft gear forces, and energy consumption to assess the performance of various control schemes.

Some of the features of this program are described below.

(a) The program models and monitors the fast and slow modes along the train caused by the dynamic characteristics of the couplers. Such dynamics are not accurately predicted in other available simulation packages such as the Association of American Railroads' Train Operation Simulator (AAR-TOS), or the Train Performance Calculator (TPC).

(b) The braking system in each car is treated individually, with a separate independent input to allow the study of electropneumatic braking systems in which the brake signal is communicated by electric wires. In the meantime, rather than a break pipe, the conventional air brake system can be simulated by providing an appropriate delay between cars via a suitable user's program. Again, such flexibility is not available in the AAR-TOS.

(c) The traction and dynamic braking data as a function of speed and throttle are accommodated with some degree of detail.

(d) Extended (or non-extended) range dynamic braking is utilized.

---

[1] It is based on previous work done by Gruber and Wong at Queen's Unversity.

(e) The simulation program computes the gravitational force on each car individually to allow study of the effects of rapidly changing grade profiles on train dynamics and train handling techniques.

(f) The asymmetrical behaviour of the brake cylinder pressure during release and build-up time is considered.

(g) The FTS accepts data in both MKS and British units.

(h) Operating commands are transferred to the FTS through a user's written subroutine. This feature is necessary to implement complex train handling techniques in which the control decisions are taken on line in accordance with train status and operating environment. This feature is also not available in the TOS, where the control commands have to be made entirely in an offline manner and then fed to the FTS as external data.

(j) The FTS has, in general, potential advantages to monitor the details of the longitudinal dynamics of trains in a changing environment for relatively short periods of time. However, if only the average behaviour of the train is the main concern or if the environmental and operational conditions are relatively stationary during the test period, then other packages such as the TOS may be faster and more appropriate to use.

B.2 TYPICAL USAGE

The FTS may be used in many situations. Some of these applications include:

(a) development of automatic control algorithms, and assessment of the performance of various schemes on intercar forces, throttle dynamic braking/electropneumatic braking coordination, and energy consumption

(b) evaluation of the performance of trains with electropneumatic braking systems, and the study of the effect of sectioning the train for different electropneumatic brake activation signals

(c) investigation of the effect of allocation and distribution of the locomotives along the train on the overall performance of the train and on energy consumption

(d) improvement of current train handling techniques over various terrains

(e) simulation and study of emergency situations initiated by remote control braking units or cars.

B.3 SOME GENERAL ASSUMPTIONS

(a) The track is assumed to be straight, i.e. the effect of lateral forces is not considered here.

(b) Locomotive throttle/dynamic braking data are approximated by linear and hyperbolic segments.

(c) Each locomotive has a single transition speed at all throttle settings, and two transition speeds at all dynamic braking settings.

(d) Cars and locomotives are assumed to be rigid bodies.

(e) Each draft gear is treated as a spring and dash-pot with a dead zone (slack).

(f) The depletion of the air reservoir of the brake cylinders due to cyclic braking is not considered.

(g) Diesel-electric locomotive units with eight power settings for traction or dynamic braking can only be simulated.

Other assumptions concerning modelling of the train will be discussed in detail in the next chapter.

B.4 LIMITATIONS

The constraints due to programming techniques are introduced in the preparation of the input data and clearly outlined later in the manual. The available computer memory and computing costs restrict the program size and consequently place limitations on train length and accuracy of analysis. Since the available computer memory and scope of analysis are variables, the arrays or tables of data may be conveniently changed in the common statements. The current sizes of the various arrays within each block are shown in Attachment B.I. The integration step and number of measurements can be specified by the user according to the specific application and the accuracy needed.

Some of the major limitations are as follows:

. maximum number of vehicles	110
. maximum number of locomotives	10
. maximum number of measurements taken and stored for plotting during one run	300
. maximum number of measurement points along the train	6
. maximum number of terrain sections (grade and/or target speed)	25

The ratio of CPU time to simulation time varies, depending on many factors such as number of cars, number of grade sections traversed during the run, number of locomotives, the integration step, sample time, and computations involved in the user's control program. The CPU time may be estimated roughly using the following empirical formula:

$$t = \alpha_0 + \beta \left(\frac{1}{15}\ \frac{n_c \times t_s}{STEP}\right)\left(1 + 0.3\,\frac{NS - 100}{100}\right)$$

where: t = CPU time

α_0 = overhead time for loading and input/output Calcomp plotting

β = constant, depending on the type of computer

n_c = number of cars in the train

t_s = simulation time

STEP = integration step in seconds

n_s = number of measurement samples during the run.

For the IBM 4341 computer, $\alpha_0 \approx 24$ seconds and $\beta = 0.01$.

B.5 THE TRAIN MODEL

The train model presentation makes use of the studies of McLane,[1] Gruber and Bayoumi,[2] and AAR Train Operation Simulator documetation.[3,4,5]

The drawbar forces, as illustrated in Figure 1, may be computed using the following equations:

$$\dot{v}_i = \frac{1}{m_i} [F_i(t) - F_{i+1}(t) - R_i(t) - G_i(t) + f_i(t) - U_i(t)]$$

$$\dot{\Delta}_i = v_{i-1}(t) - v_i(t)$$

where: F_i, F_{i+1} = draft gear forces

R_i = total rolling and aerodynamic resistance

G_i = component of gravitational force in direction of motion

f_i = dynamic braking/throttling

U_i = braking force on cars

Δ_i = dynamic displacement between cars i and i-1

v_i = velocity of car #i

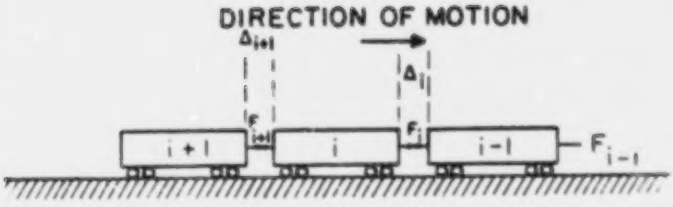


Figure B.1 Longitudinal Forces Acting on Train Member

B.5.1 Draft Gear Forces

Each coupler is represented by a spring having a spring constant, k_i, a damping coefficient, c_i, and a nonlinear dead zone, DZ. Thus, the draft gear force is given by:

$$F_i = k_i \delta_i + c_i \dot{\delta}_i \qquad \text{if } |\Delta_i| \geq DZ$$

$$= 0.0 \qquad \text{if } |\Delta_i| < DZ$$

where Δ_i is the elongation (or compression) of the front coupler of the i^{th} car.

δ_i in turn is computed via the following nonlinear differential equation:

$$\dot{\delta}_i = \frac{1}{(c_i + c_{i-1})} [- (k_i + k_{i-1}) \delta_i + d_1 (\Delta_i, k_{i-1}, DZ)$$

$$+ d_2 (\dot{\Delta}_i, c_{i-1}, DZ, \Delta_i)]$$

where: $d_1 (x,y,z) = x\, y \qquad \text{if } |x| \geq Z$

$= 0.0 \qquad \text{if } |x| < z$

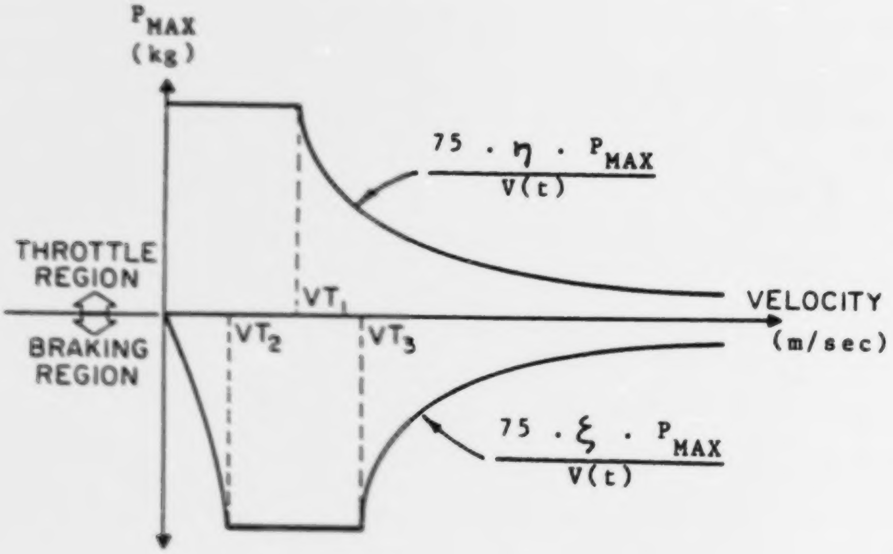
$d_2 (x,y,z,w) = y\, x \qquad \text{if } |w| \geq Z$

$= 0.0 \qquad \text{if } |w| < Z$

The FTS accepts cars and locomotives with different coupler characteristcs. Hence, the spring constant, k_i, and damping coefficient, c_i, may be independently specified for each car and locomotive in the train. However, the dead zone width, DZ, is taken to be the same for all couplers.

B.5.2 Throttling and Dynamic Braking

The locomotives are assumed to have the extended range type of dynamic braking/throtting, in which the maximum allowable throttling/dynamic braking force of the locomotive obeys approximately the curve illustrated in Figure 2.


P_{MAX} (kg)
$\frac{75 \cdot \eta \cdot P_{MAX}}{V(t)}$
THROTTLE REGION
BRAKING REGION
VT_1
VT_2
VT_3
VELOCITY (m/sec)
$\frac{75 \cdot \xi \cdot P_{MAX}}{V(t)}$

where: P_{max} = locomotive maximum horespower

η = transmission efficiency

ξ = locomotive constant

Figure B.2 Throttle-Brake Force Envelope

The values of η and ξ are taken to be 0.82 and 1.16, respectively.

For a given velocity, the throttle/dynamic braking range is quantized to eight levels. It is assumed here that the quantization is uniform, although in practice the quantization levels may not be uniform.

$$f_t = \frac{n}{8} \frac{75 \times 0.82 \times P_{max}}{v(t)}$$

$$f_{db} = -\frac{n}{8} \frac{75 \times 1.16 \times P_{max}}{v(t)}$$

$n = 0, 1, \ldots, 8$ is the throttle run level

The FTS requires P_{max} and the transition speeds, vT_1 vT_2 and vT_3 to be specified by the user for each locomotive.

B.5.3 Gravity Force

$$G_i = W_i \frac{\tan \phi_i}{\sqrt{1 + \tan^2 \phi_i}}$$

where: $\phi_i = \tan^{-1} \frac{GRADE}{100}$

W_i = weight of i^{th} vehicle

The simulation program computes the gravitational force on each car individually to allow the study of rapidly changing grade profiles. The gravitational forces on all cars are updated every 20 metres.

B.5.4 Rolling and Aerodynamic Resistance

The total rolling and aerodynamic resistance is computed via a Davis equation:

$$R = r_0 \times n_a + r_1 \times W + r_2 \times W \times v + r_3 \times v^2 \times A$$

where: R = total rolling and air resistance in pounds (kg)

n_a = number of axles on car/locomotive

W = weight of the car/locomotive in tons (kg)

v = velocity in mph (metres/second)

A = cross-section area in square feet (square metres)

The coefficients r_0, r_1, r_2 and r_3 are taken as follows:

Front Locomotive

r_0 = 18 lbs. force (8.165 kg. force)

r_1 = 1.5 lbs/ton (7.5×10^{-4})

r_2 = 0.03 lb.wt.hr/ton/mile (0.335612×10^{-4} sec/metre)

r_3 = 0.0024 lb.wt.$hr^2/mile^2\ ft^2$ (0.058657 kg sec^2/m^4)

Middle Locomotive

r_0 = 18 (8.165)

r_1 = 1.5 (7.5×10^{-4})

r_2 = 0.03 (0.335612×10^{-4})

r_3 = 0.00034 (0.00831)

All Cars

r_0 = 29 (13.1543)

r_1 = 1.3 (6.5×10^{-4})

r_2 = 0.045 (5.03412×10^{-5})

r_3 = 0.0005 (0.01222059)

The parameters r_0, r_1, r_2 and r_3 are permanently stored in the simulation program. Only the number of axles for each car and locomotive and the cross-section areas have to be provided by the user.

In our test example:

- . A_C, the cross-section area of all cars = 8.361 m^2
- . A_L, the cross-section area of all locomotives - 11.15 m^2
- . the number of axles of each locomotive = 6 or 4
- . the number of axles of all cars = 4
- . the rolling resistance is updated every 20 metres

B.5.5 Electropneumatic Braking (EP)

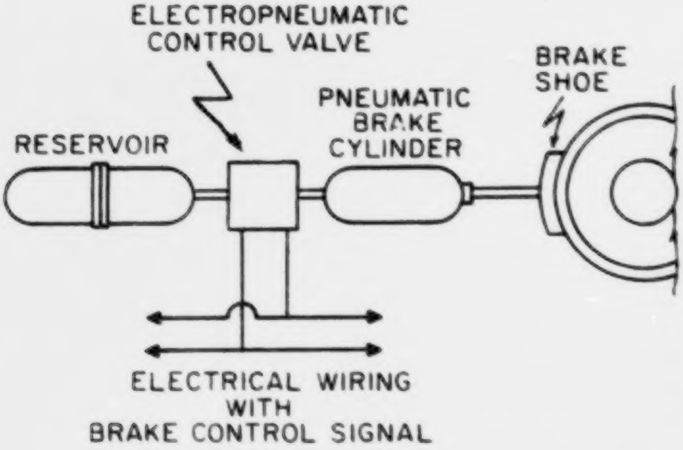

ELECTROPNEUMATIC
CONTROL VALVE
BRAKE
SHOE
PNEUMATIC
BRAKE
CYLINDER
RESERVOIR
ELECTRICAL WIRING
WITH
BRAKE CONTROL SIGNAL

Figure B.3 A Car (Locomotive) E.P. Brake System

In an analog EP braking system, as illustrated in the schematic of Figure 3,[6] the electrical control signal is usually transmitted by wires connected through the train. The variable electrical current is inversely proportional to the braking force. Additional train wires and associated equipment may be used to provide the needed braking force in an emergency situation.

Brake pressure release and build-up may be approximated by the curve shown in Figure 4,[5] where:

p_0 : previous brake cylinder pressure
t_1 : instant at which pressure increase is requested
t_2 : time at which target pressure, p_1, is reached
t_3 : instant at which pressure decreases to the target pressure, p_2, is requested
t_4 : instant at which pressure reaches target pressure, p_2

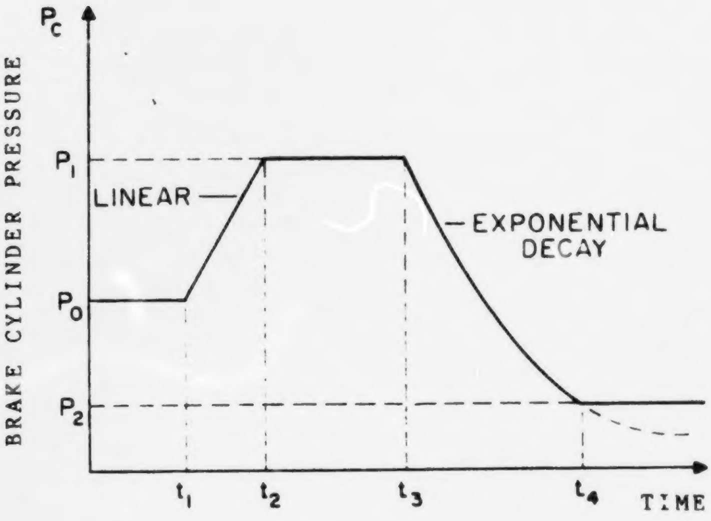


Figure B.4 Brake Cylinder Pressure Curves

It will be assumed that the brake cylinder pressure will be described by the following equations:

(i) for the release case:

$$p_C = (p_{CT}-0.703) + [p_{C0} - (p_{CT}-0.703)] \quad e^{-\frac{\Delta T}{\sigma}}$$

where: $p_{C0} \geq - p_C(t) \geq p_{CT}$

where: $p_C(t)$ = brake cylinder pressure

p_{CO} = initial cylinder pressure before reduction

p_{CT} = target cylinder pressure

σ = release time constant

(ii) for brake application:

$$p_C(t) = p_{CO} + \Delta t \qquad p_{CT} \geq p_C(t) \geq p_{CO}$$

If $p_{CT} \geq 3.5154$ kg/cm^2, it is set to 3.5154 kg/cm^2. In other words, the maximum allowable brake cylinder pressure for all cars and locomotives is taken to be 3.5154 kg/cm^2

(iii) for braking force per car:

$$BCI \times p_C(t)$$

where: BCI = braking force constant = $\frac{BF_{max}}{3.515}$

Typical values for these parameters are:

σ_L : release time constant for locomotives = 3-5 seconds

σ_C : release time constant for cars = 14-20 seconds

α_L : 16 psi/second (1.125 kg/cm^2/sec)

α_C : 4-8 psi/second (0.28-0.56 kg/cm^2/sec)

BF_{max} depends on the car/locomotive type:

for cars BF_{max} = 20-50 K.lb. ($\approx$ 9,090-22,730 kg)

for locomtives, BF_{max} = 120-300 K.lb. (54,500-136,400 kg)

The parameters σ_L, σ_C, σ_L and σ_C should be provided by the user. However, the given values are assumed to be independent of the locomotive or car types. On the other hand, BF_{max} is highly dependent on car/locomotive type, and hence should be provided for each member of the train individually.

The FTS treats the EP system on each car separately to allow the user to investigate different groupings of cars for brake application.

B.6 HOW TO USE THE FTS

The FTS is built in such a way as to simplify the data setting phase as well as to produce the output results in a convenient manner. Most of the relevant performance parameters are plotted directly using the Calcomp plotter with minimum intervention by the user.

The following steps are necessary to use this package.

Step 1:

Assign the input/output logical units as follows:

Unit No.	Function	Current Name	Description
4	output	TRN1 RESULT	Disk file containing a summary of the input data, output results, program error messages, and optionally a trace of the whole run.
5	input	TRN1 DATA	Contains all train data to be suppleid by the user.
6	output	TRN ERROR	Disk file reserved for system error messages.
7	input	TRNPERM DATA	Disk file containing permanent data needed by FTS.

9	input	TRNFB DATA	Option file recommended to store any data to be used by the user-written control subroutine. It is not used by any of the subroutines of the FTS.

Currently the Calcomp plotter uses the printionix printer for plotting. Other plotting devices may be assigned as well, but the JCL command for loading and linking the Calcomp routines may have to be replaced.

Step 2:

Introduce the train data in the file TRN1 DATA as explained in the next section (refer to Section 7 and Attachment B.I).

Step 3:

Write an appropriate train control subroutine which returns to the train operation commands at each call. For the parameters and format of this subroutine, refer to Attachment B.III.

Step 4:

Run the program. The output results will be routed to the Calcomp plotting device, and a summary of the run can be found in the file TRN1 RESULT. It should be noted that:

- The ratio of CP time to simulation time varies, depending on many factors such as train length, integration step, number of measurements to be taken during run, number of grade sections traversed during run, and computations involved in the user's conrol program. Some examples of this ratio together with test examples will be given in Attachment B.IV.
- If one wishes to specify the range of one of the plotted data, a new plot range may be introduced in the appropriate DATA statement in the subroutine PRNT.

- Initial intercar displacement and initial brake cylinder pressure are automatically set to zero. Initial car velocities are set equal to the reference velocity at the start of the first section. Other initial conditions may be introduced by modifying the subroutine INIT.

- The first terrain section should be greater than the train length. The initial train position is assumed to be such that the tail of the train coincides with the starting point of the first section.

B.7 INPUT DESCRIPTION

This section gives a detailed description of user-supplied data cards required for FTS. These data are read by the FTS via the logical unit 5. Currently this unit is assigned to the disk file TRN1 DATA. The FTS expected to find the train data in the following sequence and formats:

1. STEP, TTIME, SMPLT, ENDD in format 4F12.4, where:

   STEP = integration step in seconds
   TTIME = travelling time in seconds
   SMPLT = measurement sampling period in seconds
   ENDD = a default for the maximum travelled distance in km

2. MM, NLOC, NS, NUM, MKS in format 5I5, where:

   MM = number of train members, cars and locomtives (max. 110)
   NLOC = number of locomotives (max. 10)
   NS = number of terrain sections for different grades or reference speeds (max. 25)
   NUM = number of observation points along the train (max. 6)
   MKS = MKS=0 if train data is in British units
   MKS=1 if train data use MKS system

3. The vector ML1 of length NLOC in format 6I5, where:

   ML1 = vector of the positions of the locomotives along the train, ML1(1) should always be 1

4. The vector TCL of length MM in format 6F12.4, where:

TCL = vector of the lengths of the train members in metres (ft)

5. The vector W of length MM in format 6F12.4, where:

W = vector of weights of train members in metric tons (B-tons).

6. The vector PMAX of length NLOC in format 6F12.4, where:

PMAX = vector of the maximum power of the locomotives in metric horespower (B-hp)

7. The array TV of dimension TV(NLOC,3) in format 3F12.4, where:

TV(1,1) = transition speed of the throttle of I^{th} locomotive
TV(I,2) = first transition speed of the dynamic brake of I^{th} locomotive
TV(1,3) = second transition speed of the dynamic brake of I^{th} locomotive

8. The vector SPRGC of length MM in format 6F12.4, where:

SPRGC(1) = spring constant of couplers of I^{th} train member.

9. The vector DAMPC of length MM in format 6F12.4, where:

DAMPC(I) = damping constant of couplers of I^{th} member

10. The vector BFBPC of length MM in format 6F12.4, where:

BFBPC(1) = maximum braking force for I^{th} train member in kg (lb), or equivalently the braking force for 3.525 kg/cm^2 (50 lbs/in^2) brake cylinder pressure

11. AC, AL, in format 2F12.4, where:

AC, AL = cross-sectional area of cars and locomotives, respectively, in square metres (square feet)

12. NAXC, NAXL, in format 6I5, where:

NAXC = number of axles on cars, usually NAXC = 4
NAXL = an integer vector of length NLOC containing number of axles on each locomotive.

13. The array OFFLN of dimenion OFFLN(NS,4) in format 4F12.4, where:

OFFLN(I,1) = length of I^{th} terrain section
OFFLN(I,2) = grade of I^{th} terrain section
OFFLN(I,3) = velocity of I^{th} terrain section
OFFLN(I,4) = index for velocity of section; OFFLN(I,4) = 0.0 if velocity is constant, OFFLN(I,4) = 1.0 if velocity is changing linearly

14. The vector IVDISP of length NUM in format 6I5, where:

IVDISP = index vector observation points of the velocity of the train units. IVDISP always contains even integers, corresponding to two times the position of the train number whose velocity is to be monitored. For example, if IVDISP contains 2, 10, 50, 100, 120, 130, the velocities of train members 1, 5, 25, 50, 60 and 65 will be stored and plotted.

15. The vector IDDISP of length NUM in format 6I5, where:

IDDISP = index vector of intercar displacement observation points. IDDSIP always contains odd integers which correspond to the position of train members next to this displacement. For example, if IDDISP contains 3, 11, 51, 101, 121 and 131, the displacement between the following cars will be stored and plotted:
1st and 2nd cars, 5th and 6th cars, 25th and 26th cars, 50th and 51st cars, 60th and 61st cars, and 65th and 66th cars.

16. The vector IBDISP of length NUM in format 6I5, where:

IBDISP = index vector of postion of train members whose brakes are to be monitored and plotted.

17. The vector ITHRT of length 3 in format 3I5, where:

ITHRT = index vector of locomotives whose throttle/dynamic braking is to be monitored and plotted.

18. SIGMAC, SIGMAL, ALPHAC, ALPHAL, in format 4F12.4, where:

SIGMAC, SIGNAL = inverse of time constant of brake cylinder release for cars and locomotives, respectively
ALPHAC, ALPHAL = rate of increase of brake cylinder pressure for cars and locomotives, respectively, in kg/cm^2 sec. (lb/in^2 sec.)

19. DZ in format F12.4, where:

    DZ = dead zone of couplers (0.5 x slack) in metres (ft)

20. ITRACE in format I5. If ITRACE = 1, a summary of train status is printed out at each sampling time

21. DMAX, VMAX in format 2I5, where:

    DMAX = maximum permissible intercar displacement in metres (ft). If the displacement of any coupler exceeds this value, the program stops and a warning message is issued.
    VMAX = maximum permissible relative velocity between cars in metres/sec. (ft/sec.). Again if this value is exceeded the program stops and a warning message is issued.

A typical example of a TRN1 DATA file is given in Attachment B.I.

B.8 PROGRAM DESCRIPTION (see Attachment B.II)

Attachment B.II provides a brief description of the various subroutines constituting the FTS. The calling sequence of these subroutines is summarized in Figure B.5. The arrows go from the low-level routines to the higher-level routines. The letter M above an arrow indicates multiple calls. The calling sequence coincides with the clockwise direction.

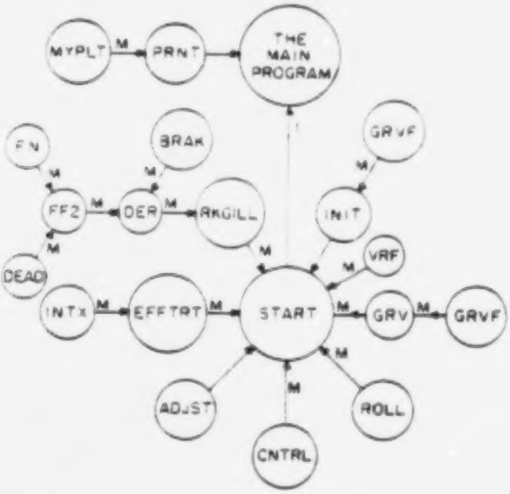

MYPLT
M
PRNT
THE MAIN PROGRAM
FN
BRAK
GRVF
M
M
M
FF2
M
DER
M
RKGILL
INIT
M
VRF
M
DEAD
M
INTX
M
EFFTRT
M
START
M
GRV
M
GRVF
M
M
ADJST
ROLL
CNTRL

Figure B.5 Schematic of Subroutines

B.9 THE CONTROL SUBROUTINE CNTRL (see Attachment B.III)

The user must provide an appropriate subroutine which returns at each call the target throttle and target brake. The format of the control program is shown in Attachment B.III. If MNL=1, the control program should return the throttle as integer numbers between -8 and +8, representing the throttle or the dynamic brake level. The target brake of each car should also be returned as numbers between 0 and 3.548, representing the target brake cylinder pressure in kg/cm^2. In the automatic mode, MNL=0, the target throttles and target brakes should be returned in Newtons.

B.10 EXAMPLES (see Attachment B.IV)

Attachment B.IV provides two illustrative examples. The train data for the two examples are given in Attachment B.I. The examples demonstrate to different train handling techniques over a severe downhill grade. In the first example, the train runs under manual operation. The air brakes are applied upon arrival at the point where the terrain changes downhill and the throttle is decreased to idle gradually. The dynamic brake plays a minor role here only to assist the air brakes. Such a technique may be used to prevent severe impact between cars. However, this techniqe has a serious drawback for long steep descents. It causes overheating of brake shoes and hence they need to be replaced more frequently due to excessive use of brakes.

In the second example, a complex feedback controller is used to maintain target speed and to minimize coupler forces. Emphasis is placed in this case on the dynamic brake to reduce wear of the brake shoes.

B.11 REFERENCES

1. P.J. McLane, L.E. Peppard, and K.K. Sundareswaran, "Decentralized Feedback Control for the Brakeless Operation of Multi-Locomotive Powered Trains," IEEE Trans. Automat. Cont., Vol. AC-21, 1976, pp. 358-363.

2. P. Gruber and M.M. Bayoumi, Train Braking Performance Studies Using Suboptimal Controllers, Part I, CIGGT Report No. 79-11, 1979.

3. Low, Garg, Train Operations Simulator - Programmer's Manual, AAR Report No. R-359.

4. Luttrell, Gupta, Low, Martin, Train Operations Simulator - User's Manual, AAR Report No. R-198.

5. Low, Garg, Train Operations Simulator - Technical Documentation, AAR Report No. R-269

6. Electro-pneumatic Train Brakes, Maughan Railway Consulting Services Inc., Report No. TP1435.

7. M.J. Romanelli, "Runge-Kutta Methods for Solution of Ordinary Differential Equations," in Mathematical Methods for Digital Computers, A. Ralson and H.S. Wilf, eds, John Wiley & Sons Inc., 1960, pp. 110-120.

ATTACHMENT B.1

TRAIN DATA AND INPUT DESCRIPTION

Shown below is a TRN1 file for the simulation of the following case:

- Integration step = 0.2 seconds. Simulation time TTIME = 150 seconds. Sample period SMPT = 1 second. Default of travelling distance ENDD = 3 miles.
- Number of vehicles = 32. Number of locomotives = 2. Number of observation points along the train = 4. Number of terrain sections = 4. MKS = 0. (British units are used for data.)
- Position of the locomotive: 1, 22
- Locomotive length is 60 ft. Car length is 65 ft.
- Locomotive weight is 150 tons. Car weight is 110 tons.
- Locomotive power is 2,000 hp
- Transition velocities of the locomotives are 5, 8 and 16 mph.
- All couplers have the same spring constant, 480,000 lb/ft, and the same damping constant, 13,900 lb.sec/ft.
- The maximum brake forcing for locomotives is 90,000 lbs, and for cars 33,000 lbs.
- The cross-sectional area of a car is 90 square feet, and of a locomotive is 120 square feet.
- The number of axles on both cars and locomotives is 4.
- The terrain profile is as follows:

Length of Terrain	Grade	Ref. Velocity	Velocity Index
0.45	0.0	40	0.0
0.60	-2.0	40	0.0
5.15	0.0	40	0.0
3.15	0.0	40	0.0

- The velocities of the following vehicles will be plotted: 1, 10, 22, 32.
- The displacements between the following vehicles will be plotted: 1 and 2, 10 and 11, 21 and 22, 22 and 23.
- The throttle of the following locomotives will be plotted: 1, 1, 2.
- SIGMAC = 16/sec, SIGMAL = 5/sec., ALPHAC = 3 lb/sq.in./sec., ALPHAL = 16 lb/sq.in./sec.
- ITRACE = 0.
- The breakdown displacement is 1 ft. The impact intercar velocity is 3 ft/sec.

```
       20        150        1 0           3
  32    2     4    4     0
  1    22
  60          65            65            65            65          65
  65          65            65            65            65          65
  65          65            65            65            65          65
  65          65            65            60            65          65
  65          65            65            65            65          65
  65          65            65            65            65          65
   150         110            110          110          110         110
   110         110            110          110          110         110
   110         110            110          110          110         110
   110         110            110          150          110         110
   110         110            110          110          110         110
   110         110            110          110          110         110
   2000        2000
5           8            16
5           8            16
480000      480000       480000       480000        480000      480000
480000      480000       480000       480000        480000      480000
480000      480000       480000       480000        480000      480000
480000      480000       480000       480000        480000      480000
480000      480000       480000       480000        480000      480000
480000      480000       480000       480000        480000      480000
13900       13900        13900        13900         13900         13900
13900       13900        13900        13900         13900         13900
13900       13900        13900        13900         13900         13900
13900       13900        13900        13900         13900         13900
13900       13900        13900        13900         13900         13900
13900       13900        13900        13900         13900         13900
90000       33000          33000      33000          33000      33000
33000       33000          33000      33000          33000      33000
33000       33000          33000      33000          33000      33000
33000       33000          33000      90000          33000      33000
33000       33000          33000      33000          33000      33000
33000       33000          33000      33000          33000      33000
   90        120
   4     4    4
  45         0 0            40           0 0
  60        -2 0            40           0 0
5 15         0 0            40           0 0
3 5          0 0            40           0 0
   2    20    44  64
   3    21    43  45
   1    10    22  32
   1     1    2
 8          4             3            10
 083
   0
  1 0          3
```

```

C
C
C     P.I. CONTROLLER + NEW OFFLINE SCHEDULE.
C
C
C
C***********************************************************************
C*                                                                     *
C*   SUBROUTINE CONTROL                                                *
C*                                                                     *
C*   FUNCTION        USER PROGRAM TO CREATE THE CONTROL COMMANDS FOR   *
C*                   THE TRAIN  .                                      *
C*                                                                     *
C*   USAGE          CALL CNTRL(S,T,VREF,MNL,ISTEP,IG,GL,TRNL)          *
C*                                                                     *
C*   PARAMETERS     (D : DATA TRANSFERED TO CNTRL.)                    *
C*              D   S    - THE TRAVELLED DISTANCE IN METERS.           *
C*              D   T    - THE TRAVELLING TIME.                        *
C*              D   VREF- THE CURRENT REFERENCE SPEED.                 *
C*                  MNL - MNL SHOULD BE RETURNED 1 IF THE TRAIN IS     *
C*                        OPERATED MANUALLY. MNL=0 IF AN AUTOMATIC     *
C*                        CONTROL SCHEME IS USED.                      *
C*                  ISTEP- POINTER FOR THE CURRENT CONTROL STEP.       *
C*                         IF ISTEP IS RETURNED =1000 THE SIMULATION   *
C*                         STOPS.                                      *
C*              D   IG   - INDEX FOR THE CURRENT TERRAIN SECTION.      *
C*              D   GL   - THE TOTAL LENGTH OF THE GRADE SECTIONS TILL *
C*                         NOW IN KM.                                  *
C*              D   TRNL - THE TRAIN LENGTH IN METERS.                 *
C*              ......... FOR OTHER PARAMETERS SEE SUBROUTINE START.   *
C*                                                                     *
C*    COMMON         BLK1,BLK2,BLK3,BLK5,BLK9,BLK10,BLK11,BLK50        *
C*                                                                     *
C*    CALLED BY       START                                            *
C*                                                                     *
C*                                                                     *
C***********************************************************************
      SUBROUTINE  CNTRL(S,T,VREF,MNL,ISTEP,IG,GL,TRNL)
C
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)
      COMMON/BLK2/ICL( 70),W( 70),PMAX(10),TV(10,3)
C     COMMON/BLK3/SPRGC( 70),DAMPC( 70),DZ,BFBPC( 70)
      COMMON/BLK5/OFFLN(25,6),NS
      COMMON/BLK9/Y(230),DYDX(230),Q(230),V( 70),R( 70)
      COMMON/BLK10/EFTHRT(10),TTHRT(10)
      COMMON/BLK11/EFABRK( 70),TBF( 70)
      COMMON/BLK50/FB(10,80),U0(20),IC,STEP,DZDT(2),Z(2),Q1(2),IFB(10)
      REAL S,ENRGY,T,STEP,TTIME,SMPLT,ENDD
      INTEGER KL,MNL,MKS,NI,INC,ITRACE
C
      DIMENSION A(4),B(4),C(4)
      DATA A/.5,.2928932,1.707107,.1666667/,
     +B/2.,1.,1.,2./,C/.5,.2928932,1.707107,.5/
      INTEGER MNL,ISTEP
      MNL=0
      S1=S+TRNL
C
      NI=6
      M1=M+1
      IF(ISTEP.NE.1) GO TO 20
5     FORMAT(5(1PE14.7,2X))
      DO 10 I=1,NI
      READ(9, 5)(FB(I,J),J=1,M)
C     WRITE(4,5)(FB(I,J),J=1,M)
10    CONTINUE
C
```

```
      READ(9,15)(IFB(I),I=1,2)                                          TRA0067
15    FORMAT(5I5)                                                       TRA0068
      Z(1)=   0.0                                                       TRA0069
      TT=1                                                              TRA0070
      Q(2)=3.525*2.2/(36000.*9.81)                                      TRA0071
      ITANDM=IFB(2)                                                     TRA0072
      UO( 7)=0.0                                                        TRA0073
      UO( 8)=0.0                                                        TRA0074
      UO(9)=0.0                                                         TRA0075
      UO(10)=0.0                                                        TRA0076
      UO(11)=GL                                                         TRA0077
      UO(12)=0.0                                                        TRA0078
      IFB(3)=1                                                          TRA0079
      WRITE(4,600)PMAX                                                  TRA0080
 600  FORMAT(' MAXIMUM  LOCOMOTIVES POWER...',/,3(1PE12.5,2X))          TRA0081
C                                                                       TRA0082
      IF(IFB(1).EQ.0) GO TO 20                                          TRA0083
      CALL PROFIL                                                       TRA0084
      UO( 7)=OFFLN(1,5)                                                 TRA0085
      UO( 8)=OFFLN(1,6)                                                 TRA0086
      UO(9)=UO(8)                                                       TRA0087
      UO(10)=UO(10)                                                     TRA0088
20    CONTINUE                                                          TRA0089
C                                                                       TRA0090
      IF(T.LT.TT) GO TO 28                                              TRA0091
      ERROR=Y(2)-VREF                                                   TRA0092
      WRITE(4,25)T,Z(1),S,ERROR                                         TRA0093
25    FORMAT(' TIME = ',F7.2,'  Z(1)= ',E10.3,' S=',F10.1,/,            TRA0094
     + ' ACTUAL SPEED - REFERENCE SPEED  = ',F7.2,/)                    TRA0095
      TT=TT+5.                                                          TRA0096
      WRITE(4,500) IG,GL                                                TRA0097
 500  FORMAT(' IG=',I5,10X,' GL=',F12.4)                                TRA0098
C                                                                       TRA0099
      WRITE(4,505) (IFB(11-I),I=1,NLOC)                                 TRA0100
505   FORMAT(' THE POWER LEVELS OF THE LOCOMOTIVES ....',/,4I5)         TRA0101
      WRITE(4,26)( UO(I)   ,I=1,8 )                                     TRA0102
26    FORMAT(' THE FEEDBACK THROTTLE :....',/,3(1PE14.7,2X))            TRA0103
      WRITE(4,27)(TTHRT(I),I=1,NLOC)                                    TRA0104
27    FORMAT(' THE TARGET   THROTTLE :....',/,3(1PE14.7,2X))            TRA0105
      WRITE(4,2702)( EFTHRT(I),I=1,NLOC)                                TRA0106
2702  FORMAT(' THE EFFECTIVE THROTTLE:....',/,3(1PE14.7,2X))            TRA0107
28    CONTINUE                                                          TRA0108
      DO 30 J=1,4                                                       TRA0109
      DZDT(1)=  2.0*(Y(2) -VREF)                                        TRA0110
C     DZDT(1)=       (Y(2) -VREF)                                       TRA0111
      AA=A(J)                                                           TRA0112
      BB=B(J)                                                           TRA0113
      CC=C(J)                                                           TRA0114
      ZZ=AA*(DZDT(1)-BB*Q1(1))                                          TRA0115
      Z(1)=Z(1)+STEP*ZZ                                                 TRA0116
30    Q1(1)=Q1(1)+3.*ZZ-CC*DZDT(1)                                      TRA0117
C                                                                       TRA0118
C                                                                       TRA0119
C                                                                       TRA0120
      DO 50 I=1,NI                                                      TRA0121
      UO(I)=0.0                                                         TRA0122
      MM1=M-1                                                           TRA0123
      DO 40 J=1,MM1                                                     TRA0124
      J1=J+1                                                            TRA0125
40    UO(I)=UO(I)+FB(I,J)*Y(J1)                                         TRA0126
      DO 45 J=1,MM1,2                                                   TRA0127
45    UO(I)=UO(I)-FB(I,J)*VREF                                          TRA0128
      UO(I)=UO(I)+FB(I,M)*Z(1)                                          TRA0129
      UO(I)=-UO(I)                                                      TRA0130
50    CONTINUE                                                          TRA0131
C                                                                       TRA0132
C                                                                       TRA0133
```

```
      DO=50.+VREF*10.
      IF(IFB(1).EQ.0) GO TO 55
      IF((GL*1000.-S1).GE.DO) GO TO 51
      UO( 7)=OFFLN(IG,5)+(S1-GL*1000.+DO)*
     +                           (OFFLN(IG+1,5)-OFFLN(IG,5))/DO
C
C
51    CONTINUE
      IF((UO(11)*1000.-S1).GE.20.) GO TO 52
      UO(9)=UO(8)
      UO(12)=UO(11)
      IFB(3)=IFB(3)+1
      UO(11)=UO(11)+OFFLN(IFB(3),1)
      UO(10)=OFFLN(IFB(3),6)
52    IF((S1-UO(12)*1000.-TRNL).GT.0.0) GO TO 55
      UO(8)=UO(9)+(S1-UO(12)*1000.)*(UO(10)-UO(9))/TRNL
55    CONTINUE
      FTOT=UO(7)+UO(8)
      IF(FTOT)56,58,58
56    IF(OFFLN(IG,2).LT.0.0) GO TO 57
      FTOT=FTOT/(MM+NLOC)
      UO(1)=UO(1)+FTOT
      UO(2)=UO(2)+FTOT
      UO(3)=UO(3)-FTOT
      UO(4)=UO(4)-FTOT
      UO(5)=-FTOT
      UO(6)=-FTOT
      GO TO 59
57    UO(1)=UO(1)+.2*FTOT/NLOC
      UO(2)=UO(2)+.2*FTOT/NLOC
      UO(3)=UO(3)-.2*FTOT/NLOC
      UO(4)=UO(4)-.2*FTOT/NLOC
      UO(5)=-.8*FTOT/MM
      UO(6)=UO(5)
      GO TO 59
58    FTOT=FTOT/NLOC

      UO(1)=UO(1)+FTOT
      UO(2)=UO(2)+FTOT
      UO(3)=UO(3)-FTOT
      UO(4)=UO(4)-FTOT
      UO(5)=0.0
      UO(6)=0.0
C
59    CONTINUE
      IF(UO(1).GE.0.0.AND.UO(2).GE.0.0)    GO TO 60
      IF(UO(3).GT.0.0.AND.UO(4).GT.0.0)   GO TO 80
      IF(UO(3).EQ.0.0.AND.UO(4).EQ.0.0)    GO TO 60
      IF(ISTEP.EQ.1)GO TO 60
      IF(IC.EQ.0) GO TO 80
60    CONTINUE
      TTHRT(1)=UO(1)
      IF(UO(1).LT.0.0) TTHRT(1)=0.0
      TTHRT(2)=UO(2)
      IF(UO(2).LT.0.0)TTHRT(2)=0.0
200   CONTINUE
      IF(ITANDB.EQ.0) GO TO 65
      IF(UO(5).LT.3000.0)UO(5)=0.0
      IF(UO(6).LT.3000.0) UO(6)=0.0
      DO 62 I=1,11
 62   TBF(I)=UO(5)
      DO 63 I=12,MM
 63   TBF(I)  =UO(6)
      GO  TO 75
 65   CONTINUE
      DO 70 I=1,MM
 70   TBF(I)=0.0                                                        TRA0200
 75   IC=1                                                              TRA0201
      ISTEP= 2                                                          TRA0202
      RETURN                                                            TRA0203
80    CONTINUE                                                          TRA0204
      TTHRT(1)=-U0(3)                                                   TRA0205
      IF(U0(3).LT.0.0) TTHRT(1)=0.0                                     TRA0206
      TTHRT(2)=-U0(4)                                                   TRA0207
      IF(U0(4).LT.0.0)TTHRT(2)=0.0                                      TRA0208
300   CONTINUE                                                          TRA0209
C                                                                       TRA0210
      IF(U0(5).LT.3000.0)U0(5)=0.0                                      TRA0211
      IF(U0(6).LT.3000.0) U0(6)=0.0                                     TRA0212
      DO 90 I=1,11                                                      TRA0213
 90   TBF(I)=U0(5)                                                      TRA0214
      DO 95 I=12,MM                                                     TRA0215
 95   TBF(I)  =U0(6)                                                    TRA0216
      IC=0                                                              TRA0217
      ISTEP=2                                                           TRA0218
C                                                                       TRA0219
C                                                                       TRA0220
      RETURN                                                            TRA0221
      END                                                               TRA0222
C                                                                       TRA0223
C                                                                       TRA0224
C                                                                       TRA0225
C                                                                       TRA0226
      SUBROUTINE PROFIL                                                 TRA0227
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                             TRA0228
      COMMON/BLK2/TCL( 70),W( 70),PMAX(10),TV(10,3)                     TRA0229
      COMMON/BLK4/ AC,AL,NAXC,NAXL(10)                                  TRA0230
      COMMON/BLK5/OFFLN(25,6),NS                                        TRA0231
      COMMON/BLK9/Y(230),DYDX(230),Q(230),V( 70),R( 70)                 TRA0232
      COMMON/BLK50/FB(10,80),U0(20),IC,STEP,DZDT(2),Z(2),Q1(2),IFB(10)  TRA0233
      WTOT=0.0                                                          TRA0234
      WL=0.                                                             TRA0235
      NC=MM-NLOC                                                        TRA0236
      DO 10 I=1,MM                                                      TRA0237
10    WTOT=WTOT+W(I)                                                    TRA0238
      WAV=WTOT/MM                                                       TRA0239
      DO 11 I=1,NLOC                                                    TRA0240
 11   WL=WL+W(ML1(I))                                                   TRA0241
      WC=WTOT-WL                                                        TRA0242
      WL=WL/NLOC                                                        TRA0243
      WC=WC/NC                                                          TRA0244
C                                                                       TRA0245
      DO 1000 I=1,NS                                                    TRA0246
      AVSP=OFFLN(I,3)/3.6                                               TRA0247
      IF(OFFLN(I,4).GT.0.0) AVSP=(OFFLN(I,3)+OFFLN(I+1,3))/7.2          TRA0248
      ACC=0.0                                                           TRA0249
      IF(OFFLN(I,4).GT.0.)ACC=                                          TRA0250
     +          AVSP*(OFFLN(I+1,3)-OFFLN(I,3))/(OFFLN(I,1)*3600.)       TRA0251
      TOTGRF=MM*GRVF(OFFLN(I,2),WAV)                                    TRA0252
      OFFLN(I,6)=TOTGRF                                                 TRA0253
C                                                                       TRA0254
C       COMPUTE THE ROLLING RESISTANCE                                  TRA0255
C                                                                       TRA0256
C     FRONT LOCO                                                        TRA0257
       RTOT =8.165*NAXL(1)+.75*W(1)/1000.+.335612*W(1)*AVSP/10000.      TRA0258
     +     +.58657*AL*AVSP*AVSP/10.0                                    TRA0259
C                                                                       TRA0260
C                                                                       TRA0261
      RTOT=( 13.1543*NAXC+.65*WC  /1000. + 5.03412*WC*AVSP/100000.      TRA0262
     +     +.1222059*AC*AVSP*AVSP/10.0)*NC +RTOT                        TRA0263
C                                                                       TRA0264
C     THE MIDDLE LOCOMOTIVES                                            TRA0265
C                                                                       TRA0266
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      IF(NLOC.EQ.1)GO TO 20                                             TRA0267
      RTOT =( 8.165*NAXL(2)+.75*WL/1000. +.335612*WL*AVSP/10000.        TRA0268
     +       +.8310*AL*AVSP*AVSP/100.)*(NLOC-1)+RTOT                    TRA0269
C                                                                       TRA0270
20    CONTINUE                                                          TRA0271
      RTOT=   9.81*RTOT                                                 TRA0272
      OFFLN(I,5)=RTOT  +ACC*WTOT                                        TRA0273
1000  CONTINUE                                                          TRA0274
      RETURN                                                            TRA0275
      END                                                               TRA0276
```

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C **********************************************************************C TRA0001
C                                                                      C TRA0002
C                 VERSION THREE OF THE F.T.S.                          C TRA0003
C                                                                      C TRA0004
C*********************************************************************** TRA0005
C*                                                                     * TRA0006
C*   SUBROUTINE    DEAD                                                * TRA0007
C*                                                                     * TRA0008
C*   FUNCTION      COMPUTE THE NONLINEAR FUNCTION OF THE SPRING        * TRA0009
C*                 PART OF THE COUPLER                                 * TRA0010
C*                                                                     * TRA0011
C*   USAGE         Z=DEAD(X,SLOPE)                                     * TRA0012
C*                                                                     * TRA0013
C*   PARAMETRES    X      - VALUE OF THE COUPLER DISPLACEMENT.         * TRA0014
C*                 SLOPE  - SPRING CONSTANT OF THE COUPLER.            * TRA0015
C*                 DEAD   - THE RETURNED COUPLER FORCE.                * TRA0016
C*                 DZ     - THE DEAD ZONE OF THE COUPLER(1/2. SLACK).  * TRA0017
C*                                                                     * TRA0018
C*   OTHER PARAMETERS    SEE THE MAIN PROGRAM .                        * TRA0019
C*                                                                     * TRA0020
C*   COMMON     BLK3                                                   * TRA0021
C*                                                                     * TRA0022
C*   ROUTINE NEEDED  NONE                                              * TRA0023
C*                                                                     * TRA0024
C*                                                                     * TRA0025
C*********************************************************************** TRA0026
      FUNCTION DEAD(X,SLOPE)                                            TRA0027
      COMMON/BLK3/SPRGC( 70),DAMPC( 70),DZ,BFBPC( 70)                   TRA0028
      IF(X.LT.(-DZ)) GO TO 12                                           TRA0029
      IF(X.LT.DZ) GO TO 13                                              TRA0030
      DEAD=(X-DZ)*SLOPE                                                 TRA0031
      RETURN                                                            TRA0032
 12   DEAD=(X+DZ)*SLOPE                                                 TRA0033
      RETURN                                                            TRA0034
 13   DEAD=0.0                                                          TRA0035
      RETURN                                                            TRA0036
      END                                                               TRA0037
C                                                                       TRA0038
C                                                                       TRA0039
```

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C***********************************************************************
C*                                                                     *
C*   SUBROUTINE    FN                                                  *
C*                                                                     *
C*   FUNCTION      COMPUTE THE NONLINEAR FUNCTION OF THE DAMPING       *
C*                 PART OF THE COUPLER                                 *
C*                                                                     *
C*   USAGE         Z=FN(X)                                             *
C*                                                                     *
C*   PARAMETER      X   ACTUAL VALUE OF THE COUPLER                    *
C*                  DZ  DEAD ZONE                                      *
C*                                                                     *
C*   COMMON         BLK10                                              *
C*                                                                     *
C*   ROUTINE NEEDED  NONE                                              *
C*                                                                     *
C***********************************************************************
      FUNCTION FN(X)
      COMMON/BLK3/SPRGC( 70),DAMPC( 70),DZ,BFBPC( 70)
      IF(ABS(X).LT.DZ) GO TO 12
      FN=1.0
      RETURN
12    FN=0.0
      RETURN
      END
C
C***********************************************************************
C*                                                                     *
C*   SUBROUTINE    BRAK                                                *
C*                                                                     *
C*   FUNCTION      SIMULATES THE UNSYMETRIC BEHAVIOR OF THHE BRAKES .  *
C*                                                                     *
C*                                                                     *
C*   USAGE         CALL BRAK(TBFF,BFBP,YY,DY,ALPHA,SIGMA)              *
C*                                                                     *
C*   PARAMETER     TBFF  - THE TARGET BRAKE FORCE.                     *
C*                 BFBP  -BRAKEFORCE/BRAKE PRESSURE CONSTANT.          *
C*                 YY    - THE CURRENT BRAKE CYLINDER PRESSURE.        *
C*                 DY    - THHE RATE OF CHANGE OF THE BRAKE CYLINDER   *
C*                           PRESSURE.                                 *
C*                 ALPHA -THE RATE OF INCREASE OF THE BRAKE CYLINDER   *
C*                           PRESSURE.                                 *
C*                 SIGMA - THHE INVERSE OF THE TIME CONSTANT OF THE    *
C*                           BRAKE CYLINDER PRESSURE RELEASE.          *
C*                                                                     *
C*   COMMON   NONE                                                     *
C*                                                                     *
C*   ROUTINES NEEDED NONE                                              *
C*                                                                     *
C*   CALLED BY   DER                                                   *
C*                                                                     *
C*   WRITTEN BY   MOUSTAFA ELSHAFEI AHMED.                             *
C*                                                                     *
C*   DATE         MARSH,1982                                           *
C*                                                                     *
C*                                                                     *
C***********************************************************************
      SUBROUTINE BRAK(TBFF,BFBP,YY,DY,ALPHA,SIGMA)
      TB=TBFF/(9.81*BFBP)
      IF(TB - YY) 10,20,30
10    DY=(TB-YY)/SIGMA
      RETURN
20    DY=0.0
      RETURN
30    DY= ALPHA
      RETURN
      END
```

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C                                                                       TRA0107
C***********************************************************************TRA0108
C*                                                                     *TRA0109
C*   SUBROUTINE     INTX                                               *TRA0110
C*                                                                     *TRA0111
C*   FUNCTION       INTEGER FUNCTION WHICH ROUNDS OFF THE ARGUMENT     *TRA0112
C*                  TO THE NEEREST INTEGER.                            *TRA0113
C*                                                                     *TRA0114
C*   USAGE          INTX(X)                                            *TRA0115
C*                                                                     *TRA0116
C*   PARAMETERS      X    - THE FLOATING POINT NUMBER TO BE ROUNDED.   *TRA0117
C*                   INTX - THE NEEREST INTEGER NUMBER TO X.           *TRA0118
C*                                                                     *TRA0119
C*   ROUTINES NEEDED   NONE                                            *TRA0120
C*                                                                     *TRA0121
C*   CALLED BY         DER                                             *TRA0122
C*                                                                     *TRA0123
C*   COMMON             NONE                                           *TRA0124
C*                                                                     *TRA0125
C***********************************************************************TRA0126
                                                                        TRA0127
      FUNCTION INTX(X)                                                  TRA0128
      XX=ABS(X)                                                         TRA0129
      I=INT(XX)                                                         TRA0130
      INTX=I+1                                                          TRA0131
      IF((XX-I).LT.(I+1-XX)) INTX=I                                     TRA0132
      IF(X.LT.0.0) INTX=-INTX                                           TRA0133
      RETURN                                                            TRA0134
      END                                                               TRA0135
C***********************************************************************TRA0136
C*                                                                     *TRA0137
C*   SUBROUTINE     EFFTRT                                             *TRA0138
C*                                                                     *TRA0139
C*   FUNCTION       COMPUTES THE EFFECTIVE THROTTLE OR DYNAMIC BRAKE.  *TRA0140
C*                  THHE THROTTLE AND DYNAMIC BRAKE ARE ASSUMED TO     *TRA0141
C*                  HAVE 8 POWER LEVELS.THE RELATION BETWEEN FORCE AND *TRA0142
C*                  VELOCITY IS APPROXIMATED BY LINEAR AND HYPARABOLIC *TRA0143
C*                  SEGMENTS.                                          *TRA0144
C*                                                                     *TRA0145
C*   USAGE          CALL EFFTRT                                        *TRA0146
C*                                                                     *TRA0147
C*   PARAMETER                                                         *TRA0148
C*    MM    - NUMBER OF TRAIN MEMBERS ,MAX 110(CARS +LOCOMOTIVES).     *TRA0149
C*    NLOC  - NUMBER OF LOCOMOTIVES,MAX 10 UNITS.                      *TRA0150
C*    M     - 2*M THE NUMBER OF THE VELOCITIES AND DISPLACEMENTS       *TRA0151
C*    ML1   - VECTOR OF THE POSITIONS OF THE LOCOMOTIVS  .             *TRA0152
C*    ML2   - INDIX VECTOR FOR THE MIDDLE LOCOMOTIVES VELOCITY         *TRA0153
C*    TCL   - VECTOR OF THE LENGTHS OF THE TRAIN MEMBERS IN METERS.    *TRA0154
C*    W     - VECTOR OF THE WEIGHTS OF THE TRAIN MEMBERS IN METRIC TONS*TRA0155
C*    PMAX  -VECTOR OF THE MAXIMUM POWERS OF THE LOCO IN METRIC H.P.   *TRA0156
C*      TV  -ARRAY  OF THE TRANSITION VELOCITIES OF THE LOCOMOTIVES    *TRA0157
C*          TV(I,1)- THE THROTTLE TRANSITION VELOCITY OF THE I-TH      *TRA0158
C*                LOCOMOTIVE                                           *TRA0159
C*      TV(I,2),TV(I,3) - THE TRANSITION VELOCITIES OF THE DYNAMIC     *TRA0160
C*                        BRAKING. THESE VELOCITIES ARE IN KM/H .      *TRA0161
C*    AC,AL - THE CROSS SECTION AREA OF THE CARS AND LOCOMOTIVES       *TRA0162
C*            RESPECTIVELY  .                                          *TRA0163
C*    NAXC  -  THE NUMBER OF CARS AXLES. USUALLY= 4  .                 *TRA0164
C*    NAXL  -  VECTOR CONAINING THE NUMBER OF AXLES OF EACH LOCOMOTIVE.*TRA0165
C*       TTHRT ;  VECTOR OF THE TARGET THROTTLING FORCE IN NEWTONS.    *TRA0166
C*       EFTHRT:  THE EFFECTIVE THROTTLE AFTER QUANTIZATION.           *TRA0167
C*                                                                     *TRA0168
C*   OTHER PARAMETERS   SEE THE MAIN PROGRAM OR START SUBROUTINE       *TRA0169
C*                                                                     *TRA0170
C*   COMMON                : BLK1,BLK2,BLK4,AND BLK9                   *TRA0171
C*                                                                     *TRA0172
C*   SUBROUTINES NEEDED :    INTX                                      *TRA0173
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C*                                                                     *TRA0174
C*   CALLED BY            : START                                      *TRA0175
C*                                                                     *TRA0176
C*   WRITTEN BY   MOUSTAFA ELSHAFEI AHMED                              *TRA0177
C*                                                                     *TRA0178
C*   DATE         APRIL,1982                                           *TRA0179
C*                                                                     *TRA0180
C***********************************************************************TRA0181
      SUBROUTINE EFFTRT                                                 TRA0182
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                             TRA0183
      COMMON/BLK2/TCL( 70),W( 70),FMAX(10),TV(10,3)                     TRA0184
      COMMON/BLK9/Y(230),DYDX(230),Q(230),V( 70),R( 70)                 TRA0185
      COMMON/BLK10/EFTHRT(10),TTHRT(10)                                 TRA0186
      COMMON/BLK50/FF(10,80),U0(20),IC,STEP,DZDT(2),Z(2),Q1(2),IFB(10)  TRA0187
      DO 60 I=1,NLOC                                                    TRA0188
      Y1=Y(ML2(I))                                                      TRA0189
      TRT=TTHRT(I)                                                      TRA0190
      YY=ABS(Y1)                                                        TRA0191
      IF(TRT) 10,20,30                                                  TRA0192
10    IF(YY.GE.TV(I,2)) GO TO 15                                        TRA0193
      F1=FMAX(I)*1.16*YY/(TV(I,2)*TV(I,3)*8)                            TRA0194
      GO TO 16                                                          TRA0195
15    IF(YY.LE.TV(I,3))YY=TV(I,3)                                       TRA0196
      F1= FMAX(I)*1.16/(8.0*YY)                                         TRA0197
16    K=INTX(-TRT/F1)                                                   TRA0198
      IF(K.GE.8) K=8                                                    TRA0199
      K=-K                                                              TRA0200
      EFT= K*F1                                                         TRA0201
      GO TO 50                                                          TRA0202
20    EFT=0.0                                                           TRA0203
      K=0                                                               TRA0204
      GO TO 50                                                          TRA0205
30    IF(YY.LE.TV(I,1))YY=TV(I,1)                                       TRA0206
      F1= FMAX(I)*0.82/(8.0*YY)                                         TRA0207
      K=INTX(TRT/F1)                                                    TRA0208
      IF(K.GE.8) K=8                                                    TRA0209
      EFT=K*F1                                                          TRA0210
50    EFTHRT(I)=EFT                                                     TRA0211
      IFB(11-I)=K                                                       TRA0212
60    CONTINUE                                                          TRA0213
      RETURN                                                            TRA0214
      END                                                               TRA0215
C                                                                       TRA0216
C***********************************************************************TRA0217
C*                                                                     *TRA0218
C*   SUBROUTINE   GRV                                                  *TRA0219
C*                                                                     *TRA0220
C*   FUNCTION     COMPUTES THE GRAVITINAL FORCE ON EACH CAR OF THE     *TRA0221
C*                TRAIN.                                               *TRA0222
C*                                                                     *TRA0223
C*   USAGE        CALL GRV(IG,S,GL)                                    *TRA0224
C*                                                                     *TRA0225
C*   PARAMETERS      :                                                 *TRA0226
C*    IG  - INDIX OF THE CURRENT GRADE SECTION                         *TRA0227
C*    S   - THE TRAVELLED DISTANCE TILL NOW IN METERS .                *TRA0228
C*    GL  - THE TOTAL LENGTH OF THE GRADE SECTIONS UP AND INCLUDING    *TRA0229
C*          THE CURRENT ONE   IN KM.                                   *TRA0230
C*                                                                     *TRA0231
C*    MM     - NUMBER OF TRAIN MEMBERS ,MAX 110(CARS +LOCOMOTIVES).    *TRA0232
C*    NLOC   - NUMBER OF LOCOMOTIVES,MAX 10 UNITS.                     *TRA0233
C*    M      - 2*M THE NUMBER OF THE VELOCITIES AND DISPLACEMENTS      *TRA0234
C*    ML1    - VECTOR OF THE POSITIONS OF THE LOCOMOTIVS  .            *TRA0235
C*    ML2    - INDIX VECTOR FOR THE MIDDLE LOCOMOTIVES VELOCITY        *TRA0236
C*    TCL    - VECTOR OF THE LENGTHS OF THE TRAIN MEMBERS IN METERS.   *TRA0237
C*    W      - VECTOR OF THE WEIGHTS OF THE TRAIN MEMBERS IN METRIC TONS*TRA0238
C*    NS     - NUMBER OF TERRAIN SECTIONS.                             *TRA0239
```

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C*     OFFLN- ARRAY OF DIMENSION(25,6)                                       *TRA0240
C*     OFFLN(I,1) LENGTH OF THE I-TH TERRAIN SECTION.                        *TRA0241
C*     OFFLN(I,2) GRADE    .........   ....  ....... ...                     *TRA0242
C*     OFFLN(I,3) VELOCITY .... .... .... ..... .... ( AT ITS START)         *TRA0243
C*     OFFLN(I,4) INDIX OF THE VELOCITY OF THE SECTION;                      *TRA0244
C*                INDIX=0.0 IF THE VELOCITY IS CONSTANT,                     *TRA0245
C*                INDIX=1.  IF THE VELOCITY IS CHANGING LINEARLY.            *TRA0246
C*                                                                           *TRA0247
C*     OTHER PARAMETERS   SEE SUBROUTINE START OR THE MAIN PROGRAM.          *TRA0248
C*                                                                           *TRA0249
C*     COMMON             BLK1,BLK2,BLK5,BLK9                                *TRA0250
C*                                                                           *TRA0251
C*     ROUTINES CALLED    GRVF                                               *TRA0252
C*                                                                           *TRA0253
C*     CALLED BY          START , INIT  .                                    *TRA0254
C*                                                                           *TRA0255
C*     WRITTEN BY         MOUSTAFA ELSHAFEI AHMED                            *TRA0256
C*                                                                           *TRA0257
C*     DATE               APRIL,1982                                         *TRA0258
C*                                                                           *TRA0259
C****************************************************************************TRA0260
      SUBROUTINE GRV(IG,S,GL)                                                 TRA0261
C                                                                             TRA0262
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                                   TRA0263
      COMMON/BLK2/TCL( 70),W( 70),PMAX(10),TV(10,3)                           TRA0264
      COMMON/BLK5/OFFLN(25,6),NS                                              TRA0265
      COMMON/BLK9/Y(230),DYDX(230),Q(230),V( 70),R( 70)                       TRA0266
10    IF(IG.EQ.1) RETURN                                                      TRA0267
C                                                                             TRA0268
C     COMPUTE THE GRAV. FORCE ON THE FRONT LOCO                               TRA0269
C                                                                             TRA0270
      V(1)=GRVF(OFFLN(IG,2),W(1))                                             TRA0271
      GL1=GL-OFFLN(IG,1)                                                      TRA0272
      JG=IG                                                                   TRA0273
      SS=S-TCL(1)                                                             TRA0274
      DO 30 K=2,MM                                                            TRA0275
      SS=SS-TCL(K)                                                            TRA0276
      IF(SS.GE.(GL1*1000.)) GO TO 20                                          TRA0277
      JG=JG-1                                                                 TRA0278
      GL1=GL1-OFFLN(JG,1)                                                     TRA0279
20    V(K)=GRVF(OFFLN(JG,2),W(K))                                             TRA0280
30    CONTINUE                                                                TRA0281
      RETURN                                                                  TRA0282
      END                                                                     TRA0283
C                                                                             TRA0284
C                                                                             TRA0285
C****************************************************************************TRA0286
C*                                                                           *TRA0287
C*    SUBROUTINE    GRVF                                                     *TRA0288
C*                                                                           *TRA0289
C*    FUNCTION      COMPUTES THE TANGENTIAL COMPONENT OF THE GRAVITA-        *TRA0290
C*                  TIONAL FORCE ON A MASS W OVER A GRADE X % .              *TRA0291
C*                                                                           *TRA0292
C*    USAGE         GRVF(X,W)                                                *TRA0293
C*                                                                           *TRA0294
C*    PARAMETERS    :                                                        *TRA0295
C*                                                                           *TRA0296
C*                  X- THE GRADE OF THE GROUND.                              *TRA0297
C*                  W- THE MASS IN KG.                                       *TRA0298
C*               GRVF- THE TANGETIAL FORCE IN NEWTONS.                       *TRA0299
C*                                                                           *TRA0300
C*    COMMON        :   NONE                                                 *TRA0301
C*                                                                           *TRA0302
C*    CALLED BY     :   GRV                                                  *TRA0303
C*                                                                           *TRA0304
C*    ROUTINES NEEDED    NONE                                                *TRA0305
C*                                                                           *TRA0306
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C***********************************************************************
      FUNCTION GRVF(X,W)
      TG=X/100.0
      GRVF=         9.81*W*TG/SQRT(1+TG*TG)
      RETURN
      END
C
C
C***********************************************************************
C*                                                                     *
C*    SUBROUTINE     VRF                                               *
C*                                                                     *
C*    FUNCTION       UPDATES THE TERRAIN SECTION AND COMPUTES THE      *
C*                   REFERENCE VELOCITY .                              *
C*                                                                     *
C*    USAGE          CALL VRF(S,VL,IV,VREF,YY22,YY11,Y1,DVDS,TRNL)     *
C*                                                                     *
C*    PARAMETERS     :                                                 *
C*                   S   - TRAVELLED DISTANCE + TRAIN LENGTH IN METERS. *
C*                   VL  - THE TOTAL LENGTH OF THE TERRAIN SECTIONS    *
C*                         TILL NOW,INCLUDING THE CURRENT ONE,IN KM.   *
C*                  IV   - POINTER TO THE CURRENT TERRAIN SECTION.     *
C*                  VREF- THE CURRENT TARGET SPEED.                    *
C*                  YY22- ACTUAL SPEED OF THE TRAIN.                   *
C*            YY11-Y1   - THE INCREMENT OF TIME SINCE THE LAST CALL.   *
C*                  DVDS- RATE OF CHANGE OF THE REFERENCE SPEED WITH   *
C*                         DISTANCE.                                   *
C*                  TRNL- TRAIN LENGTH IN METERS.                      *
C*         ............ FOR OTHER PARAMETERS SEE SUBROUTINE START.     *
C*                                                                     *
C*    COMMON            BLK5                                           *
C*                                                                     *
C*    ROUTINES NEEDED   NONE                                           *
C*                                                                     *
C*    CALLED BY         START                                          *
C*                                                                     *
C*    WRITTEN BY        MOUSTAFA ELSHAFEI AHMED                        *
C*                                                                     *
C*    DATE               APRIL,1982                                    *
C*                                                                     *
C***********************************************************************
      SUBROUTINE VRF(S,VL,IV,VREF,YY22,YY11,Y1,DVDS,TRNL)
      COMMON/BLK5/OFFLN(25,6),NS
      IF(S.LT.(VL*1000.)) GO TO 10
      IV=IV+1
      VL=VL+OFFLN(IV,1)
      DVDS=0.0
      SN=OFFLN(IV,1)
      IF(IV.EQ.1)SN=OFFLN(IV,1)-TRNL/1000.
C
      IF(OFFLN(IV,4).GT.0.)DVDS=(OFFLN(IV+1,3)-OFFLN(IV,3))/SN
      DVDS=DVDS/3600.0
C
10    VREF=VREF+YY22*(YY11-Y1)*DVDS
      RETURN
      END
C
C***********************************************************************
C*                                                                     *
C*    SUBROUTINE     ROLL                                              *
C*                                                                     *
C*    FUNCTION       COMPUTES THE ROLLING AND AIR DYNAMIC RESISTANCE ON *
C*                   EACH CAR. THE RESULT IS STORED IN THE VECTOR R.   *
C*                                                                     *
C*    USAGE          CALL ROLL                                         *
C*                                                                     *
C*    PARAMETERS     :                                                 *
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C*                  SEE SUBROUTINE START.                               *TRA0374
C*                                                                      *TRA0375
C*    COMMON          :    BLK1,BLK2,BLK4,BLK9                          *TRA0376
C*                                                                      *TRA0377
C*    CALLED BY       :    START                                        *TRA0378
C*                                                                      *TRA0379
C*    ROUTINES NEEDED      NONE                                         *TRA0380
C*                                                                      *TRA0381
C*    WRITTEN BY          MOUSTAFA ELSHAFEI AHMED.                      *TRA0382
C*                                                                      *TRA0383
C*    DATE                JUNE,1982                                     *TRA0384
C*                                                                      *TRA0385
C***********************************************************************TRA0386
      SUBROUTINE ROLL                                                    TRA0387
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                              TRA0388
      COMMON/BLK2/TCL( 70),W( 70),FMAX(10),TV(10,3)                      TRA0389
      COMMON/BLK4/ AC,AL,NAXC,NAXL(10)                                   TRA0390
      COMMON/BLK9/Y(230),DYDX(230),Q(230),V( 70),R( 70)                  TRA0391
C                                                                        TRA0392
C     FRONT LOCO                                                         TRA0393
      R(1)=8.165*NAXL(1)+.75*W(1)/1000.+.335612*W(1)*ABS(Y(2))/10000.    TRA0394
     +     +.58657*AL*Y(2)*Y(2)/10.0                                     TRA0395
C                                                                        TRA0396
C                                                                        TRA0397
      DO 10 K=2,MM                                                       TRA0398
      K2=K*2                                                             TRA0399
      YK2=ABS(Y(K2))                                                     TRA0400
      R(K)=13.1543*NAXC+.65*W(K)/1000. + 5.03412*W(K)*YK2/100000.        TRA0401
     +     +.1222059*AC*Y(K2)*Y(K2)/10.0                                 TRA0402
10    CONTINUE                                                           TRA0403
C                                                                        TRA0404
C    THE MIDDLE LOCOMOTIVES                                              TRA0405
C                                                                        TRA0406
      IF(NLOC.EQ.1)GO TO 21                                              TRA0407
      DO 20 I=2,NLOC                                                     TRA0408
      I1=ML1(I)                                                          TRA0409
      I2=ML2(I)                                                          TRA0410
      YK2=ABS(Y(I2))                                                     TRA0411
      R(I1)=8.165*NAXL(I)+.75*W(I1)/1000. +.335612*W(I1)*YK2/10000.      TRA0412
     +         +.8310*AL*YK2*YK2/100.                                    TRA0413
 20   CONTINUE                                                           TRA0414
 21   CONTINUE                                                           TRA0415
C     CONVERT THE ROLLING RESISTANCE TO NEWTONS                          TRA0416
C                                                                        TRA0417
       DO 30 J=1,MM                                                      TRA0418
       K=2*J                                                             TRA0419
       YK=0.0                                                            TRA0420
       IF(Y(K).NE.0.0) YK=        Y(K)/ABS(Y(K))                         TRA0421
       R(J)=R(J)*9.81*YK                                                 TRA0422
C                                                                        TRA0423
30    CONTINUE                                                           TRA0424
      RETURN                                                             TRA0425
      END                                                                TRA0426
C                                                                        TRA0427
C                                                                        TRA0428
C                                                                        TRA0429
C***********************************************************************TRA0430
C*                                                                      *TRA0431
C*    SUBROUTINE      ADJST                                             *TRA0432
C*                                                                      *TRA0433
C*    FUNCTION        COMPUTES THE EFFECTIVE THROTTLE OR DYNAMIC BRAKE. *TRA0434
C*                    THE  THROTTLE AND DYNAMIC BRAKE ARE ASSUMED TO    *TRA0435
C*                    HAVE 8 POWER LEVELS.THE RELATION BETWEEN FORCE AND*TRA0436
C*                    VELOCITY IS APPROXIMATED BY LINEAR AND HYPARABOLIC*TRA0437
C*                    SEGMENTS.THE TARGET THROTTLE/DYNAMIC BRAKE MUST BE*TRA0438
C*                    SPECIFIED AS INTEGER NUMBERS BETWEEN-8,,+8.       *TRA0439
C*                    INCLUSIVE. THIS SUBROUTINE IS CALLED ONLY IF MANUAL*TRA0440
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C*                  OPERATION IS USED.                                *TRA0441
C*                                                                    *TRA0442
C*    USAGE          CALL ADJST                                       *TRA0443
C*                                                                    *TRA0444
C*    PARAMETERS                                                      *TRA0445
C*                 SEE SUBROUTINE EFFTRT,OR START .                   *TRA0446
C*                                                                    *TRA0447
C*    COMMON              : BLK1,BLK2,BLK3,BLK9,BLK10                 *TRA0448
C*                                                                    *TRA0449
C*    SUBROUTINES NEEDED :   NONE                                     *TRA0450
C*                                                                    *TRA0451
C*    CALLED BY           : START                                     *TRA0452
C*                                                                    *TRA0453
C*    WRITTEN BY     MOUSTAFA ELSHAFEI AHMED                          *TRA0454
C*                                                                    *TRA0455
C*    DATE           APRIL,1982                                       *TRA0456
C*                                                                    *TRA0457
C***********************************************************************TRA0458
      SUBROUTINE ADJST                                                  TRA0459
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                             TRA0460
      COMMON/BLK2/TCL( 70),W( 70),PMAX(10),TV(10,3)                     TRA0461
      COMMON/BLK3/SPRGC( 70),DAMPC( 70),DZ,BFBPC( 70)                   TRA0462
      COMMON/BLK9/Y(230),DYDX(230),Q(230),V( 70),R( 70)                 TRA0463
      COMMON/BLK10/EFTHRT(10),TTHRT(10)                                 TRA0464
      COMMON/BLK11/FFABRK( 70),TBF( 70)                                 TRA0465
C                                                                       TRA0466
C                                                                       TRA0467
      DO 5 I=1,MM                                                       TRA0468
 5    TBF(I)   =TBF(I)*BFBPC(I)*9.81                                    TRA0469
C                                                                       TRA0470
      DO 90 I=1,NLOC                                                    TRA0471
      Y1=Y(ML2(I))                                                      TRA0472
      TRT=TTHRT(I)                                                      TRA0473
      YY=ABS(Y1)                                                        TRA0474
      IF(TRT)10,10,20                                                   TRA0475
10    IF(YY.GE.TV(I,2)) GO TO 15                                        TRA0476
      F1=PMAX(I)*1.16*YY/(8.*TV(I,2)*TV(I,3))                           TRA0477
      TRT=TRT*F1                                                        TRA0478
      GO TO 80                                                          TRA0479
15    IF(YY.LE.TV(I,3)) YY=TV(I,3)                                      TRA0480
      F1=PMAX(I)*1.16/(8.*YY)                                           TRA0481
      TRT=TRT*F1                                                        TRA0482
      GO TO 80                                                          TRA0483
20    IF(YY.LE.TV(I,1)) YY=TV(I,1)                                      TRA0484
      F1=PMAX(I)*0.82/(8.*YY)                                           TRA0485
      TRT=TRT*F1                                                        TRA0486
80    EFTHRT(I)=TRT                                                     TRA0487
90    CONTINUE                                                          TRA0488
      RETURN                                                            TRA0489
      END                                                               TRA0490
C***********************************************************************TRA0491
C*                                                                    *TRA0492
C*    SUBROUTINE START                                                *TRA0493
C*                                                                    *TRA0494
C*    FUNCTION      : THIS IS THE MAIN SUBROUTINE. IT INITIALIZES ALL  *TRA0495
C*                    PARAMETERS,COORDINATES THE MAIN INTEGRATION LOOP,*TRA0496
C*                    TAKES AND STORES THE VARIOUS MEASUREMENTS,AND    *TRA0497
C*                    COMPUTES THE TRAVELLED DISTANCE,TIME AND THE     *TRA0498
C*                    ENERGY CONSUMPTION.                              *TRA0499
C*                                                                    *TRA0500
C*    USAGE         : CALL START(STEP,TTIME,SMPLT,ENDD,S,ENRGY,T,XL,   *TRA0501
C*                     MNL,NI,ITRACE)                                  *TRA0502
C*                                                                    *TRA0503
C*    PARAMETERS : (I FOR INPUT, O FOR OUTPUT)                        *TRA0504
C* I  STEP  - THE INTEGRATION STEP IN SECONDS .                       *TRA0505
C* I  TTIME - THE TRAVELING TIME IN SECONDS .                         *TRA0506
C* I  SMPLT - THE MEASUREMENT SAMPLING PERIOD  IN SECONDS .           *TRA0507
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C* I  ENDD  - A DEFAULT FOR THE MAXIMUM TRAVELLED DISTANCE IN KM.       *TRA0508
C* 0  S     - THE TRAVELLED DISTANCE IN METERS WHEN THE SIMULATION      *TRA0509
C*            IS TERMINATED .                                           *TRA0510
C* 0  ENRGY - THE TOTAL ENERGY CONSUMPTION IN JOULES .                   *TRA0511
C* 0  T     - THE TRAVELLING TIME TO THE END OF SIMULATION              *TRA0512
C* 0  KL    - THE NUMBER OF THE SAMPLED MEASUREMENTS .                  *TRA0513
C* 0  MNL   - MNL=1 IF THE TRAIN HAS BEEN OPERATED MANUALLY, MNL=0      *TRA0514
C*            IF THE AUTOMATIC OPTION IS USED.                          *TRA0515
C* 0  NI    - THE NUMBER OF THE INTEGRATION STEPS AT TERMINATION.       *TRA0516
C* I  ITRACE- IF ITRACE=1 A SUMMARY OF THE TRAIN STATUS WILL BE         *TRA0517
C*             PRINTED OUT AT EACH SAMPLING PERIOD.                     *TRA0518
C*                                                                      *TRA0519
C*                                                                      *TRA0520
C*   OTHER PARAMETERS : (COMMON BLOCK PARAMETERS)                       *TRA0521
C*    MM    - NUMBER OF TRAIN MEMBERS ,MAX 110(CARS +LOCOMOTIVES).      *TRA0522
C*    NLOC  - NUMBER OF LOCOMOTIVES,MAX 10 UNITS.                       *TRA0523
C*    M     - 2*M THE NUMBER OF THE VELOCITIES AND DISPLACEMENTS        *TRA0524
C*    ML2   - INDEX VECTOR FOR THE MIDDLE LOCOMOTIVES VELOCITY          *TRA0525
C*    ML1   - VECTOR OF THE POSITIONS OF THE LOCOMOTIVS  .              *TRA0526
C*    TCL   - VECTOR OF THE LENGTHS OF THE TRAIN MEMBERS IN METERS.     *TRA0527
C*    W     - VECTOR OF THE WEIGHTS OF THE TRAIN MEMBERS IN METRIC TONS*TRA0528
C*    PMAX  -VECTOR OF THE MAXIMUM POWERS OF THE LOCO IN METRIC H.P.    *TRA0529
C*    TV    -ARRAY  OF THE TRANSITION VELOCITIES OF THE LOCOMOTIVES     *TRA0530
C*          TV(I,1)- THE THROTTLE TRANSITION VELOCITY OF THE I-TH       *TRA0531
C*                   LOCOMOTIVE                                         *TRA0532
C*      TV(I,2),TV(I,3) - THE TRANSITION VELOCITIES OF THE DYNAMIC      *TRA0533
C*                        BRAKING. THESE VELOCITIES ARE IN KM/H .       *TRA0534
C*    SPRGC - THE SPRING CONSTANT OF THE LOCOMOTIVES/CARS COUPLERS.     *TRA0535
C*    DAMPC - THE DAMPING CONSTANT OF THE LOCOMOTIVES/CARS COUPLERS.    *TRA0536
C*    DZ    - THE WIDTH OF THE DEAD ZONE OF THE COUPLERS .              *TRA0537
C*    BFBPC - A VECTOR CONTAINIG THE MAXIMUM BRAKE FORCE OF EACH        *TRA0538
C*            TRAIN MEMBER IN KG (IB) OR EQUIVALENTLY THE BRAKE FORCE   *TRA0539
C*            FOR 3.525 KG/SQR CM BRAKE CYLINDER PRESSURE .             *TRA0540
C*    AC,AL - THE CROSS SECTION AREA OF THE CARS AND LOCOMOTIVES        *TRA0541
C*            RESPECTIVELY  .                                           *TRA0542
C*    NAXC  -  THE NUMBER OF CARS AXLES. USUALLY= 4  .                  *TRA0543
C*    NAXL  -  VECTOR CONAINING THE NUMBER OF AXLES OF EACH LOCOMOTIVE.*TRA0544
C*    OFFLN- ARRAY OF DIMENSION(25,6)                                   *TRA0545
C*    OFFLN(I,1) LENGTH OF THE I-TH TERRAIN SECTION.                    *TRA0546
C*    OFFLN(I,2) GRADE  .........    ....  ...... ...                   *TRA0547
C*    OFFLN(I,3) VELOCITY .... .... .... ..... .... ( AT ITS START)     *TRA0548
C*    OFFLN(I,4) INDIX OF THE VELOCITY OF THE SECTION;                  *TRA0549
C*               INDIX=0.0 IF THE VELOCITY IS CONSTANT,                 *TRA0550
C*               INDIX=1.  IF THE VELOCITY IS CHANGING LINEARLY.        *TRA0551
C*    OFFLN(I,5) APPROXIMATE TOTAL AIR AND ACCELERATION RESISTANCE.     *TRA0552
C*    OFFLN(I,6) APPROXIMATE TOTAL GRAVITATIONAL FORCES .               *TRA0553
C*      NS       NUMBER OF TERRAIN SECTIONS .                            TRA0554
C*       X     : THE X AXIS VECTOR , CONTAINS THE TIME INCREMENTS .     *TRA0555
C*       VDISP : A STORAGE FOR UP TO 6 DIFFERENT VELOCITY OBSERVATION  *TRA0556
C*               POINTS,AND UP TO 300 SAMPLES FOR EACH POINT.           *TRA0557
C*      IVDISP: INDEX VECTOR OF THE VELOCITY MEASUREMENT POINTS ,       *TRA0558
C*      DDISP : A STORAGE FOR UP TO 6 DIIFERENT DIAPLACEMENT OBSERVAT-*TRA0559
C*              ION POINTS ,AND UP TO 300 SAMPLES FOR EACH POINT.       *TRA0560
C*     IDDISP :.. . . . . . . . . .DISPLACEMENTS , . . . . . . . .      *TRA0561
C*      TRDIST: A RECORD FOR THE TRAVELED DISTANCE AGAINST TIME.        *TRA0562
C*      EBDISP: A STORAGE FOR UP TO 6 DIFFERENT OBSERVATION POINTS      *TRA0563
C*              FOR THE EFFECTIVE (ACTUAL) BRAKE FORCES, AND UP TO 300*TRA0564
C*              SAMPLES FOR EACH POINT .                                *TRA0565
C*     IBDISP :   INDEX VECTOR OF   BRAKES MEASUREMENT POINTS.          *TRA0566
C*      TBDISP: SIMILAR TO EBDISP,BUT FOR THE TARGET BRAKE FORCES AT    *TRA0567
C*              THE SAME OPSEVATION POINTS.                             *TRA0568
C*      TTDISP: A STORAGE FOR THE TARGET THROTTLES DISPLAY.             *TRA0569
C*      ETDISP: A STORAGE FOR THE EFFECTIVE THROTTLE DISPLAY ,          *TRA0570
C*                                                                      *TRA0571
C*      NUM   : THE NUMBER OF THE OBSEVATION POINTS ALONG THE TRAIN.    *TRA0572
C*      ITHRT :. . . . . . . . . LOCOMOTIVES WHOSE THROTTLE IS  TO      *TRA0573
C*              BE PLOTTED.                                             *TRA0574
C*                                                                      *TRA0575
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C*     Y          : STATE VECTOR                                        *TRA0576
C*     Y(1)       : IS THE CLOCK                                        *TRA0577
C*     Y(2) TO Y(M): IS THE STATE VECTOR OF THE CARS VELOCITIES AND     *TRA0578
C*                    THE INTER CARS DISPLACEMENTS .                    *TRA0579
C*                  Y(2*K) IS THE VELOCITY OF THE K-TH TRAIN MEMBER,    *TRA0580
C*                  Y(2*K+1) IS THE DISPLACEMENT BETWEEN THE K-TH AND   *TRA0581
C*                   THE K+1-TH MEMBERS.                                *TRA0582
C* Y(M+MM+1) TO Y(M+MM+6) ARE AUXILIARY STATES APPEAR BETWEEN CARS      *TRA0583
C*  AND LOCOMOTIVES DUE TO THE DIFFERENCE IN THE COUPLERS PROPERTIES.   *TRA0584
C*                                                                      *TRA0585
C*        DYDX  : VECTOR OF THE DERIVATIVES OF THE STATES .             *TRA0586
C*        W     : AUXILIARY VECTOR NEEDED FOR INTEGRATION .             *TRA0587
C*        V     : GRAVITATIONAL VECTOR     .                            *TRA0588
C*        R     : ROLLING RESISTANCE VECTOR .                           *TRA0589
C*        TTHRT :  VECTOR OF THE TARGET THROTTLING FORCE IN NEWTONS.    *TRA0590
C*        EFTHRT:  THE EFFECTIVE THROTTLE AFTER QUANTIZATION.           *TRA0591
C*        TBF   : VECTOR OF THE TARGET BRAKE FORCE .                    *TRA0592
C*        EFABRK: VECTOR OF THE EFFECTIVE BRAKE FORCES WHEN THE BUILD   *TRA0593
C*                UP AND RELEASE TIME CONSTANTS ARE CONSIDERED.         *TRA0594
C*     SIGMAC,SIGMAL - THE INVERSE OF THE TIME CONSTANT OF THE BRAKE    *TRA0595
C*              CYLINDER RELEASE OF CARS AND LOCOMOTIVES RESPECTIVELY.  *TRA0596
C*     ALPHAC,ALPHAL - THE RATE OF INCREASE OF THE BRAKE CYLINDER       *TRA0597
C*       PRESSURE OF THE CARS AND LOCOMOTIVES RESPECTIVELY KG/SQRCM*SEC *TRA0598
C*     DMAX,VMAX- THE MAXIMUM PERMISSIBLE INTECARS DISPLACEMENT AND     *TRA0599
C*                VELOCITY RESPECTIVELY (IN METERS AND METER/SEC.)      *TRA0600
C*     DELX       - TEMPORARY STORAGE USED BY START.                    *TRA0601
C*                                                                      *TRA0602
C*     ALL THE PARAMETERS IN BLK50 ARE TEMPORARY STORAGES THAT MAY      *TRA0603
C*     BE USED BY THE USER'S TRAIN CONTROL SUBROUTINE.                  *TRA0604
C*                                                                      *TRA0605
C*                                                                      *TRA0606
C* THE INTERNAL PARAMETERS :                                            *TRA0607
C*     KL  -       COUNTER FOR THE NUMBER OF SAMPLES .                  *TRA0608
C*     NI  -       CONTAINS THE NUMBER OF THE INTEGRATION STEPS.        *TRA0609
C*     IG  - INDIX OF THE CURRENT GRADE SECTION                         *TRA0610
C*     S   - THE TRAVELLED DISTANCE TILL NOW IN METERS .                *TRA0611
C*     GL  - THE TOTAL LENGTH OF THE GRADE SECTIONS UP AND INCLUDING    *TRA0612
C*           THE CURRENT ONE   IN KM.                                   *TRA0613
C*     T   - THE TRAVELLED TIME.                                        *TRA0614
C*     VREF- THE CURRENT REFERENCE SPEED.                               *TRA0615
C*    DVDS - THE RATE OF CHANGE OF THE TARGET VELOCITY WITH DISTANCE.   *TRA0616
C*    TRNL - THE TOTAL TRAIN LENGHT IN METERS.                          *TRA0617
C*    NINT - THE REQUIRED NUMBER OF THE INEGRATION STEPS .              *TRA0618
C*    ISTEP- INDEX FOR THE CURRENT CONTROL STEP.IF ISTEP IS RETURND 1000*TRA0619
C*           BY THE USER'S CONTROL PROGRAM ,THE SIMULATION TERMINATES.  *TRA0620
C*                                                                      *TRA0621
C*                                                                      *TRA0622
C*  COMMON       : BLK1,BLK2,BLK3,BLK4,BLK5,BLK6,BLK9,BLK10,BLK11,       TRA0623
C*                 BLK12,BLK13           .                              *TRA0624
C*                                                                      *TRA0625
C*  ROUTINE NEEDED : INITT,VRF,GRV,ROLL,CNTRL,ADJST,EFFTRT,RKGILL       *TRA0626
C*                                                                      *TRA0627
C*  CALLED BY      : THE MAIN PROGRAM                                   *TRA0628
C*                                                                      *TRA0629
C*  WRITTEN BY     : MOUSTAFA ELSHAFEI AHMED                            *TRA0630
C*                                                                      *TRA0631
C*  DATE           : JULY 1982 .                                        *TRA0632
C*                                                                      *TRA0633
C***********************************************************************TRA0634
      SUBROUTINE START(STEP,TTIME,SMPLT,ENDD,S,ENRGY,T,KL,MNL,NI,        TRA0635
     +                ITRACE)                                            TRA0636
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                              TRA0637
      COMMON/BLK2/TCL( 70),W( 70),PMAX(10),TV(10,3)                      TRA0638
      COMMON/BLK3/SPRGC( 70),DAMPC( 70),DZ,BFBPC( 70)                    TRA0639
      COMMON/BLK4/ AC,AL,NAXC,NAXL(10)                                   TRA0640
      COMMON/BLK5/OFFLN(25,6),NS                                         TRA0641
      COMMON/BLK7/X(200),VDISP(200,7),IVDISP(6),DDISP(200,6),IDDISP(6),  TRA0642
     +           TRDIST(200),EBDISP(200,6),IBDISP(6),TBDISP(200,6),
     +           TTDISP(200,3),ETDISP(200,3),NUM,ITHRT(3)
      COMMON/BLK9/Y(230),DYDX(230),Q(230),V( 70),R( 70)
      COMMON/BLK10/EFTHRT(3),ITHRT(3)
      COMMON/BLK11/FLGBRK( 70),IBF( 70)
      COMMON/BLK12/SIGMAC,SIGMAL,ALPHAC,ALPHAL
      COMMON/BLK13/DMAX,VMAX,DELX
C
C
C
      REAL DVDS,VL,GL,VREF,Y1,S
      INTEGER    IG,KRUN1,KRUN2,KRUN3,KL,MNL,NI,ISTEP,ITRACE
C
C
C    CHANGE THE TRANSITION VELOCITIES TO METERS/SEC.
C
      DO 4 I=1,NLOC
      DO 4 J=1,3
      TV(I,J)=TV(I,J)/3.6
4     CONTINUE
C
      IG=0
      GL=0.0
      TRDIST(1)=0.0
      Y1=0.0
      ENDD=ENDD*1000.0
C
C    CONVERT THE LOCO MAX POWER FROM METRIC H.P. TO NEWTON M/SEC.
C
      DO 5  I=1,NLOC
5     PMAX(I)=PMAX(I)*746.0
C
C     THE BRAKE FORCE CONSTANT=MAX BRAKE FORCE/3.525 KG/SQ.CM.
C     CONVERT ALSO THE WEIGHTS FROM TONS TO KG.
C
      TRNL=0.0
      DO 6 J=1,MM
      TRNL=TRNL+TCL(J)
      W(J)=W(J)*1000.
6     BFBPC(J)=BFBPC(J)/3.525
C
      MPMM=M+MM+NLOC
      DO 7 J=1,MPMM
7     Q(J)=0.0
C
C     CONVERT THE REFERENCE VELOCITY FROM KM/H TO M/SEC.
C
      DVDS=0.
      VREF=OFFLN(1,3)/3.6
      S=0.0
      ENRGY=0.0
      ISTEP=1
      RT=TTIME/STEP
      NINT=INT(RT)
      KL=1
      DELX=0.0
C
      CALL INIT(VREF)
C
C    THE MAIN SIMULATION LOOP
C
      DO 1000 I=1,NINT
      T=Y(1)
      DO 11 J=1,NLOC
11    IF(Y(ML2(J)).LT. 0.0 ) GO TO 12
      GO TO 20
12    CONTINUE
```

```
      WRITE(4,1001)
1001  FORMAT(/,5X,' ERROR !..................',/,10X,
     +        'ONE OF THE LOCOMOTIVES IS GOING BACKWORD')
       RETURN
20    CONTINUE
      NI=I
40    CONTINUE
C
C    FIND THE CURRENT REFERENCE VELOCITY AND TERRAIN SECTION
C
      S1=S+TRNL
      CALL VRF(S1,GL,IG,VREF,Y(2),Y(1),Y1,DVDS,TRNL)
C
      IF(VREF.GE. 0.0)GO TO 50
      WRITE(4,1002)
1002  FORMAT(5X,'ERROR ! ......',/,'    THE REF. SPEED IS NEGATIVE')
      RETURN
50    CONTINUE
C     UPDATE THE  GRAVITATIONAL FORCES
C     UPDATING IS PERFORMED EVERY 20 METERS.
C
C
      IF(S.LT.DELX*20.) GO TO 51
      CALL GRV(IG,S1,GL)
       DELX=DELX+1.
       CALL ROLL
51     CONTINUE
C
C     FIND THE TARGET THROTTLE AND THE TARGET BRAKE FORCES
C     USING THE USER SUBPROG CNTRL
C
       CALL CNTRL(S,Y(1),VREF,MNL,ISTEP,IG,GL,TRNL)
       IF(ISTEP.EQ.1000)RETURN
C
C     IF MNL=1 IT IS MANUAL CONTROL
C
      IF(MNL.NE.1) GO TO 60
      CALL ADJST
      GO TO 61
60    CONTINUE
C     FIND THE EFFECTIVE THROTTLE/DYNAMIC BRAKE TAKING IN
C     CONSIDERATION THE QUANTIZED NATURE OF THE THROTTLE
C     AND THE PARABOLIC RELATION  FORCE-VELOCITY
C
      CALL EFFTRT
61    CONTINUE
C
C     FIND THE EFFECTIVE AIR BRAKE
C     EFABRK(K) IS RETURNED IN NEWTONS
C
      DO 70 K=1,MM
      IF(Y(M+K).GT.3.525) Y(M+K)=3.525
C     THE SATURATION BRAKE PRESSURE IS TAKEN = 3.525 KG/SQ.CM.
      EFABRK(K)=Y(M+K)*BFBPC(K)*9.81
70    CONTINUE
      IF(I.EQ.1) GO TO   80
      IF(Y(1).LT.(KL-1)*SMPLT) GO TO 110
80    CONTINUE
C
C      STORAGE OF DISPLAY QUANTITIES
C
      X(KL)=Y(1)
      VDISP(KL,1)=VREF
      DO 90 J=1,NUM
      J1=J+1
      VDISP(KL,J1)=Y(IVDISP(J))
      DDISP(KL,J)=Y(IDDISP(J))
```

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                                                                        TRA0777
      IBDIST(KL)=S                                                      TRA0778
      EBDISP(KL,J)=EFABRK(IBDISP(J))                                    TRA0779
      TBDISP(KL,J)=TBF(IBDISP(J))                                       TRA0780
90    CONTINUE                                                          TRA0781
      DO 100 J=1,3                                                      TRA0782
      TTDISP(KL,J)= TTHRT(ITHRT(J))                                     TRA0783
      ETDISP(KL,J)=EFTHRT(ITHRT(J))                                     TRA0784
100   CONTINUE                                                          TRA0785
      IF(ITRACE.EQ.0) GO TO 109                                         TRA0786
      WRITE(4,101) KL                                                   TRA0787
101   FORMAT(10X,'SAMPLE NUMBER   ',I4,/)                               TRA0788
C                                                                       TRA0789
      WRITE(4,102) Y(1),S                                               TRA0790
102   FORMAT(' TIME IN SECONDS = ',F8.2,10X,'DISTANCE IN METERS = ',    TRA0791
     + F12.2)                                                           TRA0792
C                                                                       TRA0793
      WRITE(4,103)GL,IG,VREF                                            TRA0794
103   FORMAT('  GL = ',F10.4,'  IG= ',I4,' VREF = ',F8.2)               TRA0795
C                                                                       TRA0796
      WRITE(4, 104) ( Y(ML2(J)),  J=1,NLOC)                             TRA0797
104   FORMAT(1X,' THE LOCOMOTIVES VELOCITY IN METERS/SEC.....',/,       TRA0798
     +  3(5X,F10.2))                                                    TRA0799
C                                                                       TRA0800
      WRITE(4, 105) (EFTHRT(J),J=1,NLOC)                                TRA0801
105   FORMAT(/,' THE LOCOMOTIVES EFFECTIVE THROTTLE ...... ',/,         TRA0802
     +  3(5X,1PE14.7))                                                  TRA0803
C                                                                       TRA0804
      WRITE(4, 106) ( EFABRK(IBDISP(J)),J=1,NUM)                        TRA0805
106   FORMAT(/,' THE BRAKE FORCE AT THE MEASURMENT POINTS.....',/,      TRA0806
     + 3(5X,1PE14.7))                                                   TRA0807
C                                                                       TRA0808
109   KL=KL+1                                                           TRA0809
110   CONTINUE                                                          TRA0810
C                                                                       TRA0811
      Y1=Y(1)                                                           TRA0812
C     Y(1)  IS TRAVELLED TIME                                           TRA0813
C                                                                       TRA0814
C     PERFORM THE INTEGRATION USING THE RUNGE-KUTA-GILL METHOD          TRA0815
C                                                                       TRA0816
      CALL RKGILL(STEP)                                                 TRA0817
250   CONTINUE                                                          TRA0818
C                                                                       TRA0819
      S=S+Y(2)*(Y(1)-Y1)                                                TRA0820
C     S  IS THE TRAVELLED DISTANCE IN METERS                            TRA0821
C                                                                       TRA0822
C     COMPUTE THE TOTAL ENERGY                                          TRA0823
C                                                                       TRA0824
      EFPR=0.0                                                          TRA0825
      DO 255 J=1,NLOC                                                   TRA0826
      IF(EFTHRT(J).GT..0 )        EFPR  =EFPR+EFTHRT(J)*Y(ML2(J))       TRA0827
255   CONTINUE                                                          TRA0828
      ENRGY=ENRGY+EFPR*(Y(1)-Y1)                                        TRA0829
      MM1=M-1                                                           TRA0830
      DO 260 K=3,MM1,2                                                  TRA0831
      K2=K-1                                                            TRA0832
      IF(ABS(Y(K2)-Y(K2+2)).GE.VMAX) GO TO 275                          TRA0833
      IF(Y(K).GE.DMAX) GO TO 270                                        TRA0834
260   CONTINUE                                                          TRA0835
      GO TO 280                                                         TRA0836
270   CONTINUE                                                          TRA0837
      WRITE(4,1003) K1                                                  TRA0838
1003  FORMAT(5X,' ERROR ! ............',/,5X,' THE DISPLACEMENT NO.',   TRA0839
     +       I5,' EXCEEDS THE DANGEROUS LEVEL . ')                      TRA0840
      RETURN                                                            TRA0841
275   K2=K2/2                                                           TRA0842
      K3=K2+1                                                           TRA0843
      WRITE(4,1004) K2,K3
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1004  FORMAT(5X,' DANGER IMPACT MAY OCCUR BETWEEN CAR NO. ',I3,          TRA0844
     +    ' AND CAR NO. ',I3)                                           TRA0845
       RETURN                                                           TRA0846
280   CONTINUE                                                          TRA0847
C                                                                       TRA0848
      IF(S .GE.ENDD) RETURN                                             TRA0849
      IF(KL.GE. 300) RETURN                                             TRA0850
      IF(IG.LT.NS) GO TO 300                                            TRA0851
      IF(S1.GE.(GL*1000.))    RETURN                                    TRA0852
300   CONTINUE                                                          TRA0853
C                                                                       TRA0854
1000  CONTINUE                                                          TRA0855
      RETURN                                                            TRA0856
      END                                                               TRA0857
C*********************************************************************** TRA0858
C*                                                                    * TRA0859
C*  SUBROUTINE   RKGILL                                               * TRA0860
C*                                                                    * TRA0861
C*  FUNCTION     PERFORMS INTEGRATION FOR THE VECTOR D.E. DYDX=F(Y,U) * TRA0862
C*               USING THE GILL-RUNGE-KUTA METHOD.                    * TRA0863
C*                                                                    * TRA0864
C*  USAGE        CALL RKGILL(STEP).                                   * TRA0865
C*                                                                    * TRA0866
C*  PARAMETRES                                                        * TRA0867
C*               STEP  - LENGTH OF THE INTEGRATION STEP IN SECONDS.   * TRA0868
C*                                                                    * TRA0869
C*  OTHER PARAMETERS   - SEE SUBROUTINE START.                        * TRA0870
C*                                                                    * TRA0871
C*  COMMON             BLK1,BLK9                                      * TRA0872
C*                                                                    * TRA0873
C*  ROUTINE NEEDED     DER                                            * TRA0874
C*                                                                    * TRA0875
C*  CALLED BY          START                                          * TRA0876
C*                                                                    * TRA0877
C*  WRITTEN BY         WONG,AND GRUBER. MODIFIED BY M.E.AHMED         * TRA0878
C*                                                                    * TRA0879
C*  DATE               LAST VERSION   APRIL,1982.                     * TRA0880
C*                                                                    * TRA0881
C*********************************************************************** TRA0882
      SUBROUTINE RKGILL(STEP)                                           TRA0883
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                             TRA0884
      COMMON/BLK9/Y(230),DYDX(230),Q(230),V( 70),R( 70)                 TRA0885
      DIMENSION A(4),B(4),C(4)                                          TRA0886
      DATA A/.5,.2928932,1.707107,.16666667/,                           TRA0887
     +B/2.0,1.0,1.0,2.0/,C/0.5,0.2928932,1.707107,0.5/                  TRA0888
      DO 10 J=1,4                                                       TRA0889
C     CALL ROLL                                                         TRA0890
      CALL DER                                                          TRA0891
      AA=A(J)                                                           TRA0892
      BB=B(J)                                                           TRA0893
      CC=C(J)                                                           TRA0894
      MMM=M+MM+2*NLOC                                                   TRA0895
      DO 10 I=1,MMM                                                     TRA0896
C                                                                       TRA0897
C                                                                       TRA0898
      ZZ=AA*(DYDX(I)-BB*Q(I))                                           TRA0899
      Y(I)=Y(I) + STEP*ZZ                                               TRA0900
10    Q(I)=Q(I) + 3.0*ZZ - CC*DYDX(I)                                   TRA0901
      RETURN                                                            TRA0902
      END                                                               TRA0903
C                                                                       TRA0904
C                                                                       TRA0905
C*********************************************************************** TRA0906
C*                                                                    * TRA0907
C*  SUBROUTINE   DER                                                  * TRA0908
C*                                                                    * TRA0909
C*  FUNCTION     COMPUTES THE RIGHT HAND SIDE OF THE NONLINEAR        * TRA0910
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C*               DIFFERENTIAL EQUATON DYDX=F(Y,Q).                       * TRA0911
C*                                                                       * TRA0912
C*   USAGE         CALL DER                                              * TRA0913
C*                                                                       * TRA0914
C*   PARAMETRES                                                          * TRA0915
C*     MM        NUMBER OF TRAIN MEMBERS ,MAX 110(CARS +LOCOMOTIVES).    *TRA0916
C*     NLOC  - NUMBER OF LOCOMOTIVES,MAX 10 UNITS.                       *TRA0917
C*     M     - 2*M THE NUMBER OF THE VELOCITIES AND DISPLACEMENTS        *TRA0918
C*     ML2   - INDIX VECTOR FOR THE MIDDLE LOCOMOTIVES VELOCITY          *TRA0919
C*     ML1   - VECTOR OF THE POSITIONS OF THE LOCOMOTIVS  .              *TRA0920
C*     TCL   - VECTOR OF THE LENGTHS OF THE TRAIN MEMBERS IN METERS.     *TRA0921
C*     W     - VECTOR OF THE WEIGHTS OF THE TRAIN MEMBERS IN METRIC TONS*TRA0922
C*     PMAX  -VECTOR OF THE MAXIMUM POWERS OF THE LOCO IN METRIC H.P.    *TRA0923
C*     TV    -ARRAY  OF THE TRANSITION VELOCITIES OF THE LOCOMOTIVES     *TRA0924
C*           TV(I,1)  THE THROTTLE TRANSITION VELOCITY OF THE I-TH       *TRA0925
C*                    LOCOMOTIVE                                         *TRA0926
C*       TV(I,2),TV(I,3) - THE TRANSITION VELOCITIES OF THE DYNAMIC      *TRA0927
C*                            BRAKING. THESE VELOCITIES ARE IN KM/H .    *TRA0928
C*     SPRGC - THE SPRING CONSTANT OF THE LOCOMOTIVES/CARS COUPLERS.     *TRA0929
C*     DAMPC - THE DAMPING CONSTANT OF THE LOCOMOTIVES/CARS COUPLERS.    *TRA0930
C*     DZ    - THE WIDTH OF THE DEAD ZONE OF THE COUPLERS .              *TRA0931
C*     BFBPC - A VECTOR CONTAINIG THE MAXIMUM BRAKE FORCE OF EACH        *TRA0932
C*             TRAIN MEMBER IN KG ( R) OR EQUIVALENTLY THE BRAKE FORCE   *TRA0933
C*             FOR 3.525 KG/SQR CM BRAKE CYLINDER PRESSURE .             *TRA0934
C*     AC,AL - THE CROSS SECTION AREA OF THE CARS AND LOCOMOTIVES        *TRA0935
C*             RESPECTIVELY   .                                          *TRA0936
C*     NAXC  -  THE NUMBER OF CARS AXLES. USUALLY= 4  .                  *TRA0937
C*     NAXL  -  VECTOR CONAINING THE NUMBER OF AXLES OF EACH LOCOMOTIVE.*TRA0938
C*     Y        : STATE VECTOR                                           *TRA0939
C*     Y(1)     : IS THE CLOCK                                           *TRA0940
C*     Y(2) TO Y(M): IS THE STATE VECTOR OF THE CARS VELOCITIES AND      *TRA0941
C*                   THE INTER CARS DISPLACEMENTS .                      *TRA0942
C*                 Y(2*K) IS THE VELOCITY OF THE K-TH TRAIN MEMBER,      *TRA0943
C*                 Y(2*K+1) IS THE DISPLACEMENT BETWEEN THE K-TH AND     *TRA0944
C*                  THE K+1-TH MEMBERS.                                  *TRA0945
C* Y(M+MM+1) TO Y(M+MM+6) ARE AUXILIARY STATES APPEAR BETWEEN CARS       *TRA0946
C*  AND LOCOMOTIVES DUE TO THE DIFFERENCE IN THE COUPLERS PROPERTIES.    *TRA0947
C*                                                                       *TRA0948
C*        DYDX  : VECTOR OF THE DERIVATIVES OF THE STATES .              *TRA0949
C*        Q     : AUXILIARY VECTOR NEEDED FOR INTEGRATION .              *TRA0950
C*        V     : GRAVITATIONAL VECTOR     .                             *TRA0951
C*        R     : ROLLING RESISTANCE VECTOR .                            *TRA0952
C*        TTHRT :  VECTOR OF THE TARGET THROTTLING FORCE IN NEWTONS.     *TRA0953
C*        EFTHRT:  THE EFFECTIVE THROTTLE AFTER QUANTIZATION.            *TRA0954
C*        TBF   : VECTOR OF THE TARGET BRAKE FORCE .                     *TRA0955
C*        EFABRK: VECTOR OF THE EFFECTIVE BRAKE FORCES WHEN THE BUILD    *TRA0956
C*                UP AND RELEASE TIME CONSTANTS ARE CONSIDERED.          *TRA0957
C*     SIGMAC,SIGMAL - THE INVERSE OF THE TIME CONSTANT OF THE BRAKE     *TRA0958
C*              CYLINDER RELEASE OF CARS AND LOCOMOTIVES RESPECTIVELY.   *TRA0959
C*     ALPHAC,ALPHAL - THE RATE OF INCREASE OF THE BRAKE CYLINDER        *TRA0960
C*                                                                       * TRA0961
C*                                                                       * TRA0962
C*   COMMON              BLK1,BLK2,BLK3,BLK4,BLK9,BLK10,BLK11,BLK12      * TRA0963
C*                                                                       * TRA0964
C*   ROUTINE NEEDED      FF2,BRAK                                        * TRA0965
C*                                                                       * TRA0966
C*   CALLED BY           RKGILL                                          * TRA0967
C*                                                                       * TRA0968
C*   WRITTEN BY          MOUSTAFA ELSHAFEI AHMED                         * TRA0969
C*                                                                       * TRA0970
C*   DATE                LAST VERSION   JULY, 1982.                      * TRA0971
C*                                                                       * TRA0972
C************************************************************************ TRA0973
      SUBROUTINE DER                                                      TRA0974
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                               TRA0975
      COMMON/BLK2/TCL( 70),W( 70),PMAX(10),TV(10,3)                       TRA0976
      COMMON/BLK3/SPRGC( 70),DAMPC( 70),DZ,BFBPC( 70)                     TRA0977
      COMMON/BLK4/ AC,AL,NAXC,NAXL(10)                                  TRA0978
      COMMON/BLK9/Y(230),DYDX(230),W(230),V( 70),R( 70)                 TRA0979
      COMMON/BLK10/EFTHRT(10),TTHRT(10)                                 TRA0980
      COMMON/BLK11/EFABRK( 70),TBF( 70)                                 TRA0981
      COMMON/BLK12/SIGMAC,SIGMAL,ALFHAC,ALFHAL                          TRA0982
      DYDX(1)=1.0                                                       TRA0983
C                                                                       TRA0984
C                                                                       TRA0985
C    FRONT LOCO                                                         TRA0986
C                                                                       TRA0987
C                                                                       TRA0988
      MFMM=M+MM                                                         TRA0989
      INN=1                                                             TRA0990
      F1=0.0                                                            TRA0991
      MMM1=MM-1                                                         TRA0992
      DO 10 I=1,MMM1                                                    TRA0993
      I2=I*2                                                            TRA0994
      CALL FF2(INN,I,F2)                                                TRA0995
      DYDX(I2)=F1-F2-V(I)-R(I)-EFABRK(I)                                TRA0996
      DYDX(I2)=DYDX(I2)/W(I)                                            TRA0997
      F1=F2                                                             TRA0998
      DYDX(I2+1)=Y(I2)-Y(I2+2)                                          TRA0999
10    CONTINUE                                                          TRA1000
      DYDX(M)=F1-V(MM)-R(MM)-EFABRK(MM)                                 TRA1001
      DYDX(M)=DYDX(M)/W(MM)                                             TRA1002
      DO 20 I=1,NLOC                                                    TRA1003
20    DYDX(ML2(I))=DYDX(ML2(I))+EFTHRT(I)/W(ML1(I))                     TRA1004
C                                                                       TRA1005
C    AIR BRAKING BUILD UP                                               TRA1006
C                                                                       TRA1007
      DO 30 I=1,MM                                                      TRA1008
      CALL BRAK(TBF(I),BFBPC(I),Y(M+I),DYDX(M+I),ALFHAC,SIGMAC)         TRA1009
30    CONTINUE                                                          TRA1010
C                                                                       TRA1011
      DO 40 I=1,NLOC                                                    TRA1012
      I1=ML1(I)                                                         TRA1013
      CALL BRAK(TBF(I1),BFBPC(I1),Y(M+I1),DYDX(M+I1),ALFHAL,SIGMAL)     TRA1014
40    CONTINUE                                                          TRA1015
C                                                                       TRA1016
      RETURN                                                            TRA1017
      END                                                               TRA1018
C                                                                       TRA1019
C*********************************************************************** TRA1020
C*                                                                    * TRA1021
C*  SUBROUTINE   FF2                                                  * TRA1022
C*                                                                    * TRA1023
C*  FUNCTION     COMPUTES COMPUTES COUPLER FORCES.AN EXTRA STATE IS   * TRA1024
C*               CREATED FOR EVERY JOINT WHOSE COUPLERS ARE NOT       * TRA1025
C*               IDENTICAL.                                           * TRA1026
C*                                                                    * TRA1027
C*  USAGE        CALL FF2(INN,I,F2)                                   * TRA1028
C*                                                                    * TRA1029
C*  PARAMETRES                                                        * TRA1030
C*       INN       - THE NUMBER OF JOINTS OF DIFFERENT COUPLERS       * TRA1031
C*                   CHARACTERISTICS PROCESSED SO FAR.                *TRA1032
C*        I        - THE POSITION OF THE COUPLER TO BE PROCESSED.     *TRA1033
C*        F2       - THE COUPLER FORCE.                               *TRA1034
C*                                                                    *TRA1035
C*  OTHER PARAMETERS   SEE SUBROUTINES DER,OR START                   * TRA1036
C*                                                                    *TRA1037
C*  COMMON             BLK1,BLK2,BLK3,BLK4,BLK9,BLK10,BLK11,BLK12     * TRA1038
C*                                                                    * TRA1039
C*  ROUTINE NEEDED     FN,DEAD                                        * TRA1040
C*                                                                    * TRA1041
C*  CALLED BY          DER                                            * TRA1042
C*                                                                    * TRA1043
C*  WRITTEN BY         MOUSTAFA ELSHAFEI AHMED                        * TRA1044
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C*                                                                      * TRA1045
C*   DATE                 JULY,198[illegible]                                       *TRA1046
C*                                                                      *TRA1047
C*                                                                      *TRA1048
C***********************************************************************TRA1049
       SUBROUTINE FF2(INN,I,F2)                                         TRA1050
C                                                                       TRA1051
C                                                                       TRA1052
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                             TRA1053
      COMMON/BLK2/TCL( 70),W( 70),PMAX(10),TV(10,3)                     TRA1054
      COMMON/BLK3/SPRGC( 70),DAMPC( 70),DZ,RFRPC( 70)                   TRA1055
      COMMON/BLK9/Y(230),DYDX(230),Q(230),V( 70),R( 70)                 TRA1056
      COMMON/BLK10/EFTHRT(10),TTHRT(10)                                 TRA1057
      COMMON/BLK11/EFABRK( 70),TBF( 70)                                 TRA1058
C                                                                       TRA1059
      MPMM=M+MM                                                         TRA1060
      I2=I*2                                                            TRA1061
      IF(SPRGC(I).NE.SPRGC(I+1).OR.DAMPC(I).NE.DAMPC(I+1))              TRA1062
     +  GO TO 50                                                        TRA1063
      F2=DAMPC(I)*(Y(I2)-Y(I2+2))*FN(Y(I2+1))+DEAD(Y(I2+1),SPRGC(I))    TRA1064
      F2=F2/2.                                                          TRA1065
      RETURN                                                            TRA1066
C                                                                       TRA1067
50    DYDX(MPMM+INN)=-(SPRGC(I)+SPRGC(I+1))*Y(MPMM+INN)+                TRA1068
     + DEAD(Y(I2+1),SPRGC(I+1))+                                        TRA1069
     +                 FN(Y(I2+1))*DAMPC(I+1)*(Y(I2)-Y(I2+2))           TRA1070
      DYDX(MPMM+INN)=DYDX(MPMM+INN)/(SPRGC(I)+SPRGC(I+1))               TRA1071
      F2= SPRGC(I)*Y(MPMM+INN)+DAMPC(I)*DYDX(MPMM+INN)                  TRA1072
      INN=INN+1                                                         TRA1073
      RETURN                                                            TRA1074
      END                                                               TRA1075
```

ATTACHMENT B.II

PROGRAM DESCRIPTION

```
C**************************************************************************1
C**************************************************************************1
C*                                                                        *1
C*                                                                        *1
C*                                                                        *1
C*            FFFFFFFFFF           TTTTTTTTTT             SSSSSS          *1
C*            FFFFFFFFFF           TTTTTTTTTT            SSSSSSSS         *1
C*            FF                       TT               SSS   SSS         *1
C*            FF                       TT                SSS              *1
C*            FF                       TT                  SSS            *1
C*            FFFFFFF                  TT                    SSS          *1
C*            FFFFFFF                  TT                      SS         *1
C*            FF                       TT               SSS   SSS         *1
C*            FF                       TT                SSSSSSSS         *1
C*            FF                       TT                 SSSSSS          *1
C*                                                                        *1
C*            FREIGHT TRAIN SIMULATOR                                     *1
C*                                                                        *1
C*            VERSION THREE OF THE F.T.S.                                 *1
C*            --------------------------                                  *1
C*                                                                        *1
C*                                                                        *1
C*                                                                        *1
C**************************************************************************1
C**************************************************************************1
C*                                                                        *1
C*    THIS IS THE MAIN PROGRAM                                            *1
C*                                                                        *1
C*    FUNCTION    :READS THE USER'S DATA ,COVERTS THE UNITS ,AND          *1
C*                 PRINIS OUT THE TITLE AND A SUMMARY OF THE RUN.         *1
C*                                                                        *1
C*    PARAMETERS : (I FOR INPUT, O FOR OUTPUT)                            *1
C* I  STEP  - THE INTEGRATION STEP IN SECONDS                             *1
C* I  TTIME - THE TRAVELING TIME IN SECONDS .                             *1
C* I  SMPLT - THE MEASUREMENT SAMPLING PERIOD  IN SECONDS                 *1
C* I  ENDD  - A DEFAULT FOR THE MAXIMUM TRAVELLED DISTANCE IN KM          *1
C* O  S     - THE TRAVELLFD DISTANCE IN METERS WHEN THE SIMULATION        *1
C*            IS TERMINATED .                                             *1
C* O  ENRGY - THE TOTAL ENERGY CONSUMPTION IN JOULES .                    *1
C* O  T     - THE TRAVELLING TIME TO THE END OF SIMULATION                *1
C* O  NI    - THE NUMBER OF THE INTEGRATION STEPS AT TERMINATION.         *1
C* O  KL    - THE NUMBER OF THE SAMPLED MEASUREMENTS .                    *1
C* I  MM    - NUMBER OF TRAIN MEMBERS ,MAX 110(CARS +LOCOMOTIVES)         *1
C* I  NLOC  - NUMBER OF LOCOMOTIVES,MAX 10 UNITS                          *1
C* I  ML1   - VECTOR OF THE POSITIONS OF THE LOCOMOTIVS .                 *1
C* I  M     - 2*M THE NUMRER OF THE VELOCITIES AND DISPLACEMENTS          *1
C* I  ML2   - INDIX VECTOR FOR THE MIDDLE LOCOMOTIVES VELOCITY            *1
C* I  MKS   - MKS=0 IF THE TRAIN DATA IS IN THE BRITSH UNITS              *1
C*            MKS=1 IF THE DATA IS IN THE MKS SYSTEM .                    *1
C* I  TCL   - VECTOR OF THE LENGTHS OF THE TRAIN MEMBERS IN METERS.       *1
C* I  W     - VECTOR OF THE WEIGHTS OF THE TRAIN MEMBERS IN METRIC TONS*1
C* I  PMAX  -VECTOR OF THE MAXIMUM POWERS OF THE LOCO IN METRIC H.P.      *T
C* I  TV    -ARRAY  OF THF TRANSITION VELOCITIES OF THE LOCOMOTIVES       *T
C*           TV(I,1)- THE THROTTLE TRANSITION VELOCITY OF THE I-TH        *T
C*                    LOCOMOTIVE                                          *T
C*      TV(I,2),TV(I,3) - THE TRANSITION VELOCITIES OF THE DYNAMIC        *T
C*                        BRAKING. THESE VELOCITIES ARE IN KM/H           *T
C* I  SPRCC - THE SPRING CONSTANT OF THE LOCOMOTIVES/CARS COUPLERS        *T
C* I  DAMPC - THE DAMPING CONSTANT OF THE LOCOMOTIVES/CARS COUPLERS       *T
C* I  DZ    - THE WIDTH OF THE DEAD ZONE OF THE COUPLERS .                *T
C* I  BFBPC - A VECTOR CONTAINIG THE MAXIMUM BRAKE FORCE OF EACH          *T
C*            TRAIN MEMBER IN KG (IB) OR EQUIVALENTLY THE BRAKE FORCE     *T
C*            FOR 3.525 KG/SQR CM BRAKE CYLINDER PRESSURE .               *T
C* I  AC,AL - THE CROSS SECTION AREA OF THE CARS AND LOCOMOTIVES          *T
C*            RESPECTIVELY                                                *T
C* I  NAXC  -  THE NUMBER OF CARS AXLES. USUALLY= 4 .                     *T
C* I  NAXL  -  VECTOR CONAINING THE NUMBER OF AXLES OF EACH LOCOMOTIVE.   *T
C*                                                                        *T
C* I  OFFLN- ARRAY OF DIMENSION(25,6)                                     *T
C*    OFFLN(I,1) LENGTH OF THE I-TH TERRAIN SECTION.                      *T
C*    OFFLN(I,2) GRADE ........ .... ........ ...                         *T
C*    OFFLN(I,3) VELOCITY ........................ ( AT ITS START)        *T
C*    OFFLN(I,4) INDIX OF THE VELOCITY OF THE SECTION;                    *T
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C*                  INDIX=0 0 IF THE VELOCITY IS CONSTANT.                  *1
C*                  INDIX=1    IF THE VELOCITY IS CHANGING LINEARLY         *1
C* O  OFFLN(I,5) APPROXIMATE TOTAL AIR AND ACCELERATION RESISTANCE          *1
C* O  OFFLN(I,6) APPROXIMATE TOTAL GRAVITATIONAL FORCES                     *1
C* I  IVDISP- INDEX VECTOR OF THE VELOCITY MEASUREMENT POINTS               *1
C* I  IDDISP-                     DISPLACEMENTS                             *1
C* I  IBDISP-                      BRAKES                                   *1
C* I  ITHRT -                     LOCOMOTIVES WHOSE THROTTLE IS             *1
C*             TO BE PLOTTED                                                *1
C* I  SIGMAC,SIGMAL - THE INVERSE OF THE TIME CONSTANT OF THE BRAKE         *1
C*             CYLINDER RELEASE OF CARS AND LOCOMOTIVES RESPECTIVELY        *1
C* I  ALPHAC,ALPHAL - THE RATE OF INCREASE OF THE BRAKE CYLINDER            *1
C*       PRESSURE OF THE CARS AND LOCOMOTIVES RESPECTIVELY KG/SQRCM*SEC     *1
C* I  ITRACE - IF ITRACE=1 , A SUMMARY OF THE TRAIN STATUS IS PRINTED       *1
C*             OUT AFTER EACH SAMPLE TIME                                   *1
C* I  DMAX,VMAX- THE MAXIMUM PERMISSIBLE INTECARS DISPLACEMENT AND          *1
C*                VELOCITY RESPECTIVELY (IN METERS AND METER/SEC )          *1
C*                                                                          *1
C*                                                                          *1
C* THE INTERNAL PARAMETERS                                                  *1
C*    Y          - STATE VECTOR                                             *1
C*    Y(1)         IS THE CLOCK                                             *1
C*    Y(2) TO Y(M)  IS THE STATE VECTOR OF THE CARS VELOCITIES AND          *1
C*                     THE INTER CARS DISPLACEMENTS                         *1
C*                   Y(2*K) IS THE VELOCITY OF THE K-TH TRAIN MEMBER.       *1
C*                   Y(2*K+1) IS THE DISPLACEMENT BETWEEN THE K-TH AND      *1
C*                    THE K+1-TH MEMBERS                                    *1
C* Y(M+MM+1) TO Y(M+MM+6) ARE AUXILIARY STATES APPEAR BETWEEN CARS          *1
C*  AND LOCOMOTIVES DUE TO THE DIFFERENCE IN THE COUPLERS PROPERTIES        *1
C*                                                                          *1
C*        DYDX     VECTOR OF THE DERIVATIVES OF THE STATES                  *1
C*        Q        AUXILIARY VECTOR NEEDED FOR INTEGRATION                  *1
C*        V        GRAVITATIONAL VECTOR                                     *1
C*        R        ROLLING RESISTANCE VECTOR                                *1
C*        TTHRT ,   VECTOR OF THE TARGET THROTTLING FORCE IN NEWTONS        *1
C*        EFTHRT    THE EFFECTIVE THROTTLE AFTER QUANTIZATION               *1
C*        TBF      VECTOR OF THE TARGET BRAKE FORCE                         *1
C*        EFABRK   VECTOR OF THE EFFECTIVE BRAKE FORCES WHEN THE BUILD      *1
C*                 UP AND RELEASE TIME CONSTANTS ARE CONSIDERED             *1
C*        KI       COUNTER FOR THE NUMBER OF SAMPLES                        *1
C*        NI       CONTAINS THE NUMBER OF THE INTEGRATION STEPS             *1
C*                                                                          *1
C*                                                                          *1
C* THE FOLLOWING ARRAYS ARE USED TO STORE THE PLOTTED PARAMETERS            *1
C*        X        THE X AXIS VECTOR , CONTAINS THE TIME INCREMENTS         *1
C*        VDISP    A STORAGE FOR UP TO 6 DIFFERENT VELOCITY OBSERVATION     *1
C*                 POINTS,AND UP TO 300 SAMPLES FOR EACH POINT              *1
C*        DDISP    A STORAGE FOR UP TO 6 DIIFERENT DIAPLACEMENT OBSERVAT-   *1
C*                 ION POINTS ,AND UP TO 300 SAMPLES FOR EACH POINT         *1
C*        THDIST   A RECORD FOR THE TRAVELED DISTANCE AGAINST TIME          *1
C*        EBDISP   A STORAGE FOR UP TO 6 DIFFERENT OBSERVATION POINTS       *1
C*                 FOR THE EFFECTIVE (ACTUAL) BRAKE FORCES, AND UP TO 300   *1
C*                 SAMPLES FOR EACH POINT                                   *1
C*        TBDISP   SIMILAR TO EBDISP,BUT FOR THE TARGET BRAKE FORCES AT     *1
C*                 THE SAME OPSEVATION POINTS                               *1
C*        TTDISP   A STORAGE FOR THE TARGET THROTTLES DISPLAY               *1
C*        ETDISP   A STORAGE FOR THE EFFECTIVE THROTTLE DISPLAY             *1
C*                                                                          *1
C*  COMMON           BLK1,BLK2,BLK3,BLK4,BLK5,BLK7,BLK8,BLK9,BLK10,BLK11,   *1
C*                   BLK12,BLK13,AND BLK50                                  *1
C*                                                                          *1
C*  ROUTINE NEEDED    START,PRNT                                            *1
C*  CALLED BY         NONE                                                  *1
C*                                                                          *1
C*  WRITTEN BY        MOUSTAFA ELSHAFEI AHMED                               *1
C*  DATE              JULY 1982                                             *1
C*                                                                          *1
C***************************************************************************1
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                                  1
      COMMON/BLK2/TCL(110),W(110),PMAX(10),TV(10,3)                          1
      COMMON/BLK3/SPRGC(110),DAMPC(110),DZ,BFBPC(110)                        1
      COMMON/BLK4/ AC,AL,NAXC,NAXL(10)                                       1
      COMMON/BLK5/OFFLN(25,6),NS                                             1
      COMMON/BLK7/X(300),VDISP(300,7),IVDISP(6),DDISP(300,6),IDDISP(6),      1
     +            THDIST(300),EBDISP(300,6),IBDISP(6),TBDISP(300,6),         1
     +            TTDISP(300,3),ETDISP(300,3),NUM,ITHRT(3)                   1
      COMMON/BLK8/DSV(10,10),DSD(10,10),VLVL(10),DLVL(10)                    1
      COMMON/BLK9/Y(338),DYDX(338),Q(338),V(110),R(110)                      1
      COMMON/BLK10/EFTHRT(10),TTHRT(10)                                      1
      COMMON/BLK11/EFABRK(110),TBF(110)                                      1
      COMMON/BLK12/SIGMAC,SIGMAL,ALPHAC,ALPHAL                               1
      COMMON/BLK13/DMAX,VMAX,DELX                                            1
      COMMON/BLK50/FB(10,80),UO(20),IC,STEP,DZDT(2),Z(2),Q1(2),IFB(10)       1
      REAL  S,ENRGY,T,STEP,TTIME,SMPLT,ENDD                                  1
      INTEGER KL,MNL,MKS,NI,INC,ITRACE,ITITL(30)                             1
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C***********************************************************************T
C*                                                                      *T
C*     SUBROUTINE START                                                 *T
C*                                                                      *T
C*     FUNCTION       : THIS IS THE MAIN SUBROUTINE. IT INITIALIZES ALL  *T
C*                      PARAMETERS,COORDINATES THE MAIN INTEGRATION LOOP.*T
C*                      TAKES AND STORES THE VARIOUS MEASUREMENTS,AND    *T
C*                      COMPLTES THE TRAVELLED DISTANCE,TIME AND THE     *T
C*                      ENERGY CONSUMPTION.                              *T
C*                                                                      *T
C*     USAGE          : CALL START(STEP,TTIME,SMPLT,ENDD,S,ENRGY,T,KL,   *T
C*                       MNL,NI,ITRACE)                                  *T
C*                                                                      *T
C*     PARAMETERS : (I FOR INPUT, O FOR OUTPUT)                          *T
C*  I  STEP  - THE INTEGRATION STEP IN SECONDS .                         *T
C*  I  TTIME - THE TRAVELING TIME IN SECONDS .                           *T
C*  I  SMPLT - THE MEASUREMENT SAMPLING PERIOD  IN SECONDS .             *T
C*  I  ENDD  - A DEFAULT FOR THE MAXIMUM TRAVELLED DISTANCE IN KM.       *T
C*  O  S     - THE TRAVELLED DISTANCE IN METERS WHEN THE SIMULATION      *T
C*             IS TERMINATED .                                           *T
C*  O  ENRGY - THE TOTAL ENERGY CONSUMPTION IN JOULES .                  *T
C*  O  T     - THE TRAVELLING TIME TO THE END OF SIMULATION              *T
C*  O  KL    - THE NUMBER OF THE SAMPLED MEASUREMENTS .                  *T
C*  O  MNL   - MNL=1 IF THE TRAIN HAS BEEN OPERATED MANUALLY, MNL=0      *T
C*             IF THE AUTOMATIC OPTION IS USED.                          *T
C*  O  NI    - THE NUMBER OF THE INTEGRATION STEPS AT TERMINATION.       *T
C*  I  ITRACE- IF ITRACE=1 A SUMMARY OF THE TRAIN STATUS WILL BE         *T
C*              PRINTED OUT AT EACH SAMPLING PERIOD.                     *T
C*                                                                      *T
C*                                                                      *T
C*    OTHER PARAMETERS : (COMMON BLOCK PARAMETERS)                      *T
C*     MM    - NUMBER OF TRAIN MEMBERS ,MAX 110(CARS +LOCOMOTIVES).      *T
C*     NLOC  - NUMBER OF LOCOMOTIVES,MAX 10 UNITS.                       *T
C*     M     - 2*M THE NUMBER OF THE VELOCITIES AND DISPLACEMENTS        *T
C*     ML2   - INDIX VECTOR FOR THE MIDDLE LOCOMOTIVES VELOCITY          *T
C*     ML1   - VECTOR OF THE POSITIONS OF THE LOCOMOTIVS  .              *T
C*     TCL   - VECTOR OF THE LENGTHS OF THE TRAIN MEMBERS IN METERS.     *T
C*     W     - VECTOR OF THE WEIGHTS OF THE TRAIN MEMBERS IN METRIC TONS*T
C*     PMAX  -VECTOR OF THE MAXIMUM POWERS OF THE LOCO IN METRIC H.P.    *T
C*     TV    -ARRAY  OF THE TRANSITION VELOCITIES OF THE LOCOMOTIVES     *T
C*           TV(I,1)- THE THROTTLE TRANSITION VELOCITY OF THE I-TH       *T
C*                  LOCOMOTIVE                                          *T
C*       TV(I,2),TV(I,3) - THE TRANSITION VELOCITIES OF THE DYNAMIC      *T
C*                         BRAKING. THESE VELOCITIES ARE IN KM/H .       *T
C*     SPRGC - THE SPRING CONSTANT OF THE LOCOMOTIVES/CARS COUPLERS.     *T
C*     DAMPC - THE DAMPING CONSTANT OF THE LOCOMOTIVES/CARS COUPLERS.    *T
C*     DZ    - THE WIDTH OF THE DEAD ZONE OF THE COUPLERS .              *T
C*     DFBPC - A VECTOR CONTAINIG THE MAXIMUM BRAKE FORCE OF EACH        *T
C*             TRAIN MEMBER IN KG (18) OR EQUIVALENTLY THE BRAKE FORCE   *T
C*             FOR 3.525 KG/SQR CM BRAKE CYLINDER PRESSURE .             *T
C*     AC,AL - THE CROSS SECTION AREA OF THE CARS AND LOCOMOTIVES        *T
C*             RESPECTIVELY   .                                         *T
C*     NAXC  -  THE NUMBER OF CARS AXLES, USUALLY= 4   .                 *T
C*     NAXL  -  VECTOR CONAINING THE NUMBER OF AXLES OF EACH LOCOMOTIVE.*T
C*     OFFLN- ARRAY OF DIMENSION(25,6)                                   *T
C*     OFFLN(I,1) LENGTH OF THE I-TH TERRAIN SECTION.                    *T
C*     OFFLN(I,2) GRADE   ........    ....  ...... ...                   *T
C*     OFFLN(I,3) VELOCITY .... .... .... ..... .... ( AT ITS START)     *T
C*     OFFLN(I,4) INDIX OF THE VELOCITY OF THE SECTION;                  *T
C*                INDIX=0.0 IF THE VELOCITY IS CONSTANT.                 *T
C*                INDIX=1.  IF THE VELOCITY IS CHANGING LINEARLY.        *T
C*     OFFLN(I,5) APPROXIMATE TOTAL AIR AND ACCELERATION RESISTANCE.     *T
C*     OFFLN(I,6) APPROXIMATE TOTAL GRAVITATIONAL FORCES .               *T
C*       NS       NUMBER OF TERRAIN SECTIONS .                           T
C*        X     : THE X AXIS VECTOR . CONTAINS THE TIME INCREMENTS .     *T
C*        VDISP : A STORAGE FOR UP TO 6 DIFFERENT VELOCITY OBSERVATION   *T
C*                POINTS,AND UP TO 300 SAMPLES FOR EACH POINT.           *T
C*        IVDISP: INDEX VECTOR OF THE VELOCITY MEASUREMENT POINTS .      *T
C*        DDISP : A STORAGE FOR UP TO 6 DIFFERENT DIAPLACEMENT OBSERVAT-*T
C*                ION POINTS ,AND UP TO 300 SAMPLES FOR EACH POINT.      *T
C*       IDDISP :.. . . . . . . . . .DISPLACEMENTS . . . . . . . .       *T
C*        TFDIST: A RECORD FOR THE TRAVELLED DISTANCE AGAINST TIME.      *T
C*        EBDISP: A STORAGE FOR UP TO 6 DIFFERENT OBSERVATION POINTS     *T
C*                FOR THE EFFECTIVE (ACTUAL) BRAKE FORCES. AND UP TO 300*T
C*                SAMPLES FOR EACH POINT .                               *T
C*       IBDISP :    INDEX VECTOR OF   BRAKES MEASUREMENT POINTS.        *T
C*        TBDISP: SIMILAR TO EBDISP,BUT FOR THE TARGET BRAKE FORCES AT   *T
C*                THE SAME OBSEVATION POINTS.                            *T
C*        TTDISP: A STORAGE FOR THE TARGET THROTTLES DISPLAY.            *T
C*        ETDISP: A STORAGE FOR THE EFFECTIVE THROTTLE DISPLAY .         *T
C*                                                                      *T
C*        NUM   : THE NUMBER OF THE OBSEVATION POINTS ALONG THE TRAIN.   *T
C*        ITHRT :. . . . . . . . . LOCOMOTIVES WHOSE THROTTLE IS  TO     *T
C*                BE PLOTTED.                                            *T
C*                                                                      *T
C*     DSV,CSD    : ARRAYES FOR STORING THE VELOCITY DEVIATION,AND THE   *T
C*                  DISPLACEMENT STATISTICAL DISTRIBUTIONS RESPECTIVELY  *T
C*                  AT THE OBSEVATION POINTS ALONG THE TRAIN.            *T
C*
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C* VLVL.DLVL    : VECTORS CONTAINING THE INCREMENTS OF THE VELOVITY        *1
C*                DEVIATION .AND THE DISPLACEMENTS RESPECTIVELY.          *1
C*    Y         : STATE VECTOR                                            *1
C*    Y(1)      : IS THE CLOCK                                            *1
C*    Y(2) TO Y(M): IS THE STATE VECTOR OF THE CARS VELOCITIES AND        *1
C*                     THE INTER CARS DISPLACEMENTS .                     *1
C*                   Y(2*K) IS THE VELOCITY OF THE K-TH TRAIN MEMBER.     *1
C*                   Y(2*K+1) IS THE DISPLACEMENT BETWEEN THE K-TH AND    *1
C*                    THE K+1-TH MEMBERS.                                 *1
C* Y(M+MM+1) TO Y(M+MM+6) ARE AUXILIARY STATES APPEAR BETWEEN CARS        *1
C*  AND LOCOMOTIVES DUE TO THE DIFFERENCE IN THE COUPLERS PROPERTIES.     *1
C*                                                                        *1
C*       DYDX   : VECTOR OF THE DERIVATIVES OF THE STATES .               *1
C*       Q      : AUXILIARY VECTOR NEEDED FOR INTEGRATION .               *1
C*       V      : GRAVITATIONAL VECTOR      .                             *1
C*       R      : ROLLING RESISTANCE VECTOR .                             *1
C*       TTHRT ;  VECTOR OF THE TARGET THROTTLING FORCE IN NEWTONS.       *1
C*       EFTHRT:  THE EFFECTIVE THROTTLE AFTER QUANTIZATION.              *1
C*       TBF    : VECTOR OF THE TARGET BRAKE FORCE .                      *1
C*       EFABRK: VECTOR OF THE EFFECTIVE BRAKE FORCES WHEN THE BUILD      *1
C*              UP AND RELEASE TIME CONSTANTS ARE CONSIDERED.             *1
C*    SIGMAC,SIGMAL - THE INVERSE OF THE TIME CONSTANT OF THE BRAKE       *1
C*            CYLINDER RELEASE OF CARS AND LOCOMOTIVES RESPECTIVELY.      *1
C*    ALPHAC,ALPHAL - THE RATE OF INCREASE OF THE BRAKE CYLINDER          *1
C*      PRESSURE OF THE CARS AND LOCOMOTIVES RESPECTIVELY KG/SQRCM*SEC    *1
C*    DMAX,VMAX- THE MAXIMUM PERMISSIBLE INTECARS DISPLACEMENT AND        *1
C*              VELOCITY RESPECTIVELY (IN METERS AND METER/SEC.)          *1
C*    DELX        - TEMPORARY STORAGE USED BY START.                      *1
C*                                                                        *1
C*    ALL THE PARAMETERS IN BLK50 ARE TEMPORARY STORAGES THAT MAY         *1
C*    BE USED BY THE USER'S TRAIN CONTROL SUBROUTINE.                     *1
C*                                                                        *1
C*                                                                        *1
C* THE INTERNAL PARAMETERS :                                              *1
C*    KL  -         COUNTER FOR THE NUMBER OF SAMPLES .                   *1
C*    NI  -         CONTAINS THE NUMBER OF THE INTEGRATION STEPS.         *1
C*    IG  - INDIX OF THE CURRENT GRADE SECTION                            *1
C*    S   - THE TRAVELLED DISTANCE TILL NOW IN METERS .                   *1
C*    GL  - THE TOTAL LENGTH OF THE GRADE SECTIONS UP AND INCLUDING       *1
C*          THE CURRENT ONE    IN KM.                                     *1
C*    T   - THE TRAVELLED TIME.                                           *1
C*    VREF- THE CURRENT REFERENCE SPEED.                                  *1
C*   DVDS - THE RATE OF CHANGE OF THE TARGET VELOCITY WITH DISTANCE.      *1
C*   TRNL - THE TOTAL TRAIN LENGHT IN METERS.                             *1
C*   NINT - THE REQUIRED NUMBER OF THE INEGRATION STEPS .                 *1
C*   ISTEP- INDEX FOR THE CURRENT CONTROL STEP.IF ISTEP IS RETURND 1000*1
C*          BY THE USER'S CONTROL PROGRAM ,THE SIMULATION TERMINATES.     *1
C*                                                                        *1
C*                                                                        *1
C*  COMMON        : BLK1.BLK2.BLK3.BLK4.BLK5.BLK7.BLK8.BLK9.BLK10.BLK11, *1
C*                  BLK12.BLK13                .                          *1
C*                                                                        *1
C*  ROUTINE NEEDED : INITT.VRF.GRV.ROLL.CNTRL.ADJST.EFFTRT.RKGILL         *1
C*                                                                        *1
C*  CALLED BY       : THE MAIN PROGRAM                                    *1
C*                                                                        *1
C*  WRITTEN BY      : MOUSTAFA ELSHAFEI AHMED                             *1
C*                                                                        *1
C*  DATE            : JULY 1982 .                                         *1
C*                                                                        *1
C*************************************************************************1
      SUBROUTINE START(ISTEP,TTIME,SMPLT,ENDD,S,ENRGY,T,KL,MML,NI,         1
     *                 ITRACE)                                             1
      COMMCN/BLK1/MM,NLCC,M,ML2(10),ML1(10)                                1
      COMMCN/BLK2/TCL(110),W(110),PMAX(10),TV(10,3)                        1
      COMMCN/BLK3/SPRGC(110),DAMPC(110),D2,BFBPC(110)                      1
      COMMCN/BLK4/ AC,AL,NAXC,NAXL(10)                                     1
      COMMCN/BLK5/CFFLN(25,6),NS                                           1
      CCMMCN/BLK7/X(300),VDISP(300,7),IVDISP(6),DDISP(300,6),IDDISP(6),    1
     *           TADIST(300),EBDISP(300,6),IBDISP(6),TBDISP(300,6),        1
     *           TTDISP(300,3),ETDISP(300,3),NUM,ITHRT(3)                  1
      CCMMCN/BLK8/DSV(10,10),DSD(10,10),VLVL(10),DLVL(10)                  1
      COMMCN/BLK9/Y(338),DYDX(338),Q(338),V(110),R(110)                    1
      COMMON/BLK10/EFTHRT(3),TTHRT(3)                                      1
      COMMCN/BLK11/EFABRK(110),TBF(110)                                    1
      COMMCN/BLK12/SIGMAC,SIGMAL,ALPHAC,ALPHAL                             1
      COMMCN/BLK13/DMAX,VMAX,DELX                                          1
C                                                                          1
C                                                                          1
C                                                                          1
      REAL DVDS,VL,GL,VREF,VI,S                                            1
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C**********************************************************************C1
C*                                                                     *1
C*   SUBROUTINE       INIT                                             *1
C*                                                                     *1
C*   FUNCTION         INITIALIZES THE VEHICLES VELOCITIES ,THE INTER CARS *1
C*                    DISPLACEMENTS, THE BRAKE CYLINDERS PRESSURE, AND *1
C*                    THE GRAVITATIONAL FORCES .                       *1
C*                                                                     *1
C*   USAGE            CALL INIT(VREF)                                  *1
C*                                                                     *1
C*   PARAMETERS                                                        *1
C*    VREF     - THE REFERENCE VELOCITY AT THE START OF                *1
C*                         THE FIRST VELOCITY SECTION                  *1
C*       ..   - FOR OTHER PARAMETERS SEE THE MAIN PROGRAM.             *1
C*                                                                     *1
C*                                                                     *1
C*   COMMON            BLK1, BLK2, BLK5, AND BLK9                      *1
C*                                                                     *1
C*   ROUTINE NEEDED   GRVF                                             *1
C*                                                                     *1
C*   CALLED BY         SUBROUTINE START .                              *1
C*                                                                     *1
C**********************************************************************C
      SUBROUTINE INIT(VREF)
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)
      COMMON/BLK2/TCL(110),W(110),PMAX(10),TV(10,3)
      COMMON/BLK5/OFFLN(25,6),NS
      COMMON/BLK9/Y(338),DYDX(338),G(338),V(110),R(110)
      DO 10 I=1,MM
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C                                                                       TF
C***********************************************************************TF
C*                                                                     *TF
C*   SUBROUTINE     VRF                                                *TF
C*                                                                     *TF
C*   FUNCTION       UPDATES THE TERRAIN SECTION AND COMPUTES THE       *TF
C*                  REFERENCE VELOCITY                                 *TF
C*                                                                     *TF
C*   USAGE          CALL VRF(S, VL, IV, VREF, YY22, YY11, Y1, DVDS, TRNL) *TF
C*                                                                     *TF
C*   PARAMETERS                                                        *TF
C*                  S   - TRAVELLED DISTANCE + TRAIN LENGTH IN METERS. *TF
C*                  VL  - THE TOTAL LENGTH OF THE TERRAIN SECTIONS     *TF
C*                        TILL NOW, INCLUDING THE CURRENT ONE, IN KM.  *TF
C*                 IV   - POINTER TO THE CURRENT TERRAIN SECTION.      *TF
C*                 VREF- THE CURRENT TARGET SPEED.                     *TF
C*                 YY22- ACTUAL SPEED OF THE TRAIN.                    *TF
C*            YY11-Y1  - THE INCREMENT OF TIME SINCE THE LAST CALL.    *TF
C*                 DVDS- RATE OF CHANGE OF THE REFERENCE SPEED WITH    *TF
C*                        DISTANCE.                                    *TF
C*                 TRNL- TRAIN LENGTH IN METERS.                       *TF
C*       ..  ........    FOR OTHER PARAMETERS SEE SUBROUTINE START.    *TF
C*                                                                     *TF
C*   COMMON              BLK5                                          *TF
C*                                                                     *TF
C*   ROUTINES NEEDED     NONE                                          *TF
C*                                                                     *TF
C*   CALLED BY           START                                         *TF
C*                                                                     *TF
C*   WRITTEN BY          MOUSTAFA ELSHAFEI AHMED                       *TF
C*                                                                     *TF
C*   DATE                 APRIL, 1982                                  *TF
C*                                                                     *TF
C***********************************************************************TF
      SUBROUTINE VRF(S, VL, IV, VREF, YY22, YY11, Y1, DVDS, TRNL)       TF
      COMMON/BLK5/OFFLN(25,6),NS                                        TF
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```
C*****************************************************************
C*                                                               *T
C*    SUBROUTINE     GRV                                         *T
C*                                                               *T
C*    FUNCTION       COMPUTES THE GRAVITINAL FORCE ON EACH CAR OF THE *T
C*                   TRAIN.                                      *T
C*                                                               *T
C*    USAGE          CALL GRV(IG,S,GL)                           *T
C*                                                               *T
C*    PARAMETERS       :                                         *T
C*     IG   - INDIX OF THE CURRENT GRADE SECTION                 *T
C*     S    - THE TRAVELLED DISTANCE TILL NOW IN METERS .        *T
C*     GL   - THE TOTAL LENGTH OF THE GRADE SECTIONS UP AND INCLUDING *T
C*            THE CURRENT ONE   IN KM.                           *T
C*                                                               *T
C*     MM     - NUMBER OF TRAIN MEMBERS ,MAX 110(CARS +LOCOMOTIVES). *T
C*     NLOC   - NUMBER OF LOCOMOTIVES,MAX 10 UNITS.              *T
C*     M      - 2*M THE NUMBER OF THE VELOCITIES AND DISPLACEMENTS *T
C*     ML1    - VECTOR OF THE POSITIONS OF THE LOCOMOTIVS  .     *T
C*     ML2    - INDIX VECTOR FOR THE MIDDLE LOCOMOTIVES VELOCITY *T
C*     TCL    - VECTOR OF THE LENGTHS OF THE TRAIN MEMBERS IN METERS. *T
C*     W      - VECTOR OF THE WEIGHTS OF THE TRAIN MEMBERS IN METRIC TONS*T
C*     NS     - NUMBER OF TERRAIN SECTIONS.                      *T
C*     OFFLN- ARRAY OF DIMENSION(25,6)                           *T
C*     OFFLN(I,1) LENGTH OF THE I-TH TERRAIN SECTION.            *T
C*     OFFLN(I,2) GRADE  ........   ....  ...... ...             *T
C*     OFFLN(I,3) VELOCITY .... .... .... ..... .... ( AT ITS START) *T
C*     OFFLN(I,4) INDIX OF THE VELOCITY OF THE SECTION;          *T
C*                INDIX=0.0 IF THE VELOCITY IS CONSTANT.         *T
C*                INDIX=1.  IF THE VELOCITY IS CHANGING LINEARLY. *T
C*                                                               *T
C*     OTHER PARAMETERS   SEE SUBROUTINE START OR THE MAIN PROGRAM. *T
C*                                                               *T
C*     COMMON             BLK1,BLK2,BLK5,BLK9                    *T
C*                                                               *T
C*     ROUTINES CALLED    GRVF                                   *T
C*                                                               *T
C*     CALLED BY          START , INIT  .                        *T
C*                                                               *T
C*     WRITTEN BY         MOUSTAFA ELSHAFEI AHMED                *T
C*                                                               *T
C*     DATE               APRIL,1982                             *T
C*                                                               *T
C****************************************************************T
      SUBROUTINE GRV(IG,S,GL)                                    T
C                                                                T
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                      T
      COMMON/BLK2/TCL(110),W(110),PMAX(10),TV(10,3)              T
      COMMON/BLK5/OFFLN(25,6),NS                                 T
      COMMON/BLK9/Y(338),DYDX(338),D(338),V(110),R(110)          T
10    IF(IG.EQ.1) RETURN                                         T
C                                                                T


C
C****************************************************************T
C*                                                              *T
C*    SUBROUTINE     GRVF                                       *T
C*                                                              *T
C*    FUNCTION       COMPUTES THE TANGENTIAL COMPONENT OF THE GRAVITA- *T
C*                   TIONAL FORCE ON A MASS W OVER A GRADE X % . *T
C*                                                              *T
C*    USAGE          GRVF(X,W)                                  *T
C*                                                              *T
C*    PARAMETERS     :                                          *T
C*                                                              *T
C*                   X- THE GRADE OF THE GROUND.                *T
C*                   W- THE MASS IN KG.                         *T
C*                GRVF- THE TANGETIAL FORCE IN NEWTONS.         *T
C*                                                              *T
C*    COMMON         :   NONE                                   *T
C*                                                              *T
C*    CALLED BY      :   GRV                                    *T
C*                                                              *T
C*    ROUTINES NEEDED    NONE                                   *T
C*                                                              *T
C****************************************************************T
      FUNCTION GRVF(X,W)                                        T
```

```
C
C*********************************************************************
C*                                                                   *
C*    SUBROUTINE     ROLL                                            *
C*                                                                   *
C*    FUNCTION       COMPUTES THE ROLLING AND AIR DYNAMIC RESISTANCE ON *
C*                   EACH CAR. THE RESULT IS STORED IN THE VECTOR R. *
C*                                                                   *
C*    USAGE          CALL ROLL                                       *
C*                                                                   *
C*    PARAMETERS     :                                               *
C*                   SEE SUBROUTINE START.                           *
C*                                                                   *
C*    COMMON         :    BLK1, BLK2, BLK4, BLK9                     *
C*                                                                   *
C*    CALLED BY      :    START                                      *
C*                                                                   *
C*    ROUTINES NEEDED      NONE                                      *
C*                                                                   *
C*    WRITTEN BY         MOUSTAFA ELSHAFEI AHMED.                    *
C*                                                                   *
C*    DATE               JUNE, 1982                                  *
C*                                                                   *
C*********************************************************************
      SUBROUTINE ROLL
      COMMON/BLK1/MM, NLOC, M, ML2(10), ML1(10)
      COMMON/BLK2/TCL(110), W(110), PMAX(10), TV(10,3)
      COMMON/BLK4/ AC, AL, NAXC, NAXL(10)
      COMMON/BLK9/Y(338), DYDX(338), Q(338), V(110), R(110)
C
```

```
C*********************************************************************T
C*                                                                  *T
C*    SUBROUTINE     ADJST                                          *T
C*                                                                  *T
C*    FUNCTION       COMPUTES THE EFFECTIVE THROTTLE OR DYNAMIC BRAKE. *T
C*                   THE  THROTTLE AND DYNAMIC BRAKE ARE ASSUMED TO *T
C*                   HAVE 8 POWER LEVELS.THE RELATION BETWEEN FORCE AND *T
C*                   VELOCITY IS APPROXIMATED BY LINEAR AND HYPARABOLIC *T
C*                   SEGMENTS.THE TARGET THROTTLE/DYNAMIC BRAKE MUST BE *T
C*                   SPECIFIED AS INTEGER NUMBERS BETWEEN-8..+8.    *T
C*                   INCLUSIVE. THIS SUBROUTINE IS CALLED ONLY IF MANUAL *T
C*                   OPERATION IS USED.                             *T
C*                                                                  *T
C*    USAGE          CALL ADJST                                     *T
C*                                                                  *T
C*    PARAMETERS                                                    *T
C*                 SEE SUBROUTINE EFFTRT,OR START .                 *T
C*                                                                  *T
C*    COMMON               : BLK1,BLK2,BLK3,BLK9,BLK10              *T
C*                                                                  *T
C*    SUBROUTINES NEEDED :    NONE                                  *T
C*                                                                  *T
C*    CALLED BY            : START                                  *T
C*                                                                  *T
C*    WRITTEN BY        MOUSTAFA ELSHAFEI  AHMED                    *T
C*                                                                  *T
C*    DATE              APRIL,1982                                  *T
C*                                                                  *T
C*********************************************************************T
      SUBROUTINE ADJST                                               T
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                          T
      COMMON/BLK2/TCL(110),W(110),PMAX(10),TV(10,3)                  T
      COMMON/BLK3/SPRGC(110),DAMPC(110),DZ,BFBPC(110)                T
      COMMON/BLK9/Y(338),DYDX(338),Q(338),V(110),R(110)              T
      COMMON/BLK10/EFTHRT(10),TTHRT(10)                              T
      COMMON/BLK11/EFABRK(110),TBF(110)                              T
C
```

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C*********************************************************************T
C*                                                                    *T
C*    SUBROUTINE     EFFTRT                                           *T
C*                                                                    *T
C*    FUNCTION       COMPUTES THE EFFECTIVE THROTTLE OR DYNAMIC BRAKE. *T
C*                   THHE THROTTLE AND DYNAMIC BRAKE ARE ASSUMED TO   *T
C*                   HAVE 8 POWER LEVELS.THE RELATION BETWEEN FORCE AND *T
C*                   VELOCITY IS APPROXIMATED BY LINEAR AND HYPARABOLIC *T
C*                   SEGMENTS.                                        *T
C*                                                                    *T
C*    USAGE          CALL EFFTRT                                      *T
C*                                                                    *T
C*    PARAMETER                                                       *T
C*     MM     - NUMBER OF TRAIN MEMBERS ,MAX 110(CARS +LOCOMOTIVES).  *T
C*     NLOC   - NUMBER OF LOCOMOTIVES.MAX 10 UNITS.                   *T
C*     N      - 2*M THE NUMBER OF THE VELOCITIES AND DISPLACEMENTS    *T
C*     ML1    - VECTOR OF THE POSITIONS OF THE LOCOMOTIVS   .         *T
C*     ML2    - INDIX VECTOR FOR THE MIDDLE LOCOMOTIVES VELOCITY      *T
C*     TCL    - VECTOR OF THE LENGTHS OF THE TRAIN MEMBERS IN METEFS. *T
C*     W      - VECTOR OF THE WEIGHTS OF THE TRAIN MEMBERS IN METRIC TONS*T
C*     PMAX   -VECTOR OF THE MAXIMUM POWERS OF THE LOCO IN METRIC H.P. *T
C*       TV   -ARRAY  OF THE TRANSITION VELOCITIES OF THE LOCOMOTIVES *T
C*             TV(I,1)- THE THROTTLE TRANSITION VELOCITY OF THE I-TH  *T
C*                   LOCOMOTIVE                                       *T
C*       TV(I,2),TV(I,3) - THE TRANSITION VELOCITIES OF THE DYNAMIC   *T
C*                         BRAKING. THESE VELOCITIES ARE IN KM/H .    *T
C*     AC,AL - THE CROSS SECTION AREA OF THE CARS AND LOCOMOTIVES     *T
C*             RESPECTIVELY   .                                       *T
C*     NAXC   -   THE NUMBER OF CARS AXLES. USUALLY= 4   .            *T
C*     NAXL   -   VECTOR CONAINING THE NUMBER OF AXLES OF EACH LOCOMOTIVE.*T
C*        TTHRT ;  VECTOR OF THE TARGET THROTTLING FORCE IN NEWTONS.  *T
C*        EFTHRT:  THE EFFECTIVE THROTTLE AFTER QUANTIZATION.         *T
C*                                                                    *T
C*    OTHER PARAMETERS    SEE THE MAIN PROGRAM OR START SUBROUTINE    *T
C*                                                                    *T
C*    COMMON                : BLK1,BLK2,BLK4,AND BLK9                 *T
C*                                                                    *T
C*    SUBROUTINES NEEDED :    INTX                                    *T
C*                                                                    *T
C*    CALLED BY             : START                                   *T
C*                                                                    *T
C*    WRITTEN BY       MOUSTAFA ELSHAFEI AHMED                        *T
C*                                                                    *T
C*    DATE             APRIL,1982                                     *T
C*                                                                    *T
C*********************************************************************T
      SUBROUTINE EFFTRT                                                T
      COMMON/BLK1/MM,NLOC,N,ML2(10),ML1(10)                            T
      COMMON/BLK2/TCL(110),W(110),PMAX(10),TV(10,3)                    T
      COMMON/BLK9/Y(338),DYDX(338),Q(338),V(110),R(110)                T
      COMMON/BLK10/EFTHRT(10),TTHRT(10)                                T
                                                                       T


C
C*********************************************************************
C*                                                                    *
C*    SUBROUTINE     INTX                                             *
C*                                                                    *
C*    FUNCTION       INTEGER FUNCTION WHICH ROUNDS OFF THE ARGUMENT   *
C*                   TO THE NEEREST INTEGER.                          *
C*                                                                    *
C*    USAGE          INTX(X)                                          *
C*                                                                    *
C*    PARAMETERS      X    _ THE FLOATING POINT NUMBER TO BE ROUNDED. *
C*                    INTX = THE NEEREST INTEGER NUMBER TO X.         *
C*                                                                    *
C*    ROUTINES NEEDED    NONE                                         *
C*                                                                    *
C*    CALLED BY         DER                                           *
C*                                                                    *
C*    COMMON             NONE                                         *
C*                                                                    *
C*********************************************************************
       FUNCTION INTX(X)
```

```
C*****************************************************************  T
C*                                                               *  T
C*   SUBROUTINE    DER                                           *  T
C*                                                               *  T
C*   FUNCTION      COMPUTES THE RIGHT HAND SIDE OF THE NONLINEAR *  T
C*                 DIFFERENTIAL EQUATON DYDX=F(Y,U).             *  T
C*                                                               *  T
C*                                                               *  T
C*   USAGE         CALL DER                                      *  T
C*                                                               *  T
C*   PARAMETRES                                                  *  T
C*     MM     - NUMBER OF TRAIN MEMBERS ,MAX 110(CARS +LOCOMOTIVES).  *T
C*     NLOC   - NUMBER OF LOCOMOTIVES,MAX 10 UNITS.                   *T
C*     M      - 2*M THE NUMBER OF THE VELOCITIES AND DISPLACEMENTS    *T
C*     ML2    - INDIX VECTOR FOR THE MIDDLE LOCOMOTIVES VELOCITY      *T
C*     ML1    - VECTOR OF THE POSITIONS OF THE LOCOMOTIVS  .          *T
C*     TCL    - VECTOR OF THE LENGTHS OF THE TRAIN MEMBERS IN METERS. *T
C*     W      - VECTOR OF THE WEIGHTS OF THE TRAIN MEMBERS IN METRIC TONS*T
C*     PMAX   -VECTOR OF THE MAXIMUM POWERS OF THE LOCO IN METRIC H.P.  *T
C*     TV     -ARRAY  OF THE TRANSITION VELOCITIES OF THE LOCOMOTIVES   *T
C*            TV(I,1)- THE THROTTLE TRANSITION VELOCITY OF THE I-TH     *T
C*                     LOCOMOTIVE                                       *T
C*       TV(I,2),TV(I,3) - THE TRANSITION VELOCITIES OF THE DYNAMIC     *T
C*                         BRAKING. THESE VELOCITIES ARE IN KM/H .      *T
C*     SPRGC - THE SPRING CONSTANT OF THE LOCOMOTIVES/CARS COUPLERS.    *T
C*     DAMPC - THE DAMPING CONSTANT OF THE LOCOMOTIVES/CARS COUPLERS.   *T
C*     DZ    - THE WIDTH OF THE DEAD ZONE OF THE COUPLERS .             *T
C*     BFBPC - A VECTOR CONTAINIG THE MAXIMUM BRAKE FORCE OF EACH       *T
C*             TRAIN MEMBER IN KG (10) OR EQUIVALENTLY THE BRAKE FORCE  *T
C*             FOR 3.525 KG/SQR CM BRAKE CYLINDER PRESSURE .            *T
C*     AC,AL - THE CROSS SECTION AREA OF THE CARS AND LOCOMOTIVES       *T
C*             RESPECTIVELY  .                                          *T
C*     NAXC  -  THE NUMBER OF CARS AXLES. USUALLY= 4   .                *T
C*     NAXL  -  VECTOR CONAINING THE NUMBER OF AXLES OF EACH LOCOMOTIVE.*T
C*     Y        : STATE VECTOR                                          *T
C*     Y(1)     : IS THE CLOCK                                          *T
C*     Y(2) TO Y(M): IS THE STATE VECTOR OF THE CARS VELOCITIES AND     *T
C*                   THE INTER CARS DISPLACEMENTS .                     *T
C*                Y(2*K) IS THE VELOCITY OF THE K-TH TRAIN MEMBER,      *T
C*                Y(2*K+1) IS THE DISPLACEMENT BETWEEN THE K-TH AND     *T
C*                  THE K+1-TH MEMBERS.                                 *T
C* Y(M+MM+1) TO Y(M+MM+6) ARE AUXILIARY STATES APPEAR BETWEEN CARS      *T
C*  AND LOCOMOTIVES DUE TO THE DIFFERENCE IN THE COUPLERS PROPERTIES.   *T
C*                                                                      *T
C*        DYDX  : VECTOR OF THE DERIVATIVES OF THE STATES .             *T
C*        Q     : AUXILIARY VECTOR NEEDED FOR INTEGRATION .             *T
C*        V     : GRAVITATIONAL VECTOR     .                            *T
C*        R     : ROLLING RESISTANCE VECTOR .                           *T
C*        TTHRT :  VECTOR OF THE TARGET THROTTLING FORCE IN NEWTONS.    *T
C*        EFTHRT:  THE EFFECTIVE THROTTLE AFTER QUANTIZATION.           *T
C*        TBF   : VECTOR OF THE TARGET BRAKE FORCE .                    *T
C*        EFABRK: VECTOR OF THE EFFECTIVE BRAKE FORCES WHEN THE BUILD   *T

                UP AND RELEASE TIME CONSTANTS ARE CONSIDERED.          *
C*     SIGMAC,SIGMAL - THE INVERSE OF THE TIME CONSTANT OF THE BRAKE    *
C*             CYLINDER RELEASE OF CARS AND LOCOMOTIVES RESPECTIVELY.   *
C*     ALPHAC,ALPHAL - THE RATE OF INCREASE OF THE BRAKE CYLINDER       *
C*                                                                     *
C*                                                                     *
C*                      BLK1,BLK2,BLK3,BLK4,BLK9,BLK10,BLK11,BLK12     *
C*    COMMON                                                           *
C*                                                                     *
C*    ROUTINE NEEDED    FF2,BRAK                                       *
C*                                                                     *
C*    CALLED BY         RKGILL                                         *
C*                                                                     *
C*    WRITTEN BY        MOUSTAFA ELSHAFEI AHMED                        *
C*                                                                     *
C*    DATE              LAST VERSION    JULY, 1982.                    *
C*                                                                     *
C***********************************************************************
      SUBROUTINE DER
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)
      COMMON/BLK2/TCL(110),W(110),PMAX(10),TV(10,3)
      COMMON/BLK3/SPRGC(110),DAMPC(110),DZ,BFBPC(110)
      COMMON/BLK4/ AC,AL,NAXC,NAXL(10)
      COMMON/BLK9/Y(338),DYDX(338),Q(338),V(110),R(110)
      COMMON/BLK10/EFTHRT(10),TTHRT(10)
      COMMON/BLK11/EFABRK(110),TBF(110)
      COMMON/BLK12/SIGMAC,SIGMAL,ALPHAC,ALPHAL
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C**********************************************************************
C*                                                                    *
C*    SUBROUTINE      BRAK                                            *
C*                                                                    *
C*    FUNCTION        SIMULATES THE UNSYMETRIC BEHAVIUR OF THHE BRAKES . *
C*                                                                    *
C*                                                                    *
C*    USAGE           CALL BRAK(TBFF,BFBP,YY,DY,ALPHA,SIGMA)          *
C*                                                                    *
C*    PARAMETER       TBFF   - THE TARGET BRAKE FORCE.                *
C*                    BFBP   -BRAKEFORCE/BRAKE PRESSURE CONSTANT.     *
C*                    YY     - THE CURRENT BRAKE CYLINDER PRESSURE.   *
C*                    DY     - THHE RATE OF CHANGE OF THE BRAKE CYLINDER *
C*                             PRESSURE.                              *
C*                    ALPHA  -THE RATE OF INCREASE OF THE BRAKE CYLINDER *
C*                             PRESSURE.                              *
C*                    SIGMA  - THHE INVERSE OF THE TIME CONSTANT OF THE *
C*                             BRAKE CYLINDER PRESSURE RELEASE.       *
C*                                                                    *
C*    COMMON    NONE                                                  *
C*                                                                    *
C*    ROUTINES NEEDED NONE                                            *
C*                                                                    *
C*    CALLED BY    DER                                                *
C*                                                                    *
C*    WRITTEN BY     MOUSTAFA ELSHAFEI AHMED.                         *
C*                                                                    *
C*    DATE           MARSH,1982                                       *
C*                                                                    *
C*                                                                    *
C**********************************************************************
      SUBROUTINE BRAK(TBFF,BFBP,YY,DY,ALPHA,SIGMA)
      T0=TBFF/(9.81*BFBP)
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```
C**********************************************************************
C*                                                                    *
C*   SUBROUTINE    FF2                                                *
C*                                                                    *
C*   FUNCTION      COMPUTES COMPUTES COUPLER FORCES.AN EXTRA STATE IS *
C*                 CREATED FOR EVERY JOINT WHOSE COUPLERS ARE NOT     *
C*                 IDENTICAL.                                         *
C*                                                                    *
C*   USAGE         CALL FF2(INN,I,F2)                                 *
C*                                                                    *
C*   PARAMETRES                                                       *
C*       INN          - THE NUMBER OF JOINTS OF DIFFERENT COUPLERS    *
C*                      CHARACTERISTICS PROCESSED SO FAR.             *
C*       I            - THE POSITION OF THE COUPLER TO BE PROCESSED.  *
C*       F2           - THE COUPLER FORCE.                            *
C*                                                                    *
C*   OTHER PARAMETERS    SEE SUBROUTINES DER,OR START                 *
C*                                                                    *
C*   COMMON              BLK1,BLK2,BLK3,BLK4,BLK9,BLK10,BLK11,BLK12   *
C*                                                                    *
C*   ROUTINE NEEDED      FN,DEAD                                      *
C*                                                                    *
C*   CALLED BY           DER                                          *
C*                                                                    *
C*   WRITTEN BY          MOUSTAFA ELSHAFEI AHMED                      *
C*                                                                    *
C*   DATE                JULY,1982.                                   *
C*                                                                    *
C*                                                                    *
C**********************************************************************
      SUBROUTINE FF2(INN,I,F2)
C
C
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)
      COMMON/BLK2/TCL(110),W(110),PMAX(10),TV(10,3)
      COMMON/BLK3/SPRGC(110),DAMPC(110),DZ,BFBPC(110)
      COMMON/BLK9/Y(338),DYDX(338),Q(338),V(110),R(110)
      COMMON/BLK10/EFTHRT(10),TTHRT(10)
      COMMON/BLK11/EFABRK(110),TBF(110)
C
```

```
C*************************************************************** 1
C*                                                              * 1
C*   SUBROUTINE    RKGILL                                       * 1
C*                                                              * 1
C*   FUNCTION      PERFORMS INTEGRATION FOR THE VECTOR D.E. DYDX=F(Y,U) * 1
C*                 USING THE GILL-RUNGE-KUTA METHOD.            * 1
C*                                                              * 1
C*   USAGE         CALL RKGILL(STEP).                           * 1
C*                                                              * 1
C*   PARAMETRES                                                 * 1
C*                 STEP  - LENGTH OF THE INTEGRATION STEP IN SECONDS.  * 1
C*                                                              * 1
C*   OTHER PARAMETERS    - SEE SUBROUTINE START.                * 1
C*                                                              * 1
C*   COMMON              BLK1,BLK9                              * 1
C*                                                              * 1
C*   ROUTINE NEEDED      DER                                    * 1
C*                                                              * 1
C*   CALLED BY           START                                  * 1
C*                                                              * 1
C*   WRITTEN BY          WONG.AND GRUBER. MODIFIED BY H.E.AHMED * 1
C*                                                              * 1
C*   DATE                LAST VERSION   APRIL.1982.             * 1
C*                                                              * 1
C*************************************************************** 1
      SUBROUTINE RKGILL(STEP)                                     1
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                       1
      COMMON/BLK9/Y(338),DYDX(338),Q(338),V(110),R(110)           1
      DIMENSION A(4),B(4),C(4)
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```
C*************************************************************** 1
C*                                                              * 1
C*   SUBROUTINE    DEAD                                         * 1
C*                                                              * 1
C*   FUNCTION      COMPUTE THE NONLINEAR FUNCTION OF THE SPRING * 1
C*                 PART OF THE COUPLER                          * 1
C*                                                              * 1
C*   USAGE         Z=DEAD(X,SLOPE)                              * 1
C*                                                              * 1
C*   PARAMETRES    X     - VALUE OF THE COUPLER DISPLACEMENT.   *
C*                 SLOPE - SPRING CONSTANT OF THE COUPLER.      * 1
C*                 DEAD  - THE RETURNED COUPLER FORCE.          *
C*                 DZ    - THE DEAD ZONE OF THE COUPLER(1/2. SLACK). *
C*                                                              * 1
C*   OTHER PARAMETERS   SEE THE MAIN PROGRAM .                  *
C*                                                              *
C*   COMMON     BLK3                                            *
C*                                                              *
C*   ROUTINE NEEDED  NONE                                       *
C*                                                              *
C*                                                              *
C***************************************************************
      FUNCTION DEAD(X,SLOPE)
      COMMON/BLK3/SPRGC(110),DAMPC(110),DZ,BFBPC(110)
```

```
C***************************************************************1
C*                                                             *1
C*    SUBROUTINE     FN                                        *1
C*                                                             *1
C*    FUNCTION       COMPUTE THE NONLINEAR FUNCTION OF THE DAMPING *T
C*                   PART OF THE COUPLER                       *T
C*                                                             *T
C*    USAGE          Z=FN(X)                                   *1
C*                                                             *T
C*    PARAMETER       X   ACTUAL VALUE OF THE COUPLER          *T
C*                    DZ  DEAD ZONE                            *T
C*                                                             *T
C*    COMMON          BLK10                                    *T
C*                                                             *1
C*    ROUTINE NEEDED  NONE                                     *T
C*                                                             *1
C***************************************************************T
      FUNCTION FN(X)                                            T
      COMMON/BLK3/SPRGC(110),DAMPC(110),DZ,BFBPC(110)
```

```
C***********************************************************************1
C*                                                                     *1
C*    SUBROUTINE     PRNT                                              *1
C*                                                                     *1
C*    FUNCTION       PRINTER PLOT OF THE FOLLOWING DIAGRAMS            *1
C*                   VELOCITY                                          *1
C*                   COUPLING DISPLACEMENTS                            *1
C*                   EFFECTIVE BRAKING FORCE                           *1
C*                   TARGET BRAKING FORCE                              *1
C*                   EFFECTIVE THROTTLING FORCES                       *1
C*                   TARGET THROTTLING FORCES                          *1
C*                   DISPLACEMENTS STATISTICAL DISTRIBUTION            *1
C*                   VELOCITY STATISTICAL DISTRIBUTION                 *1
C*                                                                     *1
C*    USAGE          CALL PRNT(KL.NI.INC)                              *1
C*                                                                     *1
C*    P4RAMFTERS     KL   - THE NUMBER OF SAMPLES STORED FOR DISPLAY.  *1
C*                   NI   - THE NUMBER OF THE INTEGRATION STEPS WHEN THE *1
C*                          SIMULATION IS TERMINATED .                 *1
C*                   INC  - INCREMENT BETWEEN THE TIME VALUES TO BE    *1
C*                          CONSIDERED .                               *1
C*                                                                     *1
C*   INIERNAL PARAMEIERS :                                             *1
C*                   ATIT - VECTOR  CNTAINS THE X-AXIS AND THE Y-AXIS  *1
C*                          LABELS OF THE DATA TO BE PLOTTED .         *1
C*                   RANG1- VECTOR OF LENGTH 4 CONTAINS THE RANGE OF THE *1
C*                          OF THE DISPLACEMENTS PLOTS. IF THE RANGE IS *1
C*                          SET TO 0.0. THE RANGE WILL BE COMPUTED.    *1
C*                   RANG2- VECTOR CONTAINING THE RANGE OF VELOCITIES  *1
C*                   RANGB- VECTOR CONTAINING THE RANGE OF BRAKES.     *1
C*                   RANGT- VECTOR CONTAINING THE RANGE OF THROTTLE.   *1
C*                   IOPT - OPTION FOR THE SIZE OF THE PLOT.0 FOR 6IN. *1
C*                          1 FOR 8 INCH.                              *1
C*                   IMAGE.ICH-   WORKING ARRAYS.                      *1
C*                   IER  - ERROR INDEX.                               *1
C*                                                                     *1
C*    COMMON                    BLK1.BLK7.BLK8                         *1
C*                                                                     *1
C*    ROUTINE NEEDED            MYPLT                                  *1
C*                                                                     *1
C*                          IF CALCOMP PLOTTER IS NOT AVAILABLE.MYPLT CAN *1
C*                          BE REPLACED CONVENIENTLY WITH THE IMSL PRINTER *1
C*                          PLOT SUBROUIINE  USPLT. IN THIS CASE THE ARRAY *1
C*                          ATIT SHOULD BE REPLACED BY ITIT            *1
C*                                                                     *1
C*                                                                     *1
C*    WRITTEN BY  MOUSTAFA ELSHAFEI AHMED                              *1
C*    DATE     JULY.1982                                               *1
C*                                                                     *1
C**********************************************************************C1
      SUBROUTINE  PRNT(KL.NI.INC)                                       1



      DIMFNSION ATIT(144).RANG1(4).ICH(10).IMAGE(100)                   1
      DIMENSION RANGV(4).RANGD(4).RANGB(4).RANG2(4).RANGT(4)            1
      COMMON/BLK1/MM.NLOC.M.ML2(10).ML1(10)                             1
      COMMON/BLK7/X(300).VDISP(300.7).IVDISP(6).DDISP(300.6).IDDISP(6). 1
     +            TNDIST(300).EBDISP(300.6).IBDISP(6).TBDISP(300.6).    1
     +            TIDISP(300.3).ETDISP(300.3).NUM.ITHRT(3)              1
      COMMON/BLK8/DSV(10.10).DSD(10.10).VLVL(10).DLVL(10)               1
*     INTEGER IER.IOPT.INC.NI.KL.IA/ INC                                1
C     INTECER ITIT(144)                                                 1
      DATA ICH(1)/1H /.IOPT/0/                                          1
      DATA RANG1/0.0.0.0.0.0.0.0/                                       1
      DATA RANG2/0.0.0.0.0.0.0.0/                                       1
      DATA RANGB/0.0.0.0.0.0.30000./                                    1
C     DATA RANGT/0.0.0.0.0.0 .500000./                                  1
      DATA RANGT/0.0.0.0.0.0.0.0/                                       1
C     DATA RANGD/0.0..2.0.0.1.0/                                        1
C     DATA RANGV/0.0.5.0.0.0.1.0/                                       1
      IA=300                                                            1
```

```
C*****************************************************************************C1
C*                                                                            *1
C*   SUBROUTINE       MYPLT                                                   *1
C*                                                                            *1
C*   FUNCTION         PERFORMS MULTIPLOTTING .  PLOTS THE DATA CONTAINED      *1
C*                    IN THE COLUMNS OF Y AGAINST THE DATA IN X. IT           *1
C*                    USES ROUTINES FROM THE CALCOMP GRAPHIC PACKAGE          *1
C*                                                                            *1
C*   USAGE            CALL MYPLT(X, Y, IA, NPNTS, NPLTS, INC, ATITL, RANG,    *1
C*                    ICH, IOPT, IMAGE, IER)                                  *1
C*                                                                            *1
C*   PARAMETERS                                                               *1
C*      X     - VECTOR OF THE X-AXIS POINTS                                    1
C       Y     - TWO DIMENSION ARRAY CONTAINING THE Y-AXIS DATA                 1
C       IA    - DIMENSION OF X AND ROW DIMENSION OF Y IN THE CALLING PROG.     1
C       NPNTS- NUMBER OF DATA POINTS.                                          1
C       NPLTS- NUMBER OF DIFFERENT PLOTS                                       1
C       ITITL- AN INTEGER VECTOR OF LENGTH 144 CONTAINING THE TITLES           1
C                1-72 THE X-AXIS TITLE, 73-144 THE Y-AXIS TITLE.               1
C       RANG - VECTOR OF LENGTH 4 CONTAINING THE RANGE OF DATA. IF THE         1
C              RANGE OF DATA IS 0 0, THE PROGRAM COMPUTES THE RANGE.           1
C       IOPT-  OPTION FOR THE SIZE OF THE PLOT. 0 FOR 6 INCH, 1 FOR 8          1
C       IMAGE-  A WORKING ARRAY OF LENGTH IA                                   1
C       IER  - ERROR INDEX                                                     1
C*                                                                            *1
C*   COMMON             NONE                                                  *1
C*                                                                            *1
C*   CALLED BY          PRNT                                                  *1
C*                                                                            *1
C*   ROUTINES NEEDED  THE FOLLOWING SUBROUTINES FROM CALCOMP PLOTTER :        *1
C*                    PLOTS, FACTOR, AXIS, SCALE, PLOT, LINE1                 *1
C*                                                                            *1
C*   WRITTEN BY    MOUSTAFA ELSHAFEI AHMED                                    *1
C*   DATA          JULY, 1982                                                 *1
C*                                                                            *1
C*****************************************************************************1
      SUBROUTINE MYPLT(X, Y, IA, NPNTS, NPLTS, INC, ATITL, RANG, ICH,
     +   IOPT, IMAGE, IER)
C
C
      DIMENSION X(302), Y(IA, NPLTS), ATITL(144), RANG(4), Y1(302)
      DIMENSION ATIT1(80)
      INTEGER ISYM(10), ICH(10), IMAGE(100)
```

ATTACHMENT B.III

CONTROL SUBROUTINE CNTRL

```
**********************************************************************
*
*   SUBROUTINE CONTROL
*
*   FUNCTION          USER PROGRAM TO CREATE THE CONTROL COMMANDS FOR
*                     THE 1RAIN
*
*   USAGE            CALL CNTHL(S. T. VREF. MNL. ISTEP. IG. GL. TRNL)
*
*   PARAMETERS       (D . DATA 1RANSFERED TO CNTRL )
*              D     S    - 1HE TRAVELLED DISTANCE IN METERS.
*              D     T    - THE 1RAVELLING TIME
*              D     VREF- THE CURRENT REFERENCE SPEED
*                    MNL - MNL SHOULD DE RETURNED 1 IF THE TRAIN IS
*                          OPERATED MANUALLY  MNL=0 IF AN AUTOMATIC
*                          CONTROL SCHEME IS USED
*                    ISTEP- POINTER FOR THE CURRENT CONTROL STEP.
*                           IF ISTEP IS RETURNED =1000 THE SIMULATION
*                           STOPS
*              D     IG    - INDEX FOR THE CURRENT TERRAIN SECTION.
*              D     GL    - THE TOTAL LENGTH OF THE GRADE SECTIONS TILL
*                            NOW IN KM
*              D     TRNL - THE TRAIN LENGTH IN METERS.
*              . . . . . . . . . FOR OTHER PARAMETERS SEE SUBROUTINE START.
*
*   COMMON          BLK1. DLK2. BLK3. BLK5. BLK9. BLK10. BLK11. BLK50
*
*   CALLED BY        START
*
*
**********************************************************************
      SUBROUTINE  CNTRL(S. T. VREF. MNL. ISTEP. IG. GL. TRNL)

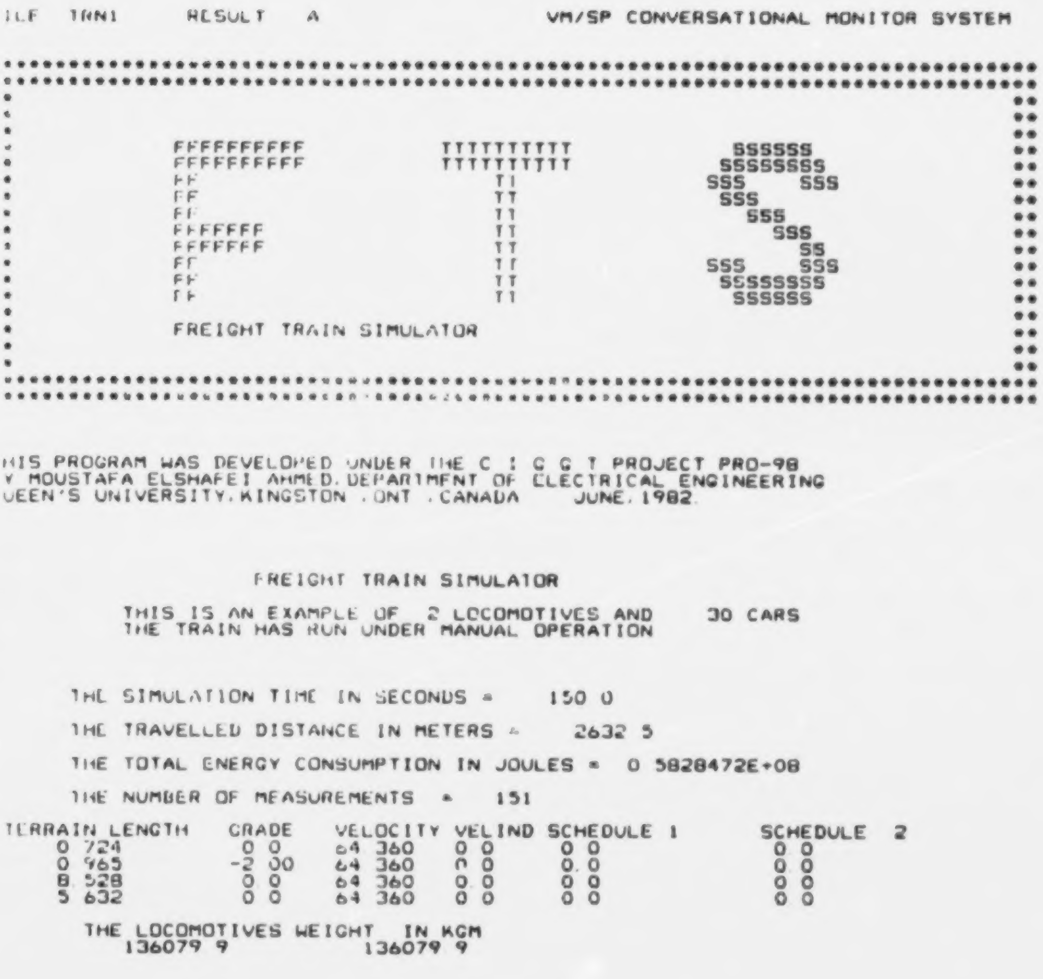
      COMMON/BLK1/MM. NLOC. M. ML2(10). ML1(10)
      COMMON/BLK2/TCL(110). W(110). PMAX(10). TV(10. 3)
      COMMON/BLK3/SPRGC(110). DAMPC(110). DZ. BFDPC(110)
      COMMON/BLK5/OFFLN(25. 6). NS
      COMMON/BLK9/Y(338). DYDX(338). Q(338). V(110). R(110)
      COMMON/BLK10/EFTHRT(10). TTHRT(10)
      COMMON/BLK11/EFABRK(110). TBF(110)
      COMMON/BLK50/FB(10. 80). U0(20). IC. STEP. DZDT(2). Z(2). Q1(2). IFB(10)
      REAL S. ENRGY. T. STEP. TTIME. SMPLT. ENDD
      INTEGER KL. MNL. MKS. NI. INC. ITRACE

      DIMENSION A(4). B(4). C(4)
      DATA A/. 5. . 2928932. 1 707107. . 1666667/.
     +B/2 . 1 . 1 . 2 /. C/ 5. 2928932. 1. 707107. 5/
      INTEGER MNL. ISTEP
      MNL=0
      S1=S+TRNL

      N1=6
      M1=M+1
      IF(ISTEP NE 1) GO TO 20
```

ATTACHMENT B.IV

EXAMPLES



```
ILF  TRN1      RESULT   A                       VM/SP CONVERSATIONAL MONITOR SYSTEM

************************************************************************************
************************************************************************************
*                                                                                 **
*                                                                                 **
*                                                                                 **
*          FFFFFFFFFF           TTTTTTTTTT              SSSSSS                    **
*          FFFFFFFFFF           TTTTTTTTTT             SSSSSSSS                   **
*          FF                       TT                SSS    SSS                  **
*          FF                       TT                 SSS                        **
*          FF                       TT                   SSS                      **
*          FFFFFFF                  TT                     SSS                    **
*          FFFFFFF                  TT                       SS                   **
*          FF                       TT                SSS    SSS                  **
*          FF                       TT                 SSSSSSSS                   **
*          FF                       TT                  SSSSSS                    **
*                                                                                 **
*          FREIGHT TRAIN SIMULATOR                                                **
*                                                                                 **
*                                                                                 **
************************************************************************************
************************************************************************************


HIS PROGRAM WAS DEVELOPED UNDER THE C I G G T PROJECT PRO-98
Y MOUSTAFA ELSHAFEI AHMED.DEPARTMENT OF ELECTRICAL ENGINEERING
UEEN'S UNIVERSITY.KINGSTON .ONT .CANADA     JUNE.1982


                    FREIGHT TRAIN SIMULATOR

          THIS IS AN EXAMPLE OF  2 LOCOMOTIVES AND     30 CARS
          THE TRAIN HAS RUN UNDER MANUAL OPERATION


      THE SIMULATION TIME IN SECONDS =    150 0

      THE TRAVELLED DISTANCE IN METERS =    2632 5

      THE TOTAL ENERGY CONSUMPTION IN JOULES =  0 5828472E+08

      THE NUMBER OF MEASUREMENTS  =   151
TERRAIN LENGTH    GRADE    VELOCITY VELIND SCHEDULE 1       SCHEDULE  2
    0 724          0 0      64 360   0 0     0 0              0 0
    0 965         -2 00     64 360   0 0     0.0              0 0
    8.528          0 0      64 360   0.0     0 0              0 0
    5 632          0 0      64 360   0 0     0 0              0 0

       THE LOCOMOTIVES WEIGHT  IN KGM
          136079 9            136079 9
```

```
C*******************************************************************TR
C*                                                                 *TR
C*    SUBROUTINE CONTROL                                           *TR
C*                                                                 *TR
C*    FUNCTION       USER PROGRAM TO CREATE THE CONTROL COMMANDS FOR *TR
C*                   THE TRAIN                                     *TH
C*                                                                 *TR
C*    USAGE          CALL CNTRL(S,T,VREF,MNL,ISTEP,IG,GL,TRNL)     *TR
C*                                                                 *TR
C*    PARAMETERS     (D    DATA TRANSFERED TO CNTRL.)              *TR
C*                 D   S    - THE TRAVELLED DISTANCE IN METERS.    *TR
C*                 D   T    - THE TRAVELLING TIME.                 *TR
C*                 D   VREF - THE CURRENT REFERENCE SPEED          *TR
C*                     MNL  - MNL SHOULD BE RETURNED 1 IF THE TRAIN IS *TR
C*                            OPERATED MANUALLY  MNL=0 IF AN AUTOMATIC *TR
C*                            CONTROL SCHEME IS USED.              *TR
C*                     ISTEP- POINTER FOR THE CURRENT CONTROL STEP. *TR
C*                             IF ISTEP IS RETURNED =1000 THE SIMULATION *TR
C*                             STOPS                               *TR
C*                 D   IG   - INDEX FOR THE CURRENT TERRAIN SECTION. *TR
C*                 D   GL   - THE TOTAL LENGTH OF THE GRADE SECTIONS TILL *TR
C*                             NOW IN KM                           *TR
C*                 D   TRNL - THE TRAIN LENGTH IN METERS.          *TR
C*                            FOR OTHER PARAMETERS SEE SUBROUTINE START *TR
C*                                                                 *TR
C*    COMMON         BLK1,BLK2,BLK3,BLK5,BLK9,BLK10,BLK11,BLK50    *TR
C*                                                                 *TR
C*    CALLED BY      START                                         *TR
C*                                                                 *TR
C*                                                                 *TR
C*******************************************************************TR
      SUBROUTINE  CNTRL(S,T,VREF,MNL,ISTEP,IG,GL,TRNL)              TR
C                                                                   TR
      COMMON/BLK1/MM,NLOC,M,ML2(10),ML1(10)                         TR
      COMMON/BLK10/EFTHRT(10),TTHRT(10)                             TR
      COMMON/BLK11/TABRK(110),TBF(110)                              TR
      REAL ENRGY,T,STEP,TTIME,SMPLT,ENDD                            TR
      INTEGER KL,MNL,MKS,NI,INC,ITRACE                              TR
C                                                                   TR
C     EXAMPLE OF MANUAL OPERATION                                   TR
C      MNL=1                                                        TR
C                                                                   TR
      MNL=1                                                         TR
C                                                                   TR
C     APPLY THROTTLE 3 FOR 5 SECONDS                                TR
C                                                                   TR
      IF(T .GT. 5.0) GO TO 10                                       TR
      TTHRT(1)=3                                                    TR
      TTHRT(2)=3                                                    TR
      DO 5 I=1,MM                                                   TR
5     TBF(I)=0.0                                                    TR
      RETURN                                                        TR
C                                                                   TR
C     REDUCE THROTTLE TO RUN 2 FOR 3 SECONDS                        TR
C                                                                   TR
10     IF(T    GT 8 ) GO TO 20                                      TR
       TTHRT(1)=2                                                   TR
       TTHRT(2)=2                                                   TR
      DO 15 I=1,MM                                                  TR
15    TBF(I)=0.0                                                    TR
       RETURN                                                       TR
C                                                                   TR
C     SET THROTTLE TO IDLE                                          TR
C     APPLY AIR BRAKE = 15 KG/SQCM                                  TR
C                                                                   TR
20     IF(T    GT  20 ) GO TO 30                                    TR
      TTHRT(1)=0                                                    TR
      TTHRT(2)=0                                                    TR
      DO 25 I=1,MM                                                  TR
25    TBF(I)=0.15                                                   TR
      RETURN                                                        TR
C                                                                   TR
C    APPLY AIR BRAKE = 3 KG/SQCM                                    TR
C    SET DYNAMIC BRAKE TO -1                                        TR
C                                                                   TR
30     IF(T .GT 60. ) GO TO 40                                      TR
      TTHRT(1)=-1                                                   TR
      TTHRT(2)=-1                                                   TR
      DO 35 I=1,MM                                                  TR
35    TBF(I)=0.3                                                    TR
      RETURN                                                        TR
C                                                                   TR
C   SET THROTTLE BACK TO IDLE                                       TR
C   REDUCE BRAKES TO .15KG/SQCM.                                    TR
C                                                                   TR
```

```
40      IF(T GT 75  ) GO TO 50                                                  TR
        TTHRT(1)=0 0                                                            TR
        TTHRT(2)=0 0                                                            TR
        DO 45 I=1,MM                                                            TR
45      TBR(I)= 15                                                              TR
        RETURN                                                                  TR
C                                                                               TR
C   RELEASE BRAKE                                                               TR
C                                                                               TR
50      IF(T GT 90  ) GO TO 60                                                  TR
        TTHRT(1)=0 0                                                            TR
        TTHRT(2)=0 0                                                            TR
        DO 55 I=1,MM                                                            TR
55      TBR(I)=0 0                                                              TR
        RETURN                                                                  TR
C                                                                               TR
C     [illegible] THROTTLE GRADUALLY .SET THROTTLE TO RUN 1 FOR 3 SECONDS.      TR
C                                                                               TR
60      IF(T GT 93  ) GO TO 70                                                  TR
        TTHRT(1)=1 0                                                            TR
        TTHRT(2)=1 0                                                            TR
        DO 65 I=1,MM                                                            TR
65      TBR(I)=0 0                                                              TR
        RETURN                                                                  TR
C                                                                               TR
C   MOVE TO RUN 2                                                               TR
C                                                                               TR
70      IF(T GT 100 ) GO TO 80                                                  TR
        TTHRT(1)=2 0                                                            TR
        TTHRT(2)=2 0                                                            TR
        DO 75 I=1,MM                                                            TR
75      TBR(I)=0 0                                                              TR
        RETURN                                                                  TR
C                                                                               TR
C    CHANGE THROTTLE LEVEL TO RUN 3                                             TR
C                                                                               TR
80      IF(T GE 150 ) ISTEP=1000                                                TR
        TTHRT(1)=3 0                                                            TR
        TTHRT(2)=3 0                                                            TR
        DO 85 I=1,MM                                                            TR
85      TBR(I)=0 0                                                              TR
        RETURN                                                                  TR
        END
```

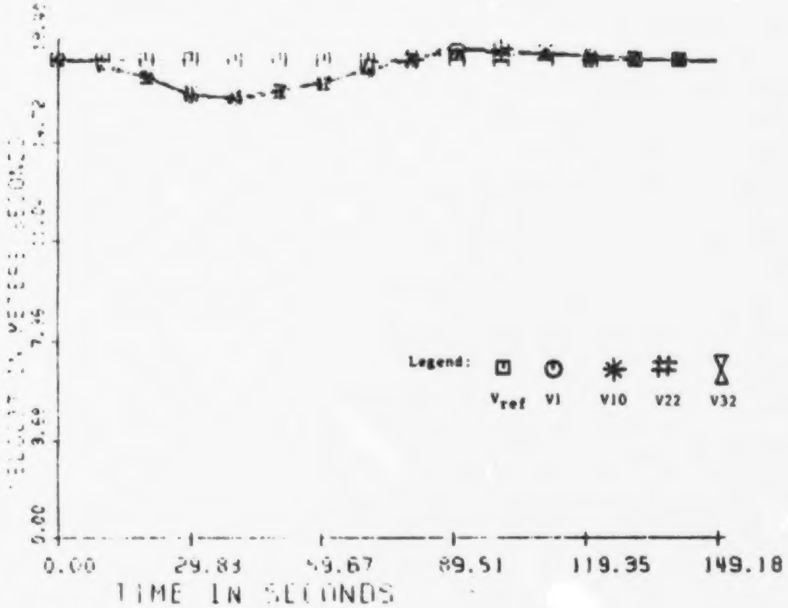


Figure B.IV.1 Velocity

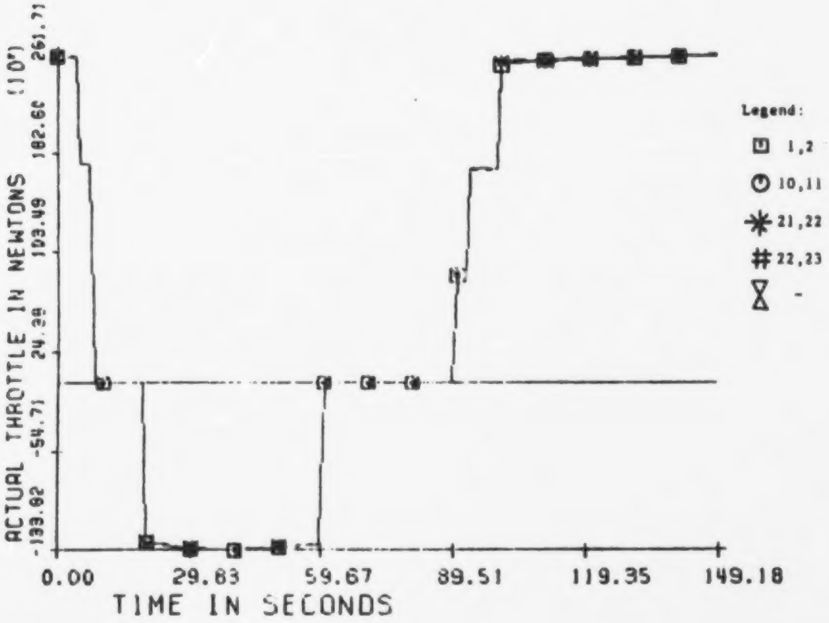


Figure B.IV.2 Coupler Displacement Between Train Members

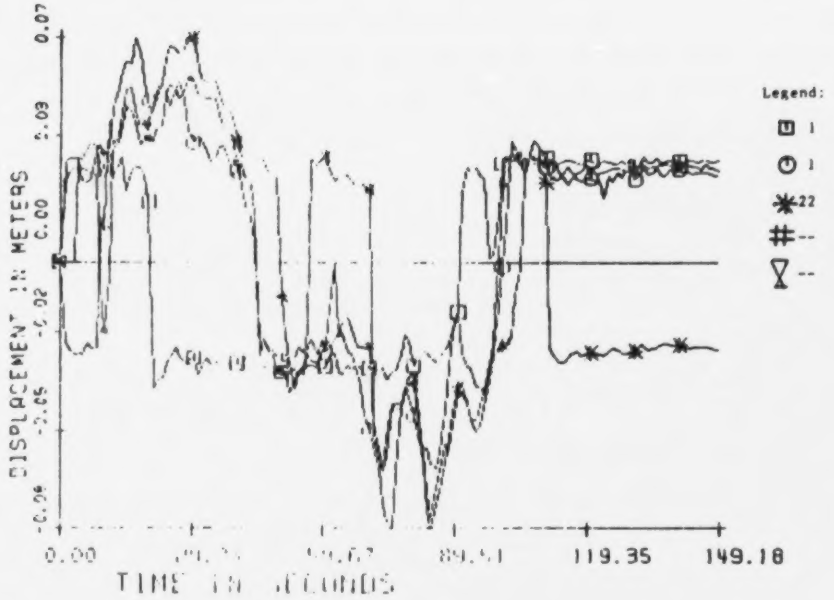


Figure B.IV.3 Throttle

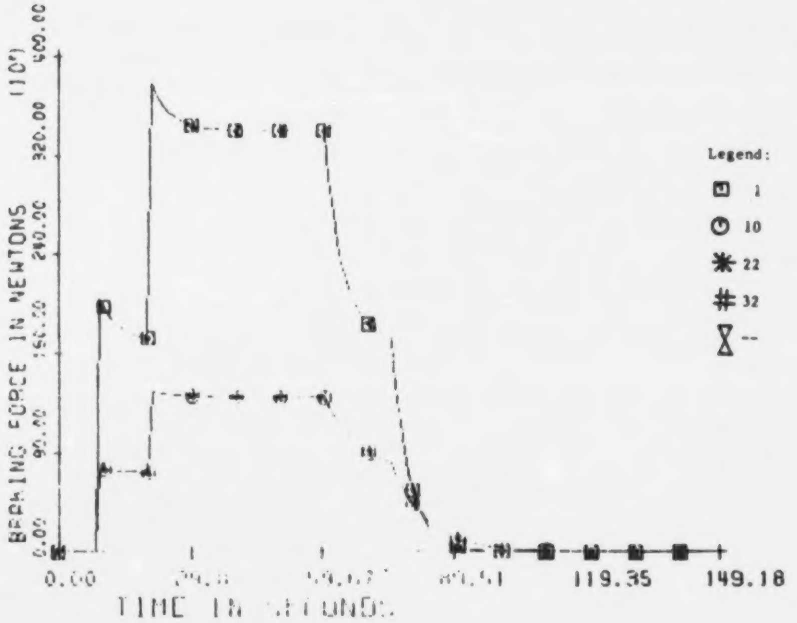


Figure B.IV.4 Brake Force

```
****************************************************************************
****************************************************************************
**                                                                        **
**                                                                        **
**                                                                        **
**           FFFFFFFFFF          TTTTTTTTTT           SSSSSS              **
**           FFFFFFFFFF          TTTTTTTTTT          SSSSSSSS             **
**           FF                      TT             SSS   SSS             **
**           FF                      TT              SSS                  **
**           FF                      TT                SSS                **
**           FFFFFFF                 TT                  SSS              **
**           FFFFFFF                 TT                    SS             **
**           FF                      TT             SSS   SSS             **
**           FF                      TT              SSSSSSSS             **
**           FF                      TT               SSSSSS              **
**                                                                        **
**           FREIGHT TRAIN SIMULATOR                                      **
**                                                                        **
**                                                                        **
****************************************************************************
****************************************************************************

THIS PROGRAM HAS DEVELOPED UNDER THE C I G G T PROJECT PRO-98
BY MOUSTAFA ELSHAFEI AHMED.DEPARTMENT OF ELECTRICAL ENGINEERING
QUEEN'S UNIVERSITY,KINGSTON ,ONT.,CANADA     JUNE,1982


                    FREIGHT TRAIN SIMULATOR

               THIS IS AN EXAMPLE OF  2 LOCOMOTIVES AND     30 CARS
               THE TRAIN HAS RUN UNDER AUTOMATIC OPERATION


          THE SIMULATION TIME IN SECONDS =     150.0

          THE TRAVELLED DISTANCE IN METERS =     2683.9

          THE TOTAL ENERGY CONSUMPTION IN JOULES =  0.9407206E+08

          THE NUMBER OF MEASUREMENTS   =    151

      TERRAIN LENGTH    GRADE    VELOCITY VELIND SCHEDULE 1       SCHEDULE  2
         0.724           0.0     64.360   0.0     7.720E+04        0.0
         0.965          -2.00    64.360   0.0     7.720E+04       -6.406E+05
         8.286           0.0     64.360   0.0     7.720E+04        0.0
         5.632           0.0     64.360   0.0     7.720E+04        0.0

           THE LOCOMOTIVES WEIGHT  IN KGM :
              136079.9            136079.9

           THE LOCOMOTIVES LENGTHS IN METERS .
                18.288              18.288

           THE AVERAGE VEIHCLE WEIGHT IN KGM = 102059.3

           THE AVERAGE VEIHCLE LENGTH IN METERS =    19.717
```

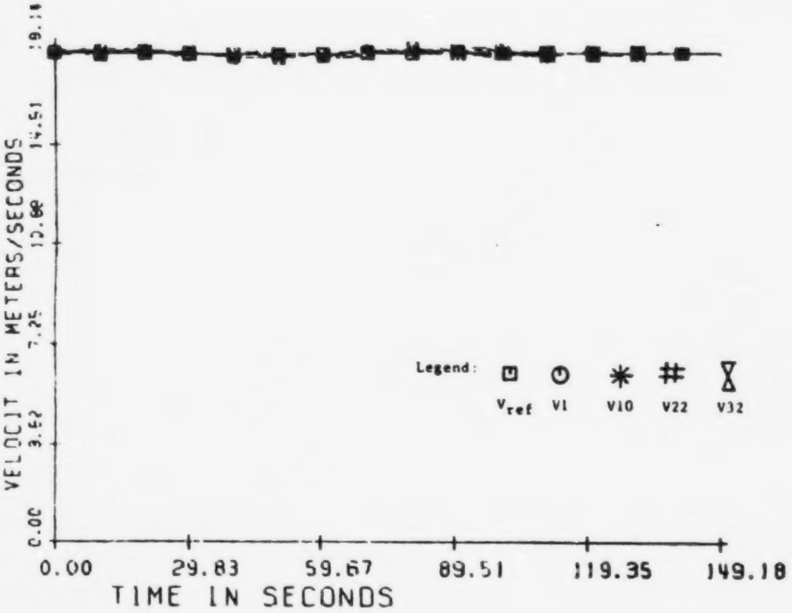


Figure B.IV.5 Velocity

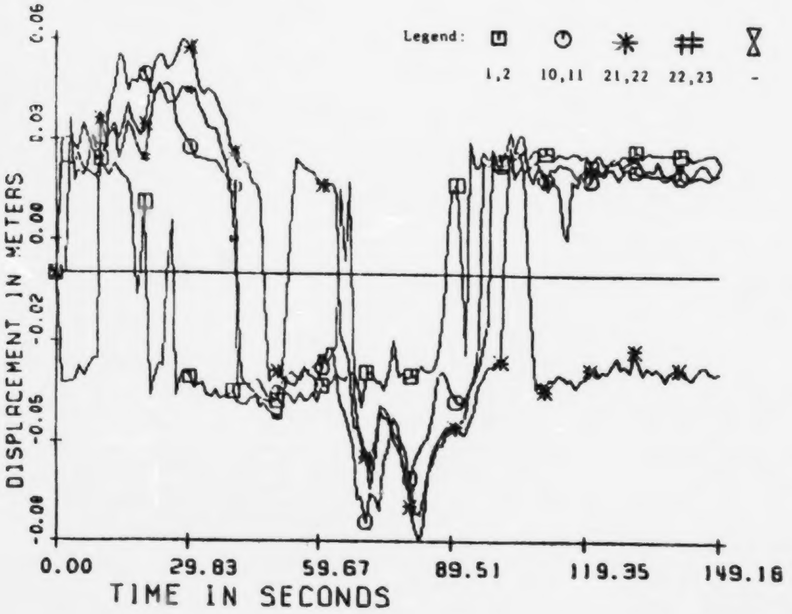


Figure B.IV.6 Coupler Displacement Between Train Members

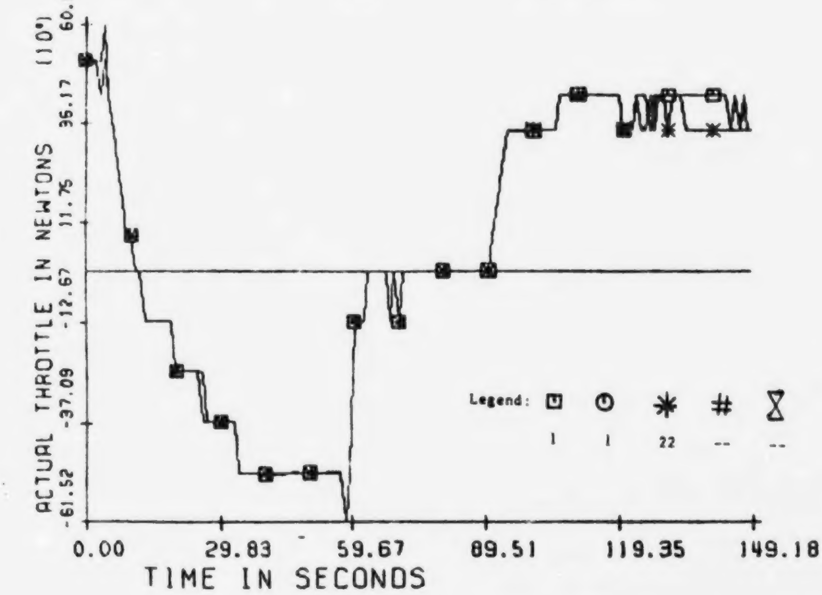


Figure B.IV.7 Throttle

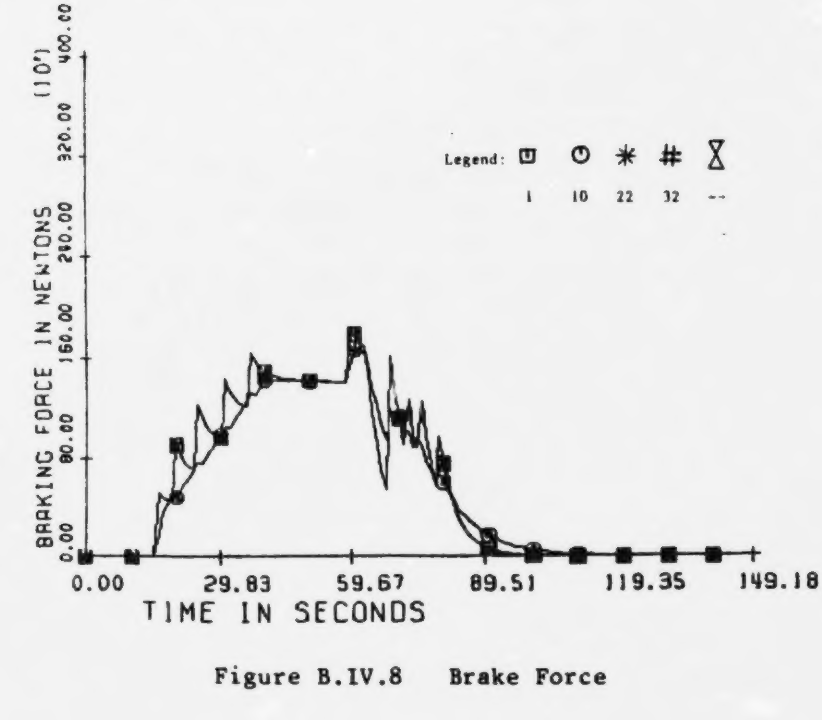


Figure B.IV.8 Brake Force

Canadian Institute of
Guided Ground Transport
Queen's University
at Kingston, Ontario

I.C. Ozsoy
M.M. Bayoumi

CIGGT Report No. 83-12
Transport Canada Report No. TP 5114E

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CIGGT Project PRO-098

CIGGT Report No. 83-12
Transport Canada No. TP-5114E

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EVALUATION OF DESIGN ALTERNATIVES FOR A FREIGHT TRAIN CONTROLLER

Appendix B

A REDUCED-ORDER MODEL FOR LONG FREIGHT TRAINS

I.C. Ozsoy
M.M. Bayoumi
Department of Electrical Engineering
Queen's University

prepared for

Transportation Development Centre, Transport Canada

Canadian Institute of Guided Ground Transport
Queen's University at Kingston, Ontario
K7L 3N6

December 1983

This report resulted from research conducted through the Canadian Institute of Guided Ground Transport. The report represents the views of the authors and does not necessarily have the endorsement of the University or the other participating organizations.

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5. Report Date
December 1983

6. Performing Organization Report No.
83-12

7. Author(s)
K.Y. Wong, M.M. Bayoumi, M.E. Ahmed, I.C. Oszoy

8. Transport Canada File No.
D1465-285-3-1

9. Performing Organization Name and Address
Canadian Institute of Guided Ground Transport (CIGGT)
Queen's University
Kingston, Ontario
K7L 3N6

10. DSS File No.
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11. DSS or Transport Canada Contract No.
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12. Sponsoring Agency Name and Address
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17. Abstract

This project on development and optimization of control schemes for freight trains was motivated by the need to find optimal or suboptimal closed-loop control schemes for long freight trains. Several on-line and off-line freight train control strategies have been studied and reported. The relation between these control strategies is discussed in this report. The structure of an overall train operation scheme which consists of an off-line energy saving strategy and an on-line control strategy to maintain the velocity profile specified by the off-line scheme is proposed. There are five separately-bound appendices: Appendix A: AUTOMATIC CONTROL OF FREIGHT TRAINS; Appendix B: A REDUCED-ORDER MODEL FOR LONG FREIGHT TRAINS; Appendix C: A COMPARATIVE STUDY OF SELF-TUNING CONTROLLERS FOR FREIGHT TRAINS; Appendix D: MINIMUM ENERGY CONSUMPTION CONTROL OF PASSENGER/FREIGHT TRAINS WITH FIXED TERMINAL TIME; Appendix E: OPTIMAL CONTROL STRATEGY FOR MINIMUM ENERGY AND MINIMUM FUEL CONSUMPTION OF FREIGHT TRAINS

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17 Resume

L'étude dont ce rapport rend compte a trait aux programmes visant l'automatisation et l'optimalisation de la conduite des longs convois de marchandises, étude motivée par le besoin pressant de trouver des techniques en boucle fermée qui soient optimales, ou tout proche. Le rapport présente un certain nombre de programmes de marche prédéterminés et de programmes d'optimalisation de la conduite; de plus, il traite de l'interaction entre eux. Il préconise un programme du premier type axé sur l'économie d'énergie, secondé par un programme du deuxième type assurant à la fois le respect des consignes imposées et une consommation minimale d'énergie. Le rapport contient cinq annexes distinctes, intitulées comme suit: A: Automatisation de la conduite des convois de marchandises; B: Logiciel en réduction approprié aux longs convois de marchandises; C: Etude comparative des logiciels tenant compte des variations dans les paramètres de conduite; D: Consommation minimale des trains de voyageurs et de marchandises à horaires fixes; E: Stratégie d'optimalisation de la consommation d'énergie et de carburant dans le cas des trains de marchandises.

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1. INTRODUCTION

During the past two decades, a number of different approaches have been under investigation to obtain reduced-order models for large-scale linear systems, and many methods have been proposed. Most of these methods can be classified into three main groups.[1] One popular approach is based on retaining either the dominant eigenvalues or the dominant states of the original system.[2,3,4] Another approach involves fitting a low-order model such that either its time response to a specified input (impulse or step) or its frequency response may match that of the original system in an optimum approximation sense, with no restriction on the location of the eigenvalues.[5,6,7,8] Other methods are based on matching some time moments or Markov parameters of the reduced models to those of the original system.[9,10,11,12,13] All of these methods try to find a low-order model which approximates, as far as possible, certain properties of the original system.

The reduction technique is usually computationally simple and can be applied to a wide class of systems. Moreover, it should yield a stable reduced-order model for a given stable large system. As a reduced-order model cannot exactly characterize the given system, it would be desirable to determine how closely the reduced-order model represents the original system and what aspects of a given large system have been retained by a particular reduced model. But the above criteria for comparison should not be the only goal. Probably more important is the need to utilize the low-order model for the design of a feedback controller for the original system. Several existing model reduction techniques were reviewed and compared for designing PID controllers that minimize the integral square error.[14,15]

In recent years, optimal control methods for designing controllers for long (more than 100 cars) multi-locomotive-powered trains were suggested and discussed in several reports.[16,17,18,19] Standard optimal control methods necessitate the solution of a Riccati equation. In large systems, this is a computationally difficult and cumbersome task. This is the case even when developing optimal controllers for trains having a number of units of the order of 30. Long freight trains have normally a number of units which is much larger than that. The need for model reduction is thus a real one, especially if we aim at the development of an optimal controller. It should be noted that the problem of designing an optimal controller for such trains has been approached using a distributed parameter model,[20] but the computational burden is still substantial.

In this study, this difficulty is avoided by obtaining two different reduced models for the system representing the original train model. These reduced-order models are compared with the original train model consisting of thirty cars and two locomotives. The original order of the system is 63 and the reduction is achieved in two steps. The first step is to select the most significant states of the original system which have been retained in the reduced model. For this purpose, the criterion proposed by Rozha, Sinha and Latsman[21] is utilized. The idea of choosing this procedure can be summarized as follows. Although, in general, the slow eigenvalues play a dominant role in the system response, some of these may be loosely coupled to the output, and may affect it only slightly. For example, if a pole is very close to a zero, its contribution to the total response energy of the output of the system may be neglected, even if it is not far to the left of the jw-axis and may represent a slow mode. In the second step, the singular perturbation method is applied in a different manner by making use of the general structure of the system equations. Singular perturbation was preferred, since the physical nature of the model is preserved and this property is necessary to run the train simulation program that is used for comparison of the models.

2. TRAIN MODEL

2.1 Non-Linear Model

A unit train is considered as a series of spring-dashpot-coupled systems of point masses where the dead zone is neglected. The system can be described by a set of non-linear differential equations of the form:[17]

$$
\begin{aligned}
m_i\,\dot{v}_i(t) &= k_i\,\Delta_i(t) - k_{i+1}(t)\,\Delta_{i+1}(t) \\
&\quad + C_i\,[v_{i-1}(t) - v_i(t)] \\
&\quad - C_{i+1}\,[v_i(t) - v_{i+1}(t)] \\
&\quad - m_i\,g\,\sin\theta_i - R_i(t) \\
&\quad + f_i(t) + U_i(t) \\
\dot{\Delta}_i(t) &= v_{i-1}(t) - v_i(t) \\
\dot{p}_i(t) &= -\frac{1}{\sigma_i}\,p_i(t) + \frac{\eta_1}{\sigma_i}\,b_i(t) \\
U_i(t) &= \eta_2\,p_i(t) \leq 0 \qquad i = 1,2,\ldots,n \qquad (1)
\end{aligned}
$$

where: n = number of train members

m_i = mass of train member i

k_i = spring constant of the coupler between train member i-1 and train member i, $k_{n+1}=0$

c_i = damping coefficient of the coupler between train member i-1 and train member i, $c_{n+1}=0$

g = gravity constant

θ_i = angle of the grade on which train member i lies

σ_i = build-up pressure time constant

η_1 = brake cylinder constant

η_2 = brake pressure to force conversion constant

v_i = velocity of train member i, $v_0(t) = v_{n+1}(t) = 0$

Δ_i = displacement of the coupler between train member i-1 and train member i, $\Delta_1 = \Delta_{n+1} = 0$

p_i = pressure in the brake cylinder of train member i, $i \notin S$

b_i = pressure in the brake pipe of train member i, $i \notin S$

f_i = dynamic braking/throttling force of train member i if it is a locomotive, i.e. $f_i = 0$ for $i \notin S$

U_i = braking force of train member i, if it is not a locomotive, i.e. $U_i = 0$ for $i \notin S$

S = the set of all positions of locomotives

R_i = non-linear rolling resistance and aerodynamic drag force of train member i.

The following simplifications are made for the controller design:

(a) the mass of each locomotive is assumed to be the same, $m_i = m_L$ for $i \in S$

(b) the mass of each car is the same, $m_i = m_c$ for $i \notin S$

(c) the coupler spring constants are equal, $k_i = k$, $i = 1,2,\ldots,n$

(d) the coupler damping coefficients have the same value, $c_i = c$, $i = 2,3,\ldots,n$

(e) build-up pressure time constants, σ_i, are set to zero

(f) quantization of throttling and dynamic braking is neglected

(g) no constraints are considered for braking and throttling except for the one-sidedness of braking conditions, i.e. $U_i(t) \leq 0$.

With these additional assumptions one can write the dynamic equations in the form:

$$m_i \dot{v}_i(t) = k\,[\Delta_i(t) - \Delta_{i+1}(t)] + c\,[v_{i-1}(t) - 2\,v_i(t) + v_{i+1}(t)]$$

$$- m_i\, g \sin \theta_i - R_i(t) + f_i(t) + U_i(t)$$

$$\dot{\Delta}_i(t) = v_{i-1}(t) - v_i(t) \qquad i = 2,3,\ldots,n \qquad (2)$$

2.2 Linearized Model

The train system can be linearized for the range of velocities under investigation if the rolling resistance is linearized around some nominal values, v^o, of velocities where:

$$\underline{v} = \underline{v}^o + \delta\underline{v} \tag{3}$$

v^o is maintained by an input, $U^o(t)$, equal to the total rolling resistance and gravity forces. Hence the linarized model is found to consist of the following equations:

$$\begin{aligned} m_i \ \delta\dot{v}_i(t) &= k\,[\Delta_i(t) - \Delta_{i+1}(t)] + c\,[\delta v_{i-1}(t) - 2\ \delta v_{i-1}(t) \\ &\quad + \delta v_{i+1}(t)] + \delta U_i(t) + \delta d_i \\ \dot{\Delta}_i(t) &= \delta v_{i-1}(t) - \delta v_i(t) \qquad i = 2,3,\dots,n \end{aligned} \tag{4}$$

where: δd_i = disturbances due to plant parameters variations and inaccuracies arising from linearization

$$\delta U_i = \begin{cases} \delta f_i \text{ for locomotive} & i = 1,m,n \\ \delta U_i \text{ for car} & i = 2,\dots,m-1,m+1,\dots,n-1 \end{cases}$$

The above equations can be written in the form of a vector state equation as given below:

$$\delta\underline{\dot{x}}(t) = A\ \delta\underline{x}(t) + B\ \delta\underline{u}(t) + \delta\underline{d}_i \tag{5}$$

with:

$$\delta\underline{x}^T = [\delta v_1, \delta v_2, \dots, \delta v_n, \Delta_2, \Delta_3, \dots, \Delta_n]$$

$$\delta\underline{u}^T = [\delta f_1, \delta U_2, \delta U_3, \dots, \delta U_{m-1}, \delta f_m, \ \delta U_{m+1}, \dots, \delta U_{m-1}, \delta f_n]$$

$$\delta\underline{d}_i^T = [\delta d_1, \delta d_2, \dots, \delta d_n, 0, \dots, 0]$$

with:

$$A = \left[\begin{array}{cc} H & G \\ \hdashline F & 0 \end{array}\right]$$

H is an n x n matrix given by:

$$H = \begin{bmatrix} -\frac{c}{m_L} & \frac{c}{m_L} & & & & \\ \frac{c}{m_c} & \frac{2c}{m_c} & \frac{c}{m_c} & & & 0 \\ & \ddots & \ddots & \ddots & & \\ & \frac{c}{m_L} & -\frac{2c}{m_L} & \frac{c}{m_L} & & \\ & & \ddots & \ddots & \ddots & \\ & & & \frac{c}{m_c} & \frac{2c}{m_c} & \frac{c}{m_c} \\ & 0 & & & \frac{c}{m_L} & -\frac{c}{m_L} \end{bmatrix}$$

G is an n x (n-1) matrix given by:

$$G = \begin{bmatrix} -\frac{k}{m_L} & & & \\ \frac{k}{m_c} & -\frac{k}{m_c} & & 0 \\ \ddots & \ddots & & \\ & \frac{k}{m_L} & -\frac{k}{m_L} & \\ & \ddots & \ddots & \\ & & \frac{k}{m_c} & -\frac{k}{m_c} \\ & 0 & & \frac{k}{m_L} \end{bmatrix}$$

F is an (n-1) x n matrix given by:

$$F = \begin{bmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & \ddots & \ddots & \\ & & & 1 & -1 \end{bmatrix}$$

and B is a (2n-1) x n matrix given by:

$$B = \left[\begin{array}{ccccccccc} \frac{1}{m_L} & & & & & & & & \\ & \frac{1}{m_c} & & & & & & & \\ & & \ddots & & & & & & \\ & & & \frac{1}{m_c} & & & & & \\ & & & & \frac{1}{m_L} & & & & \\ & & & & & \frac{1}{m_c} & & & \\ & & & & & & \ddots & & \\ & & & & & & & \frac{1}{m_c} & \\ & & & & & & & & \frac{1}{m_L} \\ \hline \multicolumn{9}{c}{0} \end{array}\right]$$

The linearized system is used in order to calculate a suboptimal control law, while the non-linear system is used in the simulation.[18]

3. METHODS OF REDUCTION

3.1 Sinha's Method for Selecting the Most Significant States

Consider a linear multivariable system described by the equations:

$$\dot{x} = Ax + Bu \tag{6}$$

$$y = Cx + Du \tag{7}$$

where $x \in R^n$, $y \in R^m$ and $U \in R^p$ are the state output and input vectors, respectively. The objective of the model reduction procedure is to obtain the equations:

$$\dot{x}_r = F\, x_r + Gu \tag{8}$$

$$\hat{y} = H\, x_r + Lu \tag{9}$$

where $x_r \in R^r$ and $r \ll n$, so that $\hat{y} \in R^m$ is a close approximation to $y(t)$ for all inputs, $u(t)$, $t \geq 0$.

It is also assumed that the components of the vector, x_r, are a subset of the components of x, so that the states of the reduced model retain their physical meaning. Hence, the main problem is the selection of the states, x_r, from x.

The Sinha method is based on the fact that the total energy of the output of the system for an impulse input can be expressed as a sum of n terms. Hence, one may select the relatively large terms to determine the most significant states. This was motivated by the earlier work of Hutton and Friedland[22] in which they utilized the concept of impulse response energy for the selection of the order of the reduced model. The same concept was used in a recent paper of Sinha[23] for selecting the eigenvalues to be retained in a reduced model based on the aggregation method. In two recent papers,[24,25] a scale-invariant criterion has been proposed for picking out the state variables which significantly participate in the construction of selected modes. The main idea behind this approach is the development of a participation matrix for evaluating the relative importance of each state. While this is a very important contribution, it does not take into account the coupling between the states and the various inputs and outputs, or else it selects the modes from the A-matrix alone. The practical importance of this criterion is thus considerably reduced. It may be noted that in Sinha's procedure both the B and C matrices will be utilized, in addition to the A-matrix. This means that the Sinha approach takes into account the relative values of the coupling of each state to the outputs and inputs.

Like the aggregation method, Sinha's method also requires the computation of the eignevalues of A. The matrix A can then be diagonalized through the relationship:

$$A = V \Omega U \tag{10}$$

where the columns of V are the right eigenvectors of A and the rows of U are the left eigenvectors of A, correspondings to the eigenvalues $_1$, $\lambda_2, \ldots, \lambda_n$. The vectors are scaled in such a manner that:

$$V U = I \tag{11}$$

Also,

$$\Omega = \mathrm{diag}\ (\lambda_1, \lambda_2, \ldots, \lambda_n) \tag{12}$$

where the eigenvalues have been ordered so that:

$$|\mathrm{Re}\ (\lambda_1)| \leq |\mathrm{Re}\ (\lambda_2)| \leq \ldots \leq |\mathrm{Re}\ (\lambda_n)| \tag{13}$$

If it is assumed that all the inputs are unit impulses, it can be shown that (for the strictly proper case, i.e., D = 0) the total energy at the output is given by:

$$E = \int_0^\infty y^T y\ dt = \mathrm{tr}\ (PW) \tag{14}$$

where: $P = C^T C$ (15)

$$W = [w_{ij}] \tag{16}$$

$$W_{ij} = - \bar{b}^T U^T \left[\frac{v_{i\mu} v_{i\nu}}{\lambda_\mu + \lambda_\nu}\right]_{\mu\nu} U \bar{b} \tag{17}$$

$\bar{b}$ = sum of the columns of B

Thus, the most significant states of the original system, to be retained in the reduced model, correspond to the larger elements on the main diagonal of PW.

3.2 Singular Perturbation Method for Reduction

This is an attractive approach for model reduction, since the physical nature of the model is preserved. It is based on partitioning the state vector into two parts, the 'slow' and the 'fast' part. Hence, equation (6) may be written as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \tag{18}$$

For a stable system, the fast modes decay much faster, so that after the initial period it is a reasonable approximation to set $\dot{x}_2 = 0$. Hence, x_2 may be eliminated from equation (18) to obtain:

$$\dot{x}_1 = (A_{11} - A_{12} A_{22}^{-1} A_{21}) x_1 + (B_1 - A_{12} A_{22}^{-1} B_2) u \tag{19}$$

The reduced model represented by equation (18) can now be solved to obtain the states approximately. If necessary, the effect of the 'fast' modes, which were neglected, can also be determined by returning to equation (18), since all is not lost.

The main difficulty with the method is the problem of deciding the proper partitioning of the state vector. The problem is complicated by the fact that, in general, the states are not decoupled, and therefore it is difficult to relate a particular state to a particular mode.

Sinha's method removes this difficulty while presenting a logical procedure for selecting the most significant states to be retained in the reduced model.

4. REDUCED-ORDER MODEL

The reduction of the original model was realized in different stages, as follows:

(1) The freight train model, consisting of thirty cars and two locomotives, is divided into blocks, each containing ten cars (see Figure 1). It is assumed that the same parallel braking input is applied to each member of the same block.

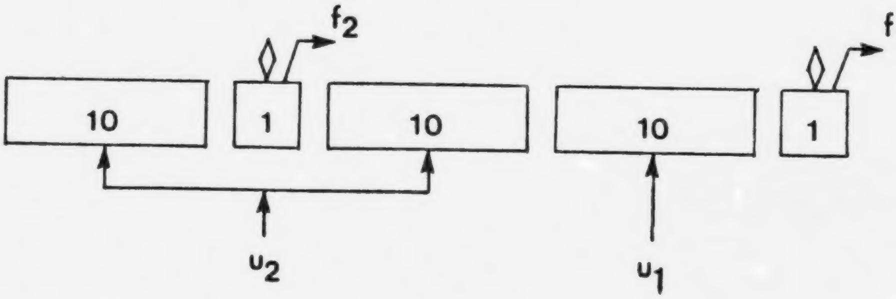

Figure 1 Configuration of Thirty Cars and Two Locomotives

(2) In the second stage, a reduced-order model is obtained by combining the method of Sinha and that based on a singular perturbation method for ten cars. This stage can be divided into two main steps. In the first step, the most significant states of the ten-car train model were selected by using the criterion proposed by Rozha, Sinha and Latsman.[21] The output matrix, C, is taken as a unity matrix. For this purpose, a computer program was written in FORTRAN IV (see Annex C). If the states are ordered according to the large elements of the main diagonal of the energy participation matrix, the system equation shown in Figure 2 is obtained (refer to Annex A). It was decided to retain the first velocity states, x_1, x_2, x_5, x_7, and the displacement states, x_{11}, x_{14}, x_{16}, depending on those velocity states that are included in x_{11}, x_{14} and x_{16}, while going down according to the order in Figure 2. Thus, the reduced model of order 7 is obtained

$$\begin{bmatrix} x_1 \\ x_2 \\ x_5 \\ x_7 \\ x_3 \\ x_4 \\ x_6 \\ x_{10} \\ x_8 \\ x_9 \\ x_{11} \\ x_{12} \\ x_{14} \\ x_{13} \\ x_{16} \\ x_{17} \\ x_{15} \\ x_{19} \\ x_{18} \end{bmatrix} = \begin{bmatrix}
-\frac{c}{m_L} & \frac{c}{m_L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k}{m_L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{c}{m_c} & -\frac{2c}{m_c} & 0 & 0 & \frac{c}{m_c} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_c} & -\frac{k}{m_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{2c}{m_L} & 0 & 0 & \frac{c}{m_L} & \frac{c}{m_L} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_L} & 0 & 0 & 0 & -\frac{k}{m_L} & 0 & 0 \\
0 & 0 & 0 & -\frac{2c}{m_c} & 0 & 0 & \frac{c}{m_c} & 0 & \frac{c}{m_c} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_c} & -\frac{k}{m_c} & 0 & 0 & 0 \\
0 & \frac{c}{m_c} & 0 & 0 & -\frac{2c}{m_c} & \frac{c}{m_c} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_c} & 0 & -\frac{k}{m_c} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{c}{m_c} & 0 & \frac{c}{m_c} & -\frac{2c}{m_c} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k}{m_c} & \frac{k}{m_c} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{c}{m_c} & \frac{c}{m_c} & 0 & 0 & -\frac{2c}{m_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k}{m_c} & 0 & \frac{k}{m_c} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{c}{m_L} & 0 & \frac{c}{m_L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_L} & 0 \\
0 & 0 & 0 & \frac{c}{m_c} & 0 & 0 & 0 & 0 & -\frac{2c}{m_c} & \frac{c}{m_c} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_c} & 0 & 0 & -\frac{k}{m_c} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{c}{m_c} & \frac{c}{m_c} & -\frac{2c}{m_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k}{m_c} & \frac{k}{m_c} \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_5 \\ x_7 \\ x_3 \\ x_4 \\ x_6 \\ x_{10} \\ x_8 \\ x_9 \\ x_{11} \\ x_{12} \\ x_{14} \\ x_{13} \\ x_{16} \\ x_{17} \\ x_{15} \\ x_{19} \\ x_{18} \end{bmatrix}$$

Figure 2 The Order of Selected States for Ten Cars

by modifying Sinha's method to keep the main structure of the system matrices and the system to be stable.

The second step is to apply a reduction method. It was intended to use singular perturbation in a different manner for making use of the structural properties of the system equations. If we write the system equations for ten cars, the simple procedure applied can be explained as follows (it is assumed that the connections of ten-car blocks with the other cars are neglected):

$$\dot{x}_1 = -\frac{c}{m_c} x_1 + \frac{c}{m_c} x_2 - \frac{k}{m_L} x_{11} + \frac{1}{m_L} u_1$$

$$\dot{x}_2 = \frac{c}{m_c} x_1 - \frac{2c}{m_c} x_2 + \frac{c}{m_c} x_3 + \frac{k}{m_c} x_{11} - \frac{k}{m_c} x_{12} + \frac{1}{m_c} u_2$$

$$\dot{x}_3 = \frac{c}{m_c} x_2 - \frac{2c}{m_c} x_3 + \frac{c}{m_c} x_4 + \frac{k}{m_c} x_{12} - \frac{k}{m_c} x_{13} + \frac{1}{m_c} u_3$$

$$\dot{x}_4 = \frac{c}{m_c} x_3 - \frac{2c}{m_c} x_4 + \frac{c}{m_c} x_5 + \frac{k}{m_c} x_{13} - \frac{k}{m_c} x_{14} + \frac{1}{m_c} u_4$$

$$\dot{x}_5 = \frac{c}{m_c} x_4 - \frac{2c}{m_c} x_5 + \frac{c}{m_c} x_6 + \frac{k}{m_c} x_{14} - \frac{k}{m_c} x_{15} + \frac{1}{m_c} u_5$$

$$\dot{x}_6 = \frac{c}{m_c} x_5 - \frac{2c}{m_c} x_6 + \frac{c}{m_c} x_7 + \frac{k}{m_c} x_{15} - \frac{k}{m_c} x_{16} + \frac{1}{m_c} u_6$$

$$\dot{x}_7 = \frac{c}{m_c} x_6 - \frac{2c}{m_c} x_7 + \frac{c}{m_c} x_8 + \frac{k}{m_c} x_{16} - \frac{k}{m_c} x_{17} + \frac{1}{m_c} u_7$$

$$\dot{x}_8 = \frac{c}{m_c} x_7 - \frac{2c}{m_c} x_8 + \frac{c}{m_c} x_9 + \frac{k}{m_c} x_{17} - \frac{k}{m_c} x_{18} + \frac{1}{m_c} u_8$$

$$\dot{x}_9 = \frac{c}{m_c} x_8 - \frac{2c}{m_c} x_9 + \frac{c}{m_c} x_{10} + \frac{k}{m_c} x_{18} - \frac{k}{m_c} x_{19} + \frac{1}{m_c} u_9$$

$$\dot{x}_{10} = \frac{c}{m_c} x_9 - \frac{c}{m_c} x_{10} + \frac{k}{m_c} x_{19} + \frac{1}{m_c} u_{10}$$

$$\dot{x}_{11} = x_1 - x_2$$

$$\dot{x}_{12} = x_2 - x_3$$

$$\dot{x}_{13} = x_3 - x_4$$

$$\dot{x}_{14} = x_4 - x_5$$

$$\dot{x}_{15} = x_5 - x_6$$

$$\dot{x}_{16} = x_6 - x_7$$

$$\dot{x}_{17} = x_7 - x_8$$

$$\dot{x}_{18} = x_8 - x_9$$

$$\dot{x}_{19} = x_9 - x_{10}$$

Setting $\dot{x}_4 = \dot{x}_3 = 0$, taking $x_2 = x_3 = x_4$ from $\dot{x}_{12} = \dot{x}_{13} = 0$ and eliminating $-\frac{k}{m_c} x_{13}$, $-\frac{k}{m_c} x_{12}$, we get:

$$\dot{x}_2 = \frac{c}{m_c} x_1 - \frac{2c}{m_c} x_2 + \frac{c}{m_c} x_5 + \frac{k}{m_c} x_{11} - \frac{k}{m_c} x_{14} + \frac{1}{m_c} u_2 + \frac{1}{m_c} u_3 + \frac{1}{m_c} u_4$$

Setting $\dot{x}_6 = 0$,, taking $x_5 = x_6$ from $\dot{x}_{15} = 0$ and eliminating $-\frac{k}{m_c} x_{15}$ yields:

$$\dot{x}_5 = \frac{c}{m_c} x_2 - \frac{2c}{m_c} x_5 + \frac{c}{m_c} x_7 + \frac{k}{m_c} x_{14} - \frac{k}{m_c} x_{16} + \frac{1}{m_c} u_5 + \frac{1}{m_c} u_6$$

Setting $\dot{x}_{10} = \dot{x}_9 = \dot{x}_8 = 0$, taking $x_7 = x_8 = x_9 = x_{10}$ from $\dot{x}_{17} = \dot{x}_{18} = \dot{x}_{19} = 0$, and eliminating $-\frac{k}{m_c} x_{19}$, $-\frac{k}{m_c} x_{18}$, $-\frac{k}{m_c} x_{17}$, we get:

$$\dot{x}_7 = \frac{c}{m_c} x_5 - \frac{c}{m_c} x_7 + \frac{k}{m_c} x_{16} + \frac{1}{m_c} u_7 + \frac{1}{m_c} u_8 + \frac{1}{m_c} u_9 + \frac{1}{m_c} u_{10}$$

and

$$\dot{x}_{14} = x_2 - x_5$$

$$\dot{x}_{16} = x_5 - x_7$$

are found. Hence, the following new state equations are obtained:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_5 \\ \dot{x}_7 \\ \dot{x}_{11} \\ \dot{x}_{14} \\ \dot{x}_{16} \end{bmatrix} = \begin{bmatrix} -\frac{c}{m_c} & \frac{c}{m_c} & 0 & 0 & -\frac{k}{m_c} & 0 & 0 \\ \frac{c}{m_c} & -\frac{2c}{m_c} & \frac{c}{m_c} & 0 & \frac{k}{m_c} & -\frac{k}{m_c} & 0 \\ 0 & \frac{c}{m_c} & -\frac{2c}{m_c} & \frac{c}{m_c} & 0 & \frac{k}{m_c} & -\frac{k}{m_c} \\ 0 & 0 & \frac{c}{m_c} & -\frac{c}{m_c} & 0 & 0 & \frac{k}{m_c} \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_5 \\ x_7 \\ x_{11} \\ x_{14} \\ x_{16} \end{bmatrix} +$$

$$\left[\begin{array}{cccccccccc} \frac{1}{m_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{m_c} & \frac{1}{m_c} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} \\ \hdashline \\ \multicolumn{10}{c}{0} \\ \\ \end{array}\right] \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \end{bmatrix}$$

If we consider Figure 2, the physical meaning of setting $\dot{x}_{12} = \dot{x}_{13} = 0$, $\dot{x}_{15} = 0$, and $\dot{x}_{17} = \dot{x}_{18} = \dot{x}_{19} = 0$ is that the blocks with dashed lines are assumed as the rigid bodies.

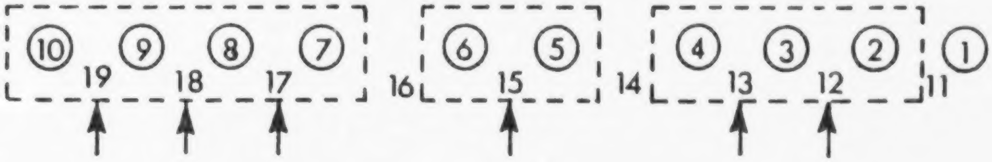

Figure 3 The Rigid Bodies Assumed in the Ten-Car, Seven-State Model

As is seen, the original structure of the A matrix is not destroyed and for this reason the reduced system matrix has the same eigenvalues as the A matrix of the original model. Hence, the reduced model is stable like the system.

If we keep the velocity states, $\dot{x}_1$ and $\dot{x}_2$, and take $\dot{x}_{11}$ depending on those states and apply the same procedure as explained above, the reduced model of order 3 for ten cars is obtained as follows:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_{11} \end{bmatrix} = \begin{bmatrix} -\frac{c}{m_c} & \frac{c}{m_c} & -\frac{k}{m_c} \\ \frac{c}{m_c} & -\frac{c}{m_c} & \frac{k}{m_c} \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_{11} \end{bmatrix} + \tag{22}
$$

$$
\left[\begin{array}{cccccccccc} \frac{1}{m_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} & \frac{1}{m_c} \\ \hdashline & & & & & & & & & \\ & & & & 0 & & & & & \\ & & & & & & & & & \end{array}\right] \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \end{bmatrix}
$$

(3) Since each train block will contain the same selected states, by taking these and the adjoining states of the blocks, the combination of train blocks is completed in this stage. The selected states are illustrated in Figures 4 and 5 for two reduced models of the whole train of order 27 and 15, respectively.

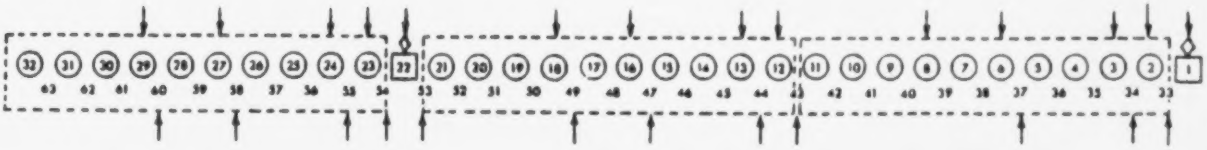
32 31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
63 62 61 60 59 58 57 56 55 54 53 52 51 50 49 48 47 46 45 44 43 42 41 40 39 38 37 36 35 34 33

Figure 4 Selected States for Thirty Cars and Two Locomotives by Using the Reduced Model of Order 7 for Each Ten-Car Block

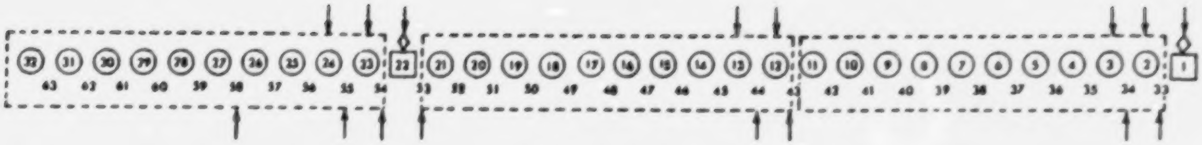

Figure 5 Selected States for the Whole Train by Using the Reduced Model of Order 3 for Each Ten-Car Block

If we consider Figures 4 and 5, the following state vector matrix of ten-cars,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \\ x_7 \\ x_{11} \\ x_{14} \\ x_{16} \end{bmatrix}$$

is replaced by the state vector matrices which are given for each ten-car block of reduced model of order 27 as follows:

$$\underline{x}' = \begin{bmatrix} x_2 \\ x_3 \\ x_6 \\ x_8 \\ x_{34} \\ x_{37} \\ x_{39} \end{bmatrix}, \quad \underline{x}'' = \begin{bmatrix} x_{12} \\ x_{13} \\ x_{16} \\ x_{18} \\ x_{44} \\ x_{47} \\ x_{49} \end{bmatrix}, \quad \underline{x}''' = \begin{bmatrix} x_{23} \\ x_{24} \\ x_{27} \\ x_{29} \\ x_{55} \\ x_{58} \\ x_{60} \end{bmatrix}$$

and for the reduced model of order 15, are given as follows:

$$\underline{x}' = \begin{bmatrix} x_2 \\ x_3 \\ x_{34} \end{bmatrix}, \quad \underline{x}'' = \begin{bmatrix} x_{12} \\ x_{13} \\ x_{44} \end{bmatrix}, \quad \underline{x}''' = \begin{bmatrix} x_{23} \\ x_{24} \\ x_{55} \end{bmatrix}$$

If the system equations are written for thirty cars and two locomotives and the procedure explained in the second stage is applied:

$$\dot{x}_1 = -\frac{c}{m_L} x_1 + \frac{c}{m_L} x_2 - \frac{k}{m_L} x_{33} + \frac{1}{m_L} u_1$$

$$\dot{x}_2 = \frac{c}{m_c} x_1 - \frac{2c}{m_c} x_2 + \frac{c}{m_c} x_3 + \frac{k}{m_c} x_{32} - \frac{k}{m_c} x_{34} + \frac{1}{m_c} u_2$$

$$\vdots$$

$$\dot{x}_{22} = \frac{c}{m_L} x_{21} - \frac{2c}{m_L} x_{22} + \frac{c}{m_L} x_{23} + \frac{k}{m_L} x_{53} - \frac{k}{m_L} x_{54} + \frac{1}{m_L} u_{22}$$

$$\vdots$$

$$\dot{x}_{31} = \frac{c}{m_c} x_{30} - \frac{2c}{m_c} x_{31} + \frac{c}{m_c} x_{32} + \frac{k}{m_c} x_{62} - \frac{k}{m_c} x_{63} + \frac{1}{m_c} u_{31} \qquad (23)$$

$$\dot{x}_{32} = - \frac{c}{m_c} x_{31} + \frac{c}{m_c} x_{32} + \frac{k}{m_c} x_{63} + \frac{1}{m_c} u_{32}$$

$$\dot{x}_{33} = x_1 - x_2$$

$$\dot{x}_{34} = x_2 - x_3$$

$$\vdots$$

$$\dot{x}_{53} = x_{21} - x_{22}$$

$$\dot{x}_{54} = x_{22} - x_{23}$$

$$\vdots$$

$$\dot{x}_{62} = x_{30} - x_{31}$$

$$\dot{x}_{63} = x_{31} - x_{32}$$

Setting $\dot{x}_4 = \dot{x}_5 = 0$, taking $x_3 = x_4 = x_5$ from $\dot{x}_{35} = \dot{x}_{36} = 0$ and eliminating $- \frac{k}{m_c} x_{36}$, $- \frac{k}{m_c} x_{35}$ gives:

$$\dot{x}_3 = \frac{c}{m_c} x_2 - \frac{2c}{m_c} x_3 + \frac{c}{m_c} x_6 + \frac{k}{m_c} x_{34} - \frac{k}{m_c} x_{37} + \frac{1}{m_c} u_3 + \frac{1}{m_c} u_4 + \frac{1}{m_c} u_5$$

Setting $\dot{x}_7 = 0$, taking $x_6 = x_7$ from $\dot{x}_{38} = 0$ and eliminating $- \frac{k}{m_c} x_{38}$ yields:

$$\dot{x}_6 = \frac{c}{m_c} x_3 - \frac{2c}{m_c} x_6 + \frac{c}{m_c} x_8 + \frac{k}{m_c} x_{37} - \frac{k}{m_c} x_{39} + \frac{1}{m_c} u_6 + \frac{1}{m_c} u_7$$

and continuing in this manner, the reduced model of order 27 is obtained. If we keep the states $\dot{x}_1$, $\dot{x}_2$, $\dot{x}_3$, $\dot{x}_{12}$, $\dot{x}_{13}$, $\dot{x}_{22}$, $\dot{x}_{23}$, $\dot{x}_{24}$, $\dot{x}_{43}$, $\dot{x}_{44}$, $\dot{x}_{53}$, $\dot{x}_{54}$ and $\dot{x}_{55}$ and set the other states to zero, the reduced model of order 15 is found. The reduced system matrices of the model of order 27 and 15 are given as follows:

$$\tilde{A}(27) = \begin{bmatrix}
-\frac{c}{m_L} & \frac{c}{m_L} & & -\frac{k}{m_L} & & & & & & & & & & \\
\frac{c}{m_c} & -\frac{2c}{m_c} & & \frac{c}{m_c} & \frac{k}{m_c} & -\frac{k}{m_c} & & & & & & & & \\
& \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & & & & 0 & & & \\
& & \frac{c}{m_c} & -\frac{2c}{m_c} & & \frac{c}{m_c} & \frac{k}{m_c} & -\frac{k}{m_c} & & & & & & \\
& & & \frac{c}{m_L} & -\frac{2c}{m_L} & \frac{c}{m_L} & 0 & \frac{k}{m_L} & -\frac{k}{m_L} & & & & & \\
& & & & \frac{c}{m_c} & \frac{2c}{m_c} & & \frac{c}{m_c} & \frac{k}{m_c} & -\frac{k}{m_c} & & & & \\
& & 0 & & & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & & & \\
& & & & & & \frac{c}{m_c} & -\frac{2c}{m_c} & & \frac{c}{m_c} & \frac{k}{m_c} & -\frac{k}{m_c} & & \\
& & & & & & & & \frac{c}{m_c} & -\frac{c}{m_c} & & & \frac{k}{m_c} & \\
1 & -1 & & & & & & & & & & & & \\
& 1 & -1 & & & & & & \vdots & \vdots & & & & \\
& & \ddots & \ddots & & & & & \vdots & \vdots & & & 0 & \\
& 0 & & & \ddots & \ddots & & & \vdots & \vdots & & & & \\
& & & & & & 1 & -1 & & & & & & \\
& & & & & & & 1 & -1 & & & & &
\end{bmatrix}$$

$$\tilde{B}_{(27)} = \begin{bmatrix} \frac{1}{m_L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m_c} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{m_c} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{m_c} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{m_c} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{m_c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{m_c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{m_c} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_L} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{m_c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{m_c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{4}{m_c} \end{bmatrix}$$

$$\tilde{A}(15) = \begin{bmatrix}
-\frac{c}{m_L} & \frac{c}{m_L} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{k}{m_L} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{c}{m_c} & -\frac{2c}{m_c} & \frac{c}{m_c} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_c} & -\frac{k}{m_c} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{c}{m_c} & -\frac{2c}{m_c} & \frac{c}{m_c} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_c} & -\frac{k}{m_c} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{c}{m_c} & -\frac{2c}{m_c} & \frac{c}{m_c} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_c} & -\frac{k}{m_c} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{c}{m_c} & -\frac{2c}{m_c} & \frac{c}{m_c} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_c} & -\frac{k}{m_c} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{c}{m_L} & -\frac{2c}{m_L} & \frac{c}{m_L} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_L} & -\frac{k}{m_L} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{c}{m_c} & -\frac{2c}{m_c} & \frac{c}{m_c} & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_c} & -\frac{k}{m_c} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{c}{m_c} & -\frac{c}{m_c} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{k}{m_c} \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$\tilde{B}_{(15)} = \begin{bmatrix} \frac{1}{m_L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m_c} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{9}{m_c} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{9}{m_c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_L} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_c} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{9}{m_c} \end{bmatrix}$$

(4) In this stage, the Riccati equation is solved for the original as well as for the reduced-order models and the feedback gain matrices are obtained.

(5) In the last stage, the reduced-order models are tested and compared with the original model by using those feedback gain matrices in the train simulation program with the same data.[26]

5. NUMERICAL EXAMPLE

Consider a two-locomotive, thirty-car train with the following data:

m_c = 102 tons

m_L = 139 tons

$k = 70 \times 10^5$ Newton/metre

$c = 2.04 \times 10^5$ N.sec./metre

The performance index may be taken as:

$$J = \int_0^\infty (x^T Q x + x^T E u + u^T R u) \, dt$$

The matrices Q and R are chosen using an argument similar to that used in the ordinary LQR approach.[18,19] The matrix E accounts for the energy consumption that is determined solely by the locomotive's throttle and velocity.[26] The computation times and the total energy consumptions are given in Table 1 for the original and reduced-order models.

TABLE 1

COMPARISON OF COMPUTATION TIMES AND TOTAL ENERGY CONSUMPTIONS

	Original Model	Reduced Model of Order 27	Reduced Model of Order 15
Total Energy Consumption (J)	0.9283037×10^8 joules	0.9841318×10^8 joules	0.9988624×10^8 joules
Computation Time for Solving the Riccati Equation	40 min.	12 min.	7 min.

6. SIMULATION RESULTS

The data used for the simulation and test conditions are given in Tables 2 and 3.

TABLE 2

DATA OF THE TWO-LOCOMOTIVE, THIRTY-CAR TRAIN

	Locomotives	Cars
Type	U25	gondola
Weight	139 tons	102 tons
Length	18.3 metres	19.7 metres
Maximum Brake Force	4×10^5 Newtons	1.47×10^5 Newtons
Maximum Power	2000 hp	--
Locomotive Transition Velocities	8,13,26 km/h	--

TABLE 3

DATA AND SCHEDULE VALUES FOR SIMULATION

	Train Section		
	1	2	3
Length of Train Section (m)	724	965	5,632
Grade (per cent slope)	0.0	-2.0	0.0
Target Velocity (km/h)	64.36	64.36	64.36
Total Rolling and Air Resistance (N)	7.72×10^4	7.72×10^4	7.72×10^4
Total Gravitational Forces (N)	0.0	-6.4×10^5	0.0

The simulation results are presented in Table 4 and Figures B.1, B.2 and B.3.

TABLE 4

COMPARISON OF RUNNING TIMES OF THE TRAIN SIMULATION PROGRAM

	Original Model	Reduced Model of Order 27	Reduced Model of Order 15
Running Times of the Train Simulation Program	14 mins.	10 mins.	4 mins.

If we examine Figures B.1, B.2 and B.3, it is seen that the gain matrix obtained from the reduced model of order 27 by solving the Riccati equation is sufficient to design a near-optimal controller for the original feedback controller.

7. CONCLUSION

Reduced-order models have been designed and evaluated for two-locomotive, thirty-car powered trains. It made use of the criterion proposed by Rozha, Sinha and Latsman[21] for selecting the most significant states of the system. As a reduction method, singular perturbation was applied in a different manner to use the structural properties of the general system equations. Simulation results show that the obtained reduced model of order 27 can be used to build a near-optimal controller for the original feedback controller.

Further investigation is needed to build the reduced-order models for long freight trains having number of members greater than 100.

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ANNEX A

RIGHT (V) AND LEFT (U) EIGENVALUES OF A MATRIX OF A TEN-CAR MODEL, ENERGY PARTICIPATION MATRIX (P)

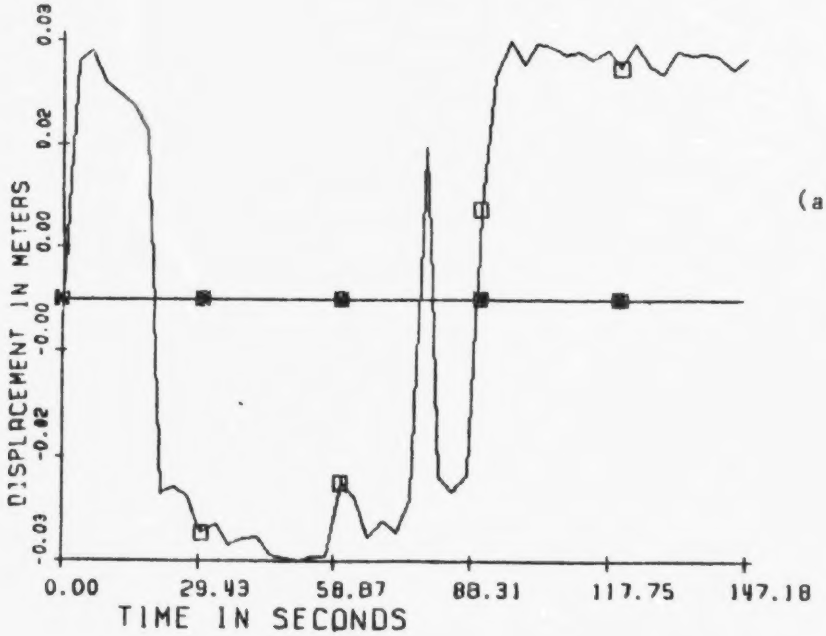
$$V = \left[\begin{array}{rrrrrrrrrrrrrrrrrrr}
0.07 & -0.37 & -0.22 & 0.71 & 0.34 & -0.98 & -0.43 & 1.15 & 0.47 & -1.21 & -0.47 & 1.15 & 0.43 & -0.98 & -0.34 & 0.74 & 0.22 & -0.37 & -0.08 \\
-1.13 & 0.04 & 3.27 & -0.07 & -5.10 & 0.10 & 6.42 & -0.12 & -7.12 & 0.13 & 7.12 & -0.12 & -6.42 & 0.10 & 5.10 & -0.07 & -3.27 & 0.04 & 1.13 \\
-0.01 & 0.70 & 0.02 & -1.13 & -0.03 & 1.13 & 0.02 & -0.70 & -0.00 & -0.00 & -0.00 & 0.70 & 0.02 & -1.14 & -0.03 & 1.13 & 0.02 & -0.70 & -0.01 \\
2.21 & -0.11 & -5.79 & 0.18 & 7.16 & -0.18 & -5.79 & 0.11 & 2.21 & 0.00 & 2.21 & -0.11 & -5.79 & 0.18 & 7.16 & -0.18 & -5.79 & 0.41 & 2.21 \\
-1.19 & -0.00 & 2.59 & 0.00 & -1.85 & -0.00 & -0.41 & -0.00 & 2.33 & 0.00 & -2.33 & -0.00 & 0.41 & -0.00 & 1.85 & 0.00 & -2.59 & -0.00 & 1.19 \\
-0.18 & -0.36 & 0.40 & -0.43 & -0.29 & 0.14 & -0.06 & 0.26 & 0.36 & -0.45 & -0.36 & 0.26 & 0.06 & 0.14 & 0.28 & -0.43 & -0.40 & 0.36 & 0.18 \\
0.37 & -1.16 & -0.61 & 0.72 & 0.00 & 0.72 & 0.61 & -1.16 & -0.37 & 0.00 & -0.37 & 1.16 & 0.61 & -0.72 & 0.00 & -0.72 & -0.61 & 1.16 & 0.37 \\
-4.23 & 0.06 & 6.84 & -0.04 & -0.00 & -0.04 & -6.84 & 0.06 & 4.23 & -0.00 & 4.23 & -0.06 & -6.84 & 0.04 & 0.00 & 0.04 & 6.84 & -0.06 & -4.23 \\
2.33 & -0.16 & -2.33 & 0.00 & -2.33 & 0.16 & 2.33 & -0.00 & 2.33 & 0.16 & -2.33 & 0.00 & -2.33 & 0.16 & 2.33 & -0.00 & 2.33 & -0.16 & -2.33 \\
-0.39 & -0.54 & 0.39 & -0.00 & 0.39 & 0.54 & -0.39 & -0.00 & -0.39 & -0.54 & 0.39 & 0.00 & 0.39 & 0.54 & -0.39 & 0.00 & -0.39 & -0.54 & 0.39 \\
2.49 & 0.85 & -0.84 & 0.53 & -2.71 & -0.53 & -0.84 & -0.85 & 2.19 & 0.00 & 2.19 & 0.85 & -0.84 & 0.53 & -2.71 & -0.53 & -0.04 & -0.05 & 2.19 \\
4.54 & -0.52 & -1.74 & 0.32 & -5.61 & 0.32 & -1.73 & 0.52 & 4.54 & -0.00 & 4.54 & -0.52 & -1.74 & -0.32 & -5.61 & 0.32 & -1.73 & 0.52 & 4.54 \\
2.70 & 0.13 & 0.47 & 0.15 & -2.14 & 0.05 & -2.99 & -0.10 & -1.37 & -0.16 & 1.37 & -0.10 & 2.99 & 0.05 & 2.14 & 0.15 & -0.47 & 0.13 & -2.20 \\
1.07 & -0.42 & 0.19 & -0.50 & -0.85 & -0.16 & -1.19 & 0.31 & -0.55 & 0.53 & 0.55 & 0.31 & 1.19 & -0.16 & 0.85 & -0.50 & -0.19 & -0.42 & -1.07 \\
5.06 & 0.00 & 5.06 & 0.00 & 5.06 & 0.00 & 5.06 & 0.00 & 5.06 & -0.00 & 5.06 & -0.00 & 5.06 & -.00 & 5.06 & -0.00 & 5.06 & -0.00 & 5.06 \\
3.15 & -0.05 & 2.84 & -0.10 & 2.26 & -0.13 & 1.45 & -0.16 & 0.50 & -0.16 & -0.50 & -0.16 & -1.45 & -0.13 & -2.26 & -0.10 & -2.04 & -0.05 & -3.15 \\
-0.88 & -0.17 & -0.79 & -0.31 & -0.63 & -0.43 & -0.40 & -0.51 & -0.14 & -0.53 & 0.14 & -0.51 & 0.10 & -0.43 & 0.63 & -0.31 & 0.79 & -0.17 & 0.88 \\
-2.96 & 0.59 & -1.83 & 0.96 & 0.00 & 0.96 & 1.83 & 0.59 & 2.96 & -0.00 & 2.96 & -0.59 & 1.83 & -0.96 & 0.00 & -0.96 & -1.83 & -0.59 & -2.96 \\
5.55 & 0.28 & 3.43 & 0.45 & 0.00 & 0.45 & -3.43 & 0.28 & -5.55 & -0.00 & -5.55 & -0.28 & -3.43 & -0.45 & 0.00 & -0.45 & 3.43 & 0.28 & 5.55
\end{array}\right]$$

$$U = \begin{bmatrix}
0.00 & 0.01 & -0.02 & 0.01 & -0.03 & -0.01 & -0.00 & -0.02 & -0.03 & 0.03 & -0.05 & 0.03 & -0.02 & -0.07 & 0.02 & 0.07 & 0.01 & 0.02 & 0.03 \\
-0.06 & 0.06 & -0.15 & 0.19 & 0.07 & -0.33 & 0.15 & -0.03 & -0.27 & -0.23 & -0.21 & -0.33 & 0.00 & -0.04 & -0.13 & -0.04 & 0.25 & -0.17 & 0.11 \\
-0.01 & -0.02 & 0.05 & -0.03 & 0.07 & 0.03 & 0.00 & 0.03 & 0.03 & -0.03 & 0.02 & -0.01 & -0.02 & -0.06 & 0.02 & 0.04 & 0.01 & 0.00 & 0.01 \\
0.12 & -0.11 & 0.24 & 0.31 & -0.09 & 0.39 & -0.10 & 0.02 & 0.00 & 0.00 & -0.13 & -0.20 & 0.00 & -0.07 & -0.25 & -0.06 & 0.41 & -0.20 & 0.13 \\
0.02 & 0.03 & -0.06 & 0.03 & -0.05 & -0.02 & 0.00 & -0.00 & 0.03 & -0.03 & 0.06 & -0.03 & -0.02 & -0.05 & 0.01 & 0.00 & -0.00 & -0.01 & -0.03 \\
-0.16 & 0.15 & -0.24 & -0.31 & 0.03 & -0.13 & -0.10 & 0.02 & 0.27 & 0.23 & 0.13 & 0.20 & -0.00 & -0.10 & -0.34 & -0.06 & 0.41 & -0.07 & -0.04 \\
-0.02 & -0.03 & 0.05 & -0.03 & -0.01 & -0.00 & -0.00 & -0.03 & -0.03 & 0.03 & 0.02 & -0.01 & -0.02 & -0.03 & 0.01 & -0.04 & -0.01 & -0.02 & -0.04 \\
0.19 & -0.18 & 0.15 & 0.19 & 0.05 & -0.24 & 0.15 & 0.03 & -0.00 & -0.00 & 0.21 & 0.33 & -0.00 & -0.11 & -0.40 & -0.04 & 0.26 & 0.13 & -0.08 \\
0.02 & 0.04 & -0.02 & 0.01 & 0.06 & 0.02 & 0.00 & 0.02 & -0.03 & 0.03 & -0.05 & 0.03 & -0.02 & -0.01 & 0.00 & -0.07 & -0.01 & -0.01 & -0.02 \\
-0.20 & 0.18 & 0.00 & 0.00 & -0.09 & 0.41 & -0.00 & -0.00 & -0.27 & -0.23 & -0.00 & -0.00 & -0.00 & -0.12 & -0.42 & -0.00 & -0.00 & 0.21 & -0.14 \\
-0.02 & -0.04 & -0.02 & 0.01 & -0.06 & -0.02 & 0.00 & 0.02 & 0.03 & -0.03 & -0.05 & 0.03 & -0.02 & 0.01 & -0.00 & -0.07 & -0.01 & 0.01 & 0.02 \\
0.19 & -0.18 & -0.15 & -0.19 & 0.05 & -0.24 & -0.16 & 0.03 & 0.00 & 0.00 & -0.21 & -0.33 & -0.00 & -0.11 & -0.40 & 0.04 & -0.26 & 0.13 & -0.08 \\
0.02 & 0.03 & 0.05 & -0.03 & 0.01 & 0.00 & -0.00 & -0.03 & 0.03 & -0.05 & 0.02 & -0.01 & -0.02 & 0.03 & -0.01 & -0.04 & -0.01 & 0.02 & 0.04 \\
0.16 & 0.15 & 0.24 & 0.31 & 0.03 & -0.13 & 0.10 & -0.02 & 0.27 & 0.23 & -0.13 & -0.20 & -0.00 & -0.10 & -0.34 & 0.06 & -0.41 & -0.07 & 0.04 \\
-0.02 & -0.03 & -0.06 & 0.03 & 0.05 & 0.02 & 0.00 & 0.00 & -0.03 & 0.03 & 0.06 & -0.03 & -0.02 & 0.05 & -0.01 & 0.00 & -0.00 & 0.01 & 0.03 \\
0.12 & -0.11 & -0.23 & -0.31 & -0.09 & 0.39 & 0.10 & -0.02 & -0.00 & -0.00 & 0.13 & 0.20 & -0.00 & -0.07 & -0.25 & 0.06 & 0.41 & -0.20 & 0.13 \\
0.01 & 0.02 & 0.05 & -0.03 & -0.07 & -0.03 & 0.00 & 0.03 & -0.03 & 0.03 & 0.02 & -0.01 & -0.02 & 0.06 & -0.02 & 0.04 & 0.01 & -0.00 & -0.01 \\
0.06 & 0.06 & 0.15 & 0.19 & 0.07 & -0.33 & -0.15 & 0.03 & -0.27 & -0.23 & 0.21 & 0.33 & -0.00 & -0.04 & -0.13 & 0.04 & -0.27 & -0.17 & 0.11 \\
0.00 & -0.01 & -0.02 & 0.01 & 0.03 & 0.01 & -0.00 & -0.02 & 0.03 & 0.03 & -0.05 & 0.03 & -0.02 & 0.07 & -0.02 & 0.07 & 0.01 & -0.02 & -0.03
\end{bmatrix}$$

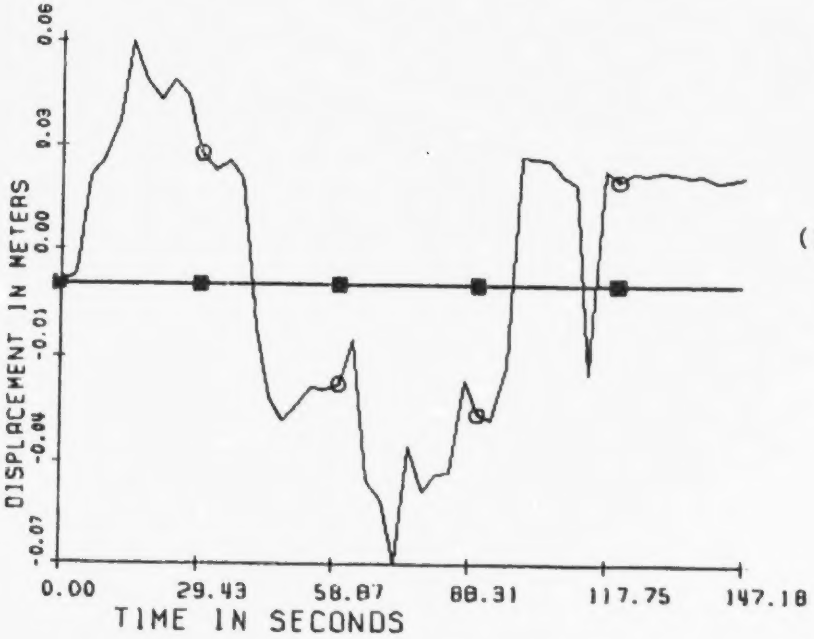
$$= \begin{bmatrix}
-3275.269 & -1.398 & 0.520 & 0.232 & 0.123 & 0.266 & -1.009 & -0.317 & 0.701 & 0.123 & 0.361 & 0.211 & 0.298 & -0.123 & 0.168 & -0.007 & 0.520 & 0.227 & -0.102 \\
-1.182 & 0.149 & -29.53 & -0.046 & -0.371 & -0.060 & 0.606 & 0.049 & -0.614 & -0.029 & -0.198 & -0.052 & 0.114 & 0.028 & -0.992 & 0.002 & -0.266 & -0.049 & 0.152 \\
20.573 & 61.731 & -9.211 & -2.225 & -2.023 & -1.968 & 4.826 & 1.289 & -5.994 & -0.741 & -2.090 & -1.211 & 1.490 & 0.621 & -1.162 & 0.051 & -3.160 & -1.048 & 1.914 \\
-0.033 & 0.353 & -3.580 & -0.080 & -8.749 & -0.072 & 1.656 & 0.168 & 0.186 & -0.043 & -0.171 & -0.078 & -0.998 & 0.054 & -0.019 & 0.002 & -0.167 & -0.112 & -0.286 \\
13.774 & 0.518 & 12.710 & 9.104 & 3.044 & 4.658 & -5.140 & -0.948 & 8.483 & 1.102 & 2.525 & 1.663 & -4.153 & 0.720 & 1.554 & -0.082 & 3.961 & 1.048 & -3.351 \\
0.683 & -0.130 & -0.346 & 0.056 & 0.047 & -0.054 & -4.240 & -0.086 & 4.171 & 0.045 & 0.867 & 0.080 & -0.417 & -0.046 & 0.327 & -0.003 & 0.834 & 0.084 & -0.441 \\
3.943 & -1.720 & -1.761 & 1.150 & -0.459 & 1.775 & 1.892 & 2.984 & -1.854 & -0.770 & -0.790 & -1.164 & -0.005 & 0.621 & -0.428 & 0.036 & -1.252 & -1.056 & 0.519 \\
0.604 & 0.066 & -1.592 & 0.015 & -0.785 & 0.023 & 2.910 & 0.063 & -4.216 & 0.009 & -0.197 & 0.014 & -1.768 & 0.0006 & 0.200 & -0.002 & 0.399 & -0.013 & -0.751 \\
-3.574 & -1.105 & 6.113 & 0.038 & 1.604 & 1.477 & -1.971 & -5.672 & 4.473 & 2.075 & 1.149 & 2.213 & -3.216 & -0.702 & 0.801 & -0.092 & 1.910 & 0.692 & -2.155 \\
-0.363 & 0.080 & 1.246 & -0.053 & 0.508 & -0.080 & -1.877 & 0.030 & 2.239 & -0.032 & -0.094 & -0.078 & 3.115 & 0.038 & -0.070 & 0.004 & -1.555 & -0.061 & 1.552 \\
6.604 & -0.946 & -6.138 & 0.965 & -1.658 & 2.095 & 3.670 & 2.253 & -5.576 & -1.248 & 1.748 & -3.773 & 2.166 & 2.105 & -1.162 & 0.135 & -3.121 & -3.299 & 2.249 \\
0.312 & -0.127 & 0.026 & 0.037 & 0.040 & 0.057 & 0.275 & -0.074 & 1.137 & 0.034 & 0.746 & 0.051 & -2.554 & -0.038 & 0.796 & -0.003 & 1.885 & 0.072 & -1.312 \\
-2.906 & -1.097 & 5.582 & 0.071 & 1.610 & 0.480 & -1.885 & -3.682 & 4.656 & 2.268 & 1.176 & 5.080 & -3.612 & -2.315 & 0.854 & -0.204 & 2.023 & 2.722 & -2.397 \\
-0.708 & 0.0004 & 1.274 & -0.034 & -0.500 & -0.053 & 1.374 & -0.019 & 3.176 & -0.027 & 1.316 & -0.047 & -0.040 & 5.044 & 0.355 & 0.003 & 0.603 & -0.021 & 0.599 \\
-1.400 & 0.829 & -0.028 & -0.368 & -0.008 & -0.858 & -0.757 & 1.187 & 0.355 & -1.007 & 0.309 & -1.757 & 0.688 & 0.524 & 0.388 & 0.069 & 0.457 & -0.015 & 0.099 \\
0.520 & 0.190 & 0.083 & 0.060 & 0.0003 & 0.009 & 0.538 & -0.113 & 0.530 & 0.057 & 0.276 & 0.105 & -3.235 & -0.062 & 0.945 & -0.004 & 2.661 & 0.117 & -2.602 \\
-0.709 & 0.283 & 0.275 & -0.142 & 0.076 & -0.312 & -0.410 & 0.328 & 0.383 & -0.315 & 0.189 & -0.667 & 0.106 & 0.170 & 0.099 & 0.024 & 0.149 & 0.032 & -0.080 \\
-0.253 & 0.045 & 0.262 & -0.021 & 0.089 & -0.032 & -0.357 & 0.023 & 0.430 & 0.013 & 0.177 & -0.034 & 0.551 & 0.018 & -0.078 & 0.002 & -0.301 & 0.018 & 0.587 \\
1.356 & 0.361 & -2.544 & 0.013 & -0.713 & -0.046 & 0.931 & 0.982 & -2.168 & -0.397 & -0.579 & -1.265 & 1.600 & 0.979 & -0.416 & 0.077 & -1.008 & -1.884 & 1.116
\end{bmatrix}$$

ANNEX B

DIAGRAMS FOR THE ORIGINAL AND REDUCED-ORDER MODELS

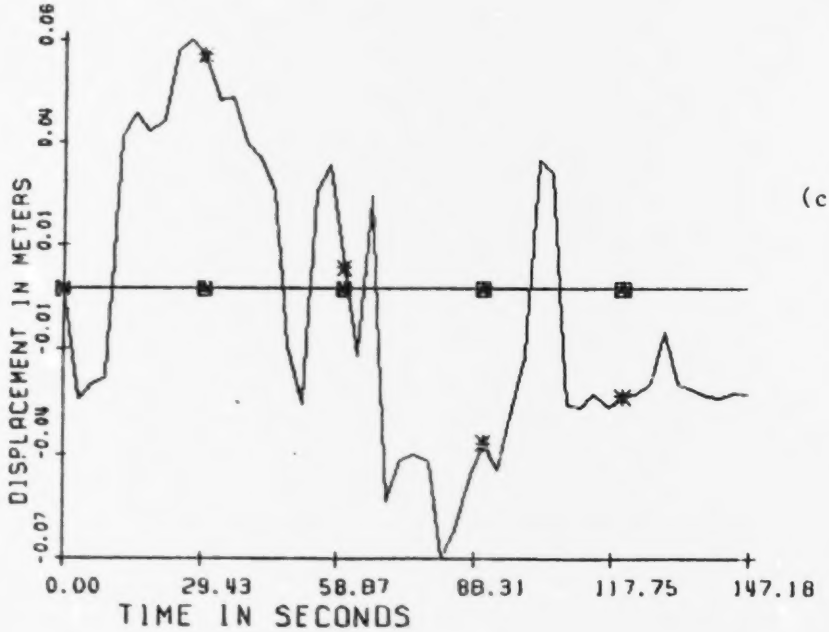


(a) Coupling Displacement Between Train Members 1 and 2

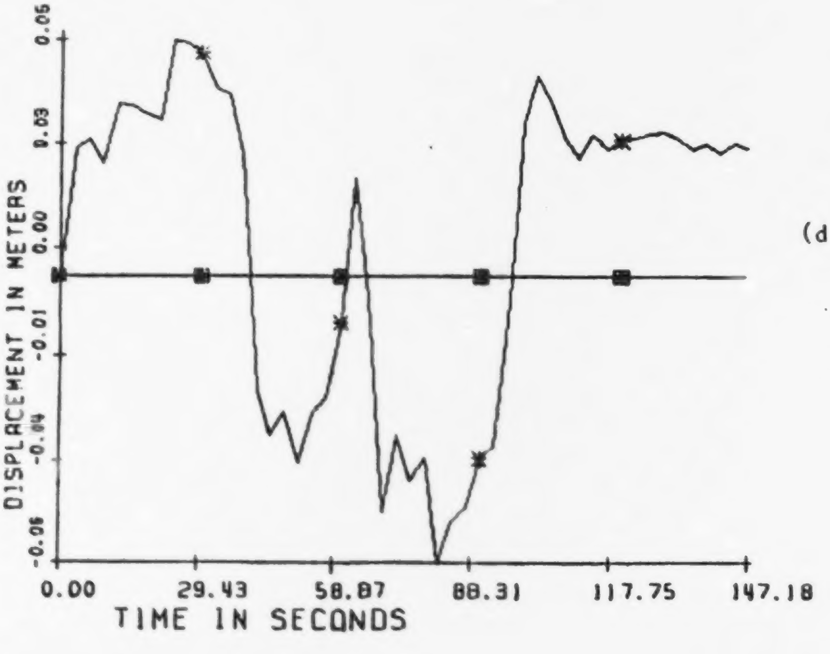


(b) Coupling Displacement Between Train Members 10 and 11

Figure B.1 Simulation Results for the Original Model

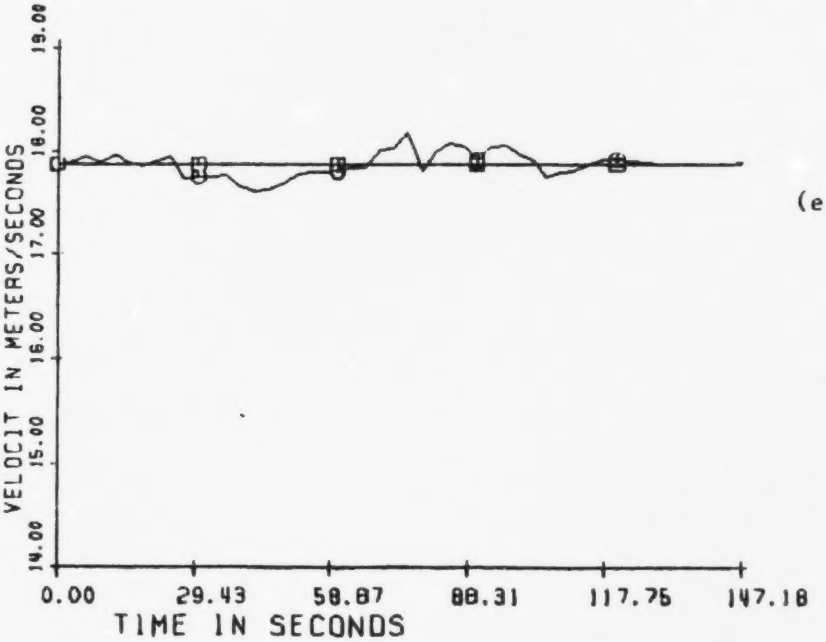


(c) Coupling Displacement Between Train Members 21 and 22

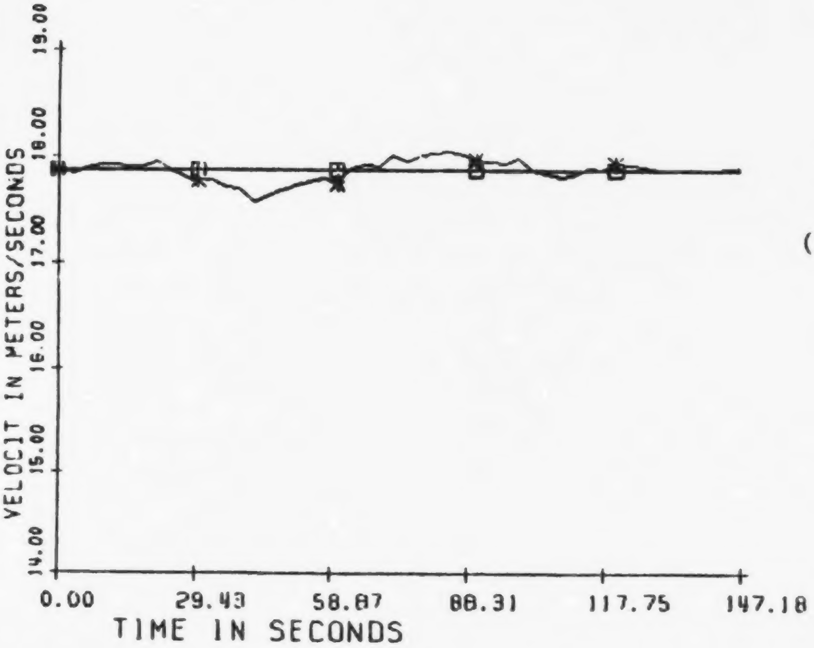


(d) Coupling Displacement Between Train Members 22 and 23

Figure B.1 Simulation Results for the Original Model (cont'd)

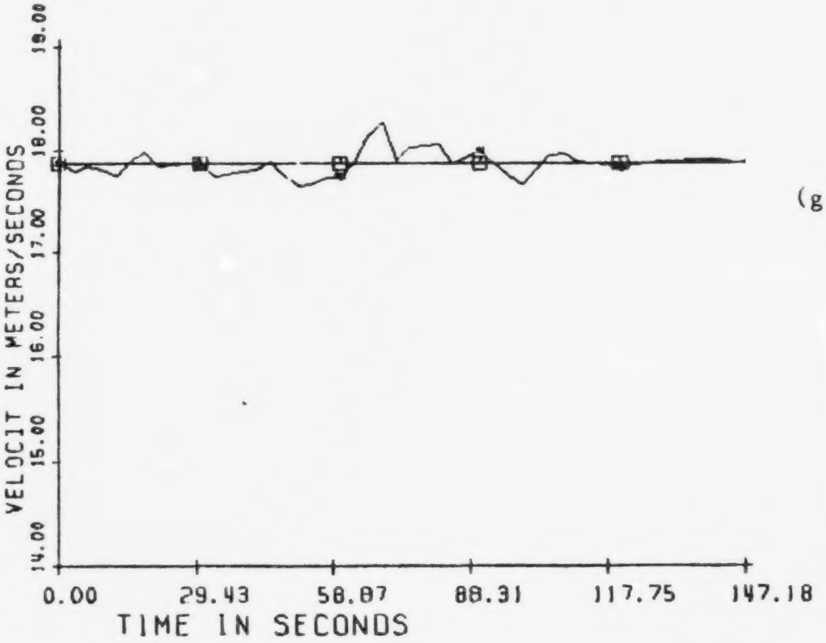


(e) Velocity of Train Member 1

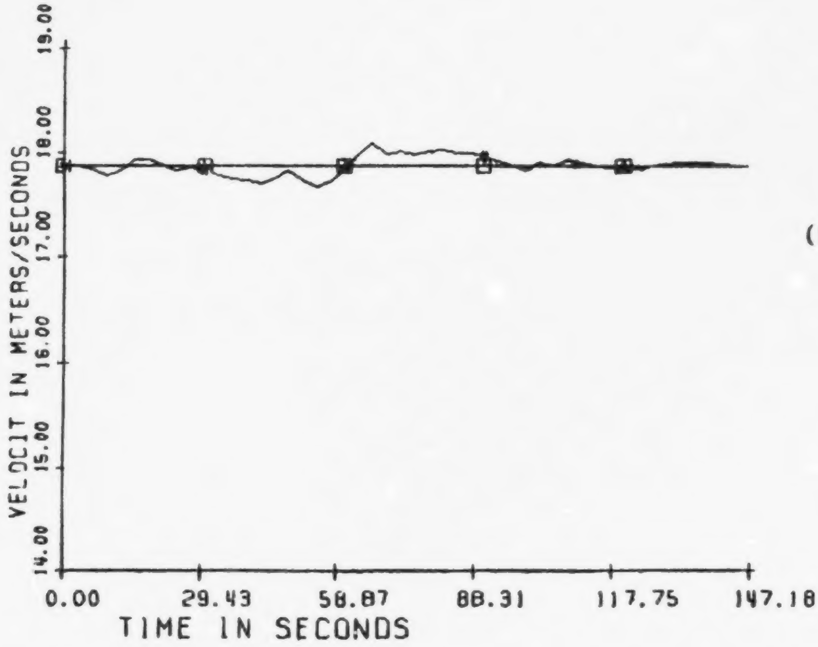


(f) Velocity of Train Member 10

Figure B.1 Simulation Results for the Original Model (cont'd)

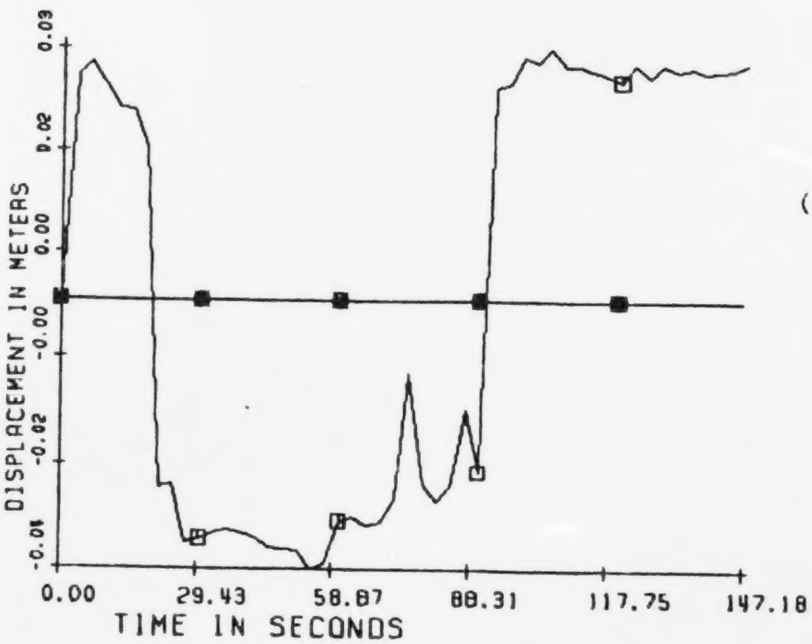


(g) Velocity of Train Member 22

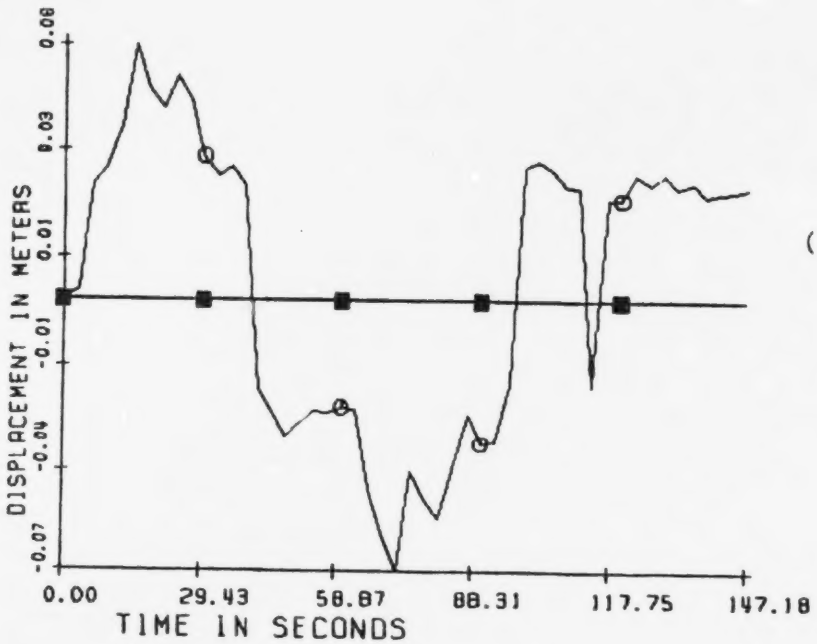


(h) Velocity of Train Member 32

Figure B.1 Simulation Results for the Original Model (cont'd)

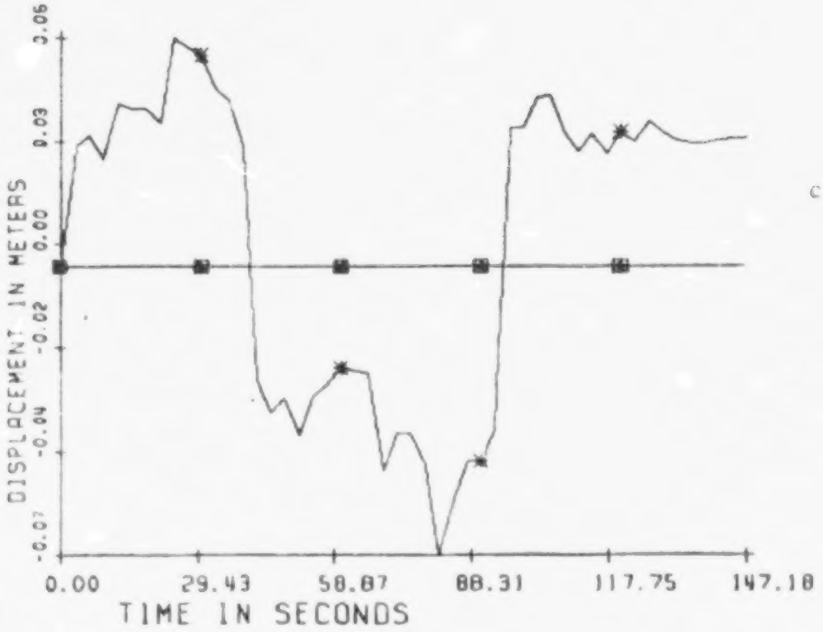


(a) Coupling Displacement Between Train Members 1 and 2

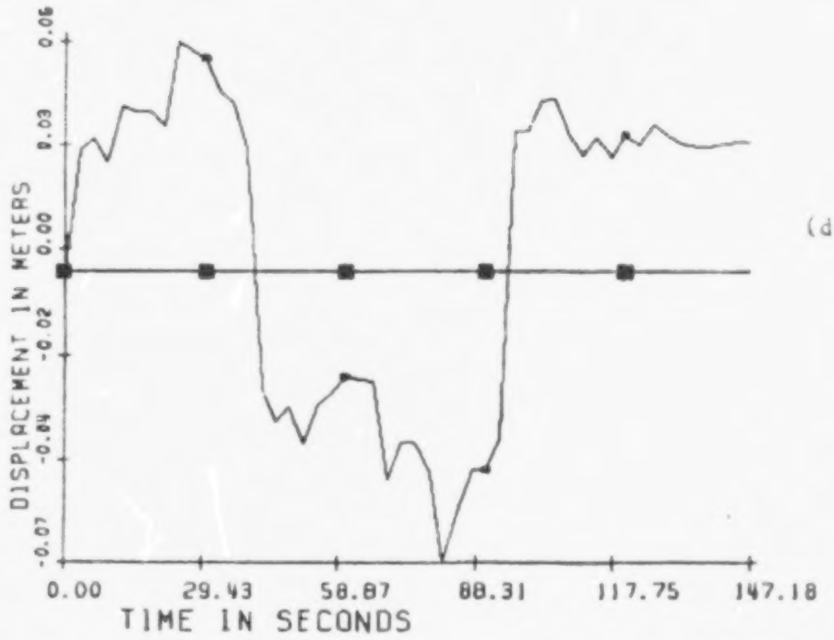


(b) Coupling Displacement Between Train Members 10 and 11

Figure B.2 Simulation Results for the Reduced Model of Order 27

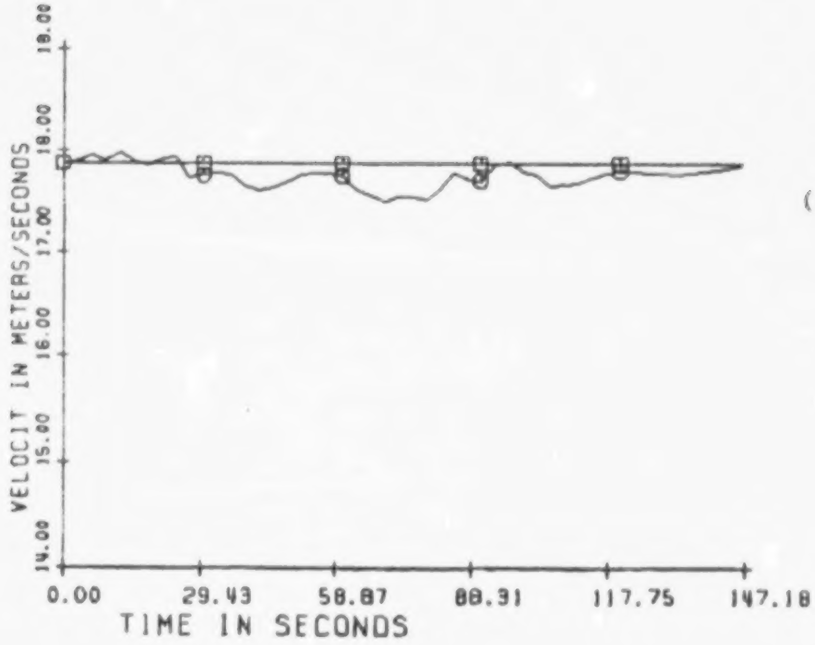


c) Coupling Displacement Between Train Members 21 and 22

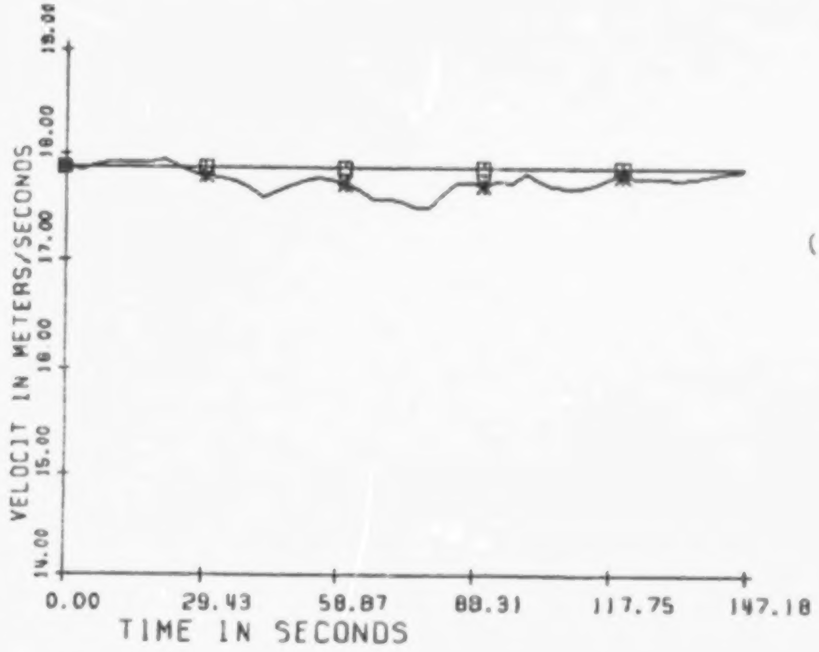


(d) Coupling Displacement Between Train Members 22 and 23

Figure B.2 Simulation Results for the Reduced Model of Order 27 (cont'd)

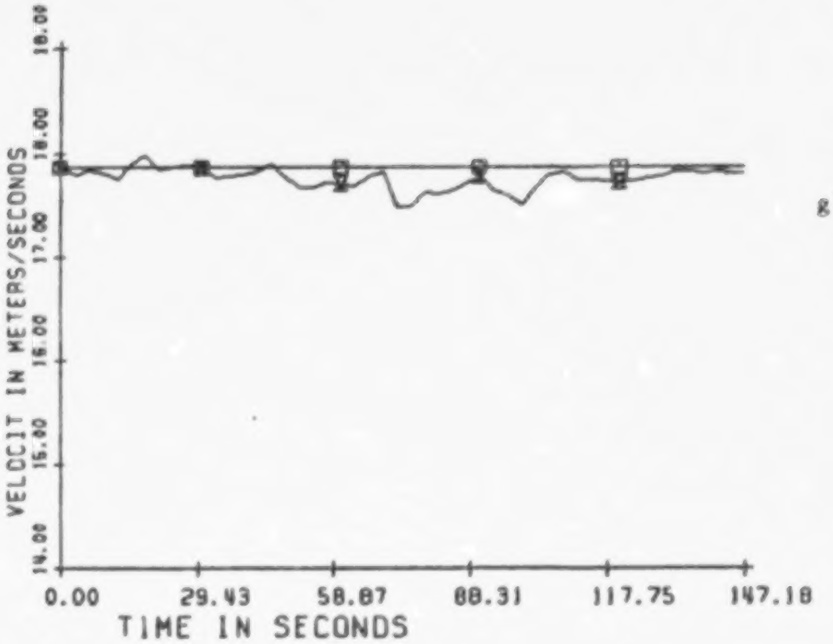


(e) Velocity
of
Train
Member 1

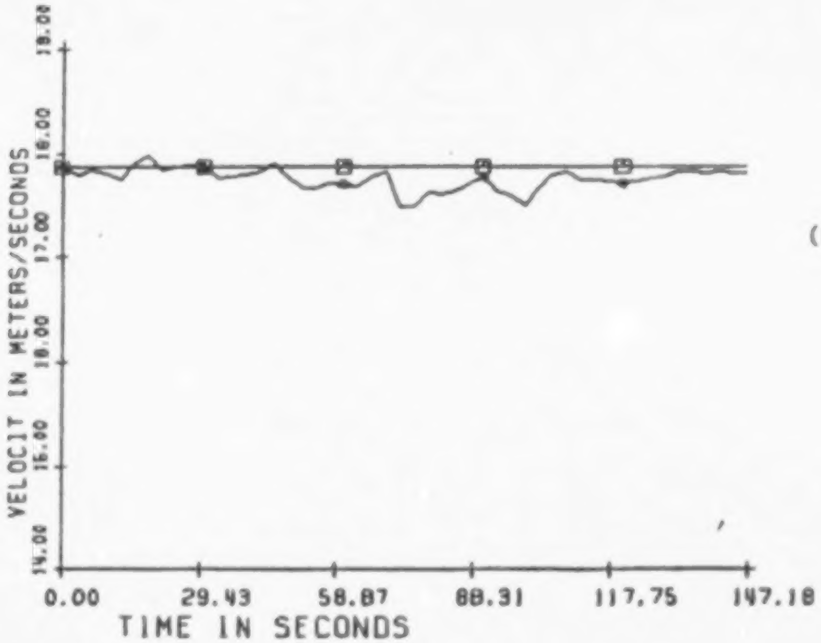


(f) Velocity
of
Train
Member 10

Figure B.2 Simulation Results for the Reduced Model of Order 27 (cont'd)

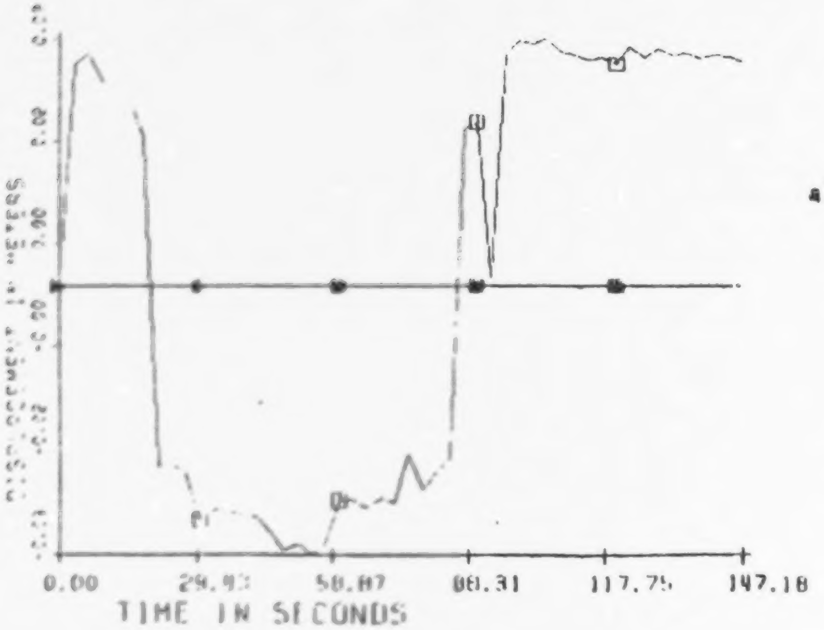


g) Velocity of Train Member 22

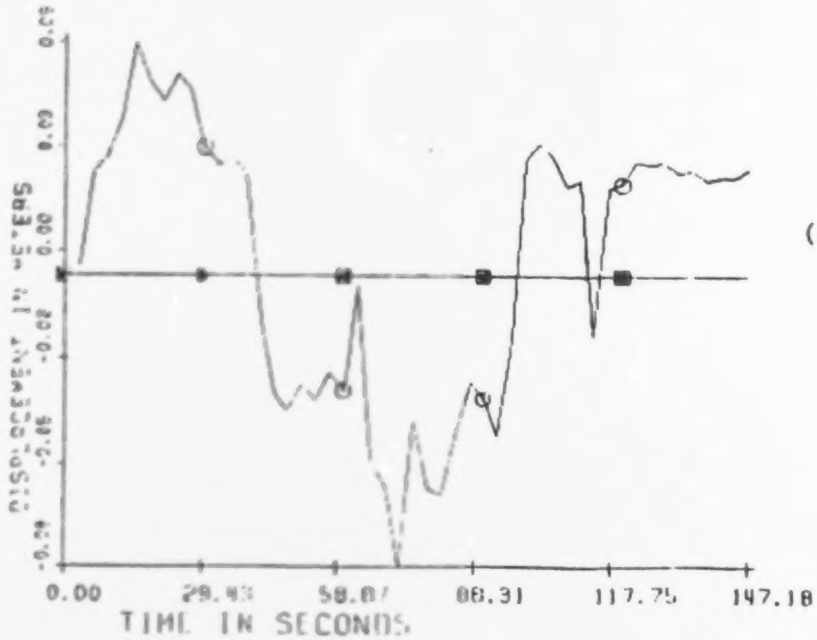


(h) Velocity of Train Member 32

Figure B.2 Simulation Results for the Reduced Model of Order 27 (cont'd)

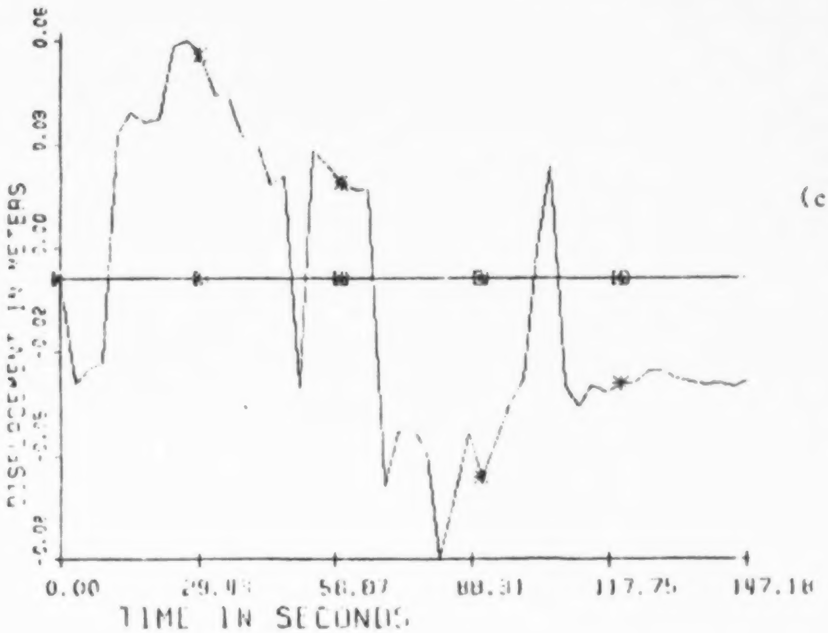


a) Coupling Displacement Between Train Members 1 and 2

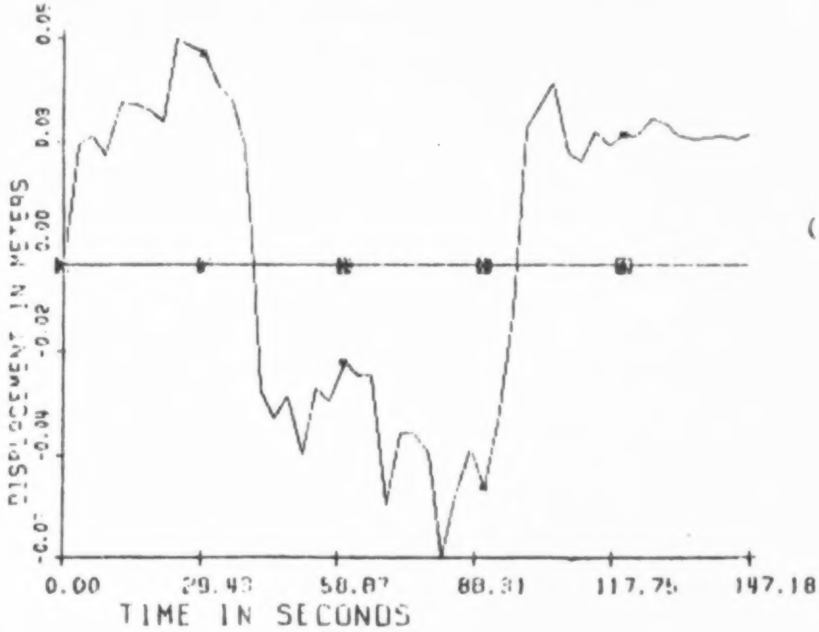


(b) Coupling Displacement Between Train Members 10 and 11

Figure B.3 Simulation Results for the Reduced Model of Order 15

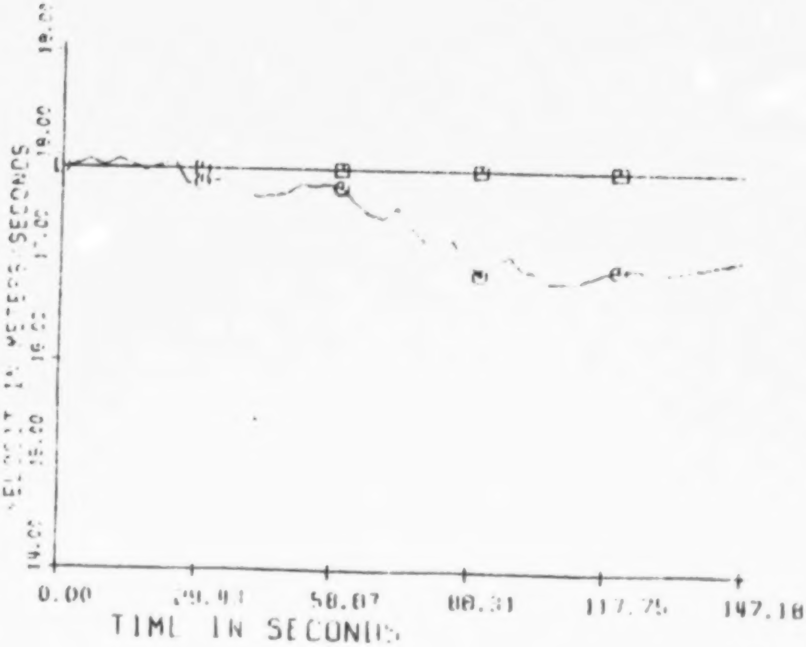


(c) Coupling Displacement Between Train Members 21 and 22

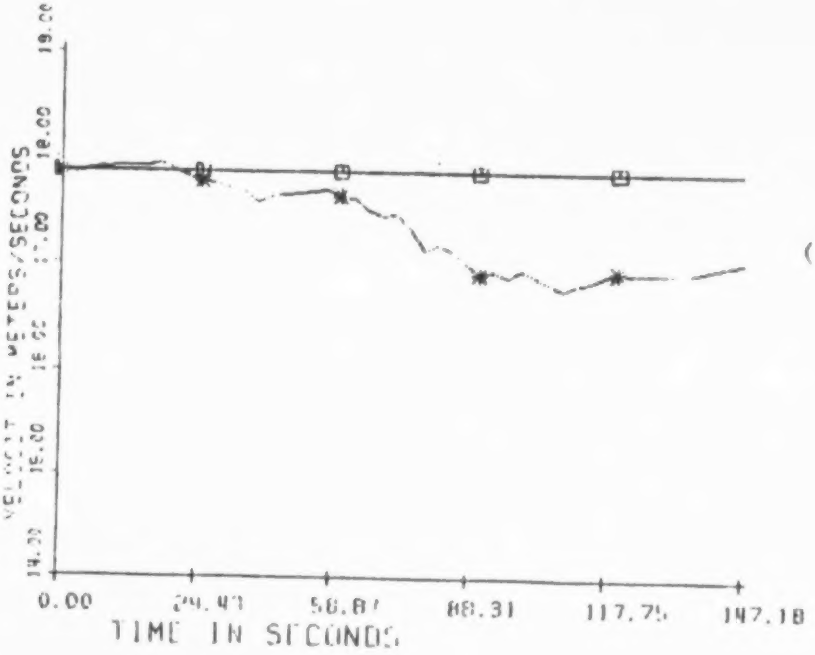


(d) Coupling Displacement Between Train Members 22 and 23

Figure B.3 Simulation Results for the Reduced Model of Order 15 (cont'd)

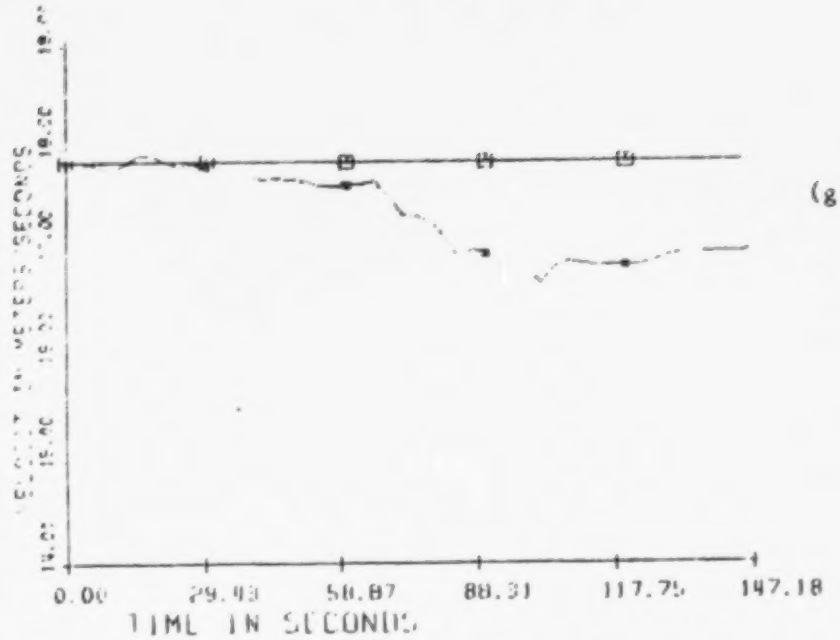


e) Velocity of Train Member 1

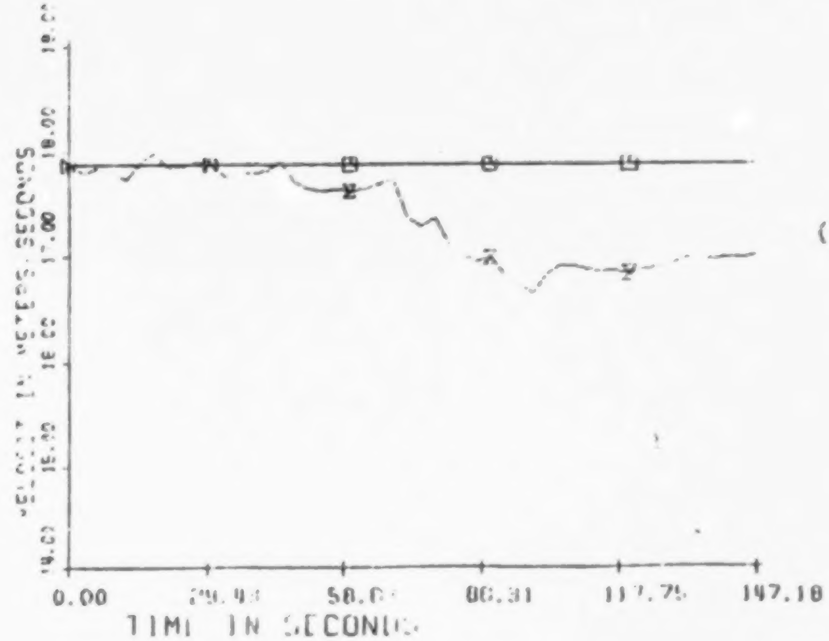


(f) Velocity of Train Member 10

Figure B.3 Simulation Results for the Reduced Model of Order 15 (cont'd)



(g) Velocity of Train Member 22



(h) Velocity of Train Member 32

Figure B.3 Simulation Results for the Reduced Model of Order 15 (cont'd)

ANNEX C

COMPUTER PROGRAM FOR SELECTING THE MOST SIGNIFICANT STATES ACCORDING TO SINHA'S METHOD

```
      DIMENSION  VV(19,19),UU(19,19),BT(19),BB(19,19),
     1AGWW(19,19),CC(19,19),DT(19,19),CTRAN(19,19)
      REAL A(19,19),Z(19,19),D(19),DD(19),WR(19),WI(19),RR(19,19),
     1UIMAG(19,19),VVUU(19,19),AA(19,19)
      COMPLEX ZZ,ZZN,VRIGT(19,19),ULEFT(19,19),VU(19,19),
     1GG(19,19),BUVV(19),BULEF(19),WW(19,19),ULEFB(19),RLAMDA(19),
     2TRLEF(19,19),ULEFN(19,19),PP(19,19)
C
      N=19
      MM=19
      IA=19
      IZ=19
      ICONT=0
C
C     READ(5,4441) ICTR
C4441 FORMAT(I5)
C
      DO 11 I=1,N
      READ(5,22)  (A(I,J),J=1,N)
 22   FORMAT(5(D14.7,2X))
 11   CONTINUE
C
C
      DO 152 I=1,N
      READ(5,22)   (BB(I,J),J=1,3)
 152  CONTINUE
C
C
      DO 156 I=1,N
      READ(5,154)   RLAMDA(I)
 154  FORMAT(2F10.6)
 156  CONTINUE
C
C
      DO 791 I=1,N
      READ(5,792)  (CC(I,J),J=1,N)
 792  FORMAT(7F10.3)
 791  CONTINUE
C
C
      DO 13 I=1,N
      DO 13 J=1,N
      AA(I,J)=A(I,J)
 13   CONTINUE
C
      WRITE(6,99) ((I,J,A(I,J),J=1,N),I=1,N)
 99   FORMAT(1H1,3(1X,'A(',I2,',',I2,') = ',F14.7))
C
C
      WRITE(6,251)((I,J,BB(I,J),J=1,6),I=1,N)
 251  FORMAT(/ ,3(1X,'B(',I2,',',I2,') = ',E17.10))
C
      WRITE(6,255) ((I,J,CC(I,J),J=1,N),I=1,N)
 255  FORMAT(/,4(1X,'C(',I2,',',I2,') = ',E17.10))
C
```

```
C       WRITE(6,252)
C252    FORMAT(//,'EIGENVALUES OF A MATRIX')
C       WRITE(6,253) (RLAMDA(I),I=1,N)
C253    FORMAT(//,2(F14.6,F14.6))
C
        IF(ICONT.EQ.0) GO TO  888
C
 777    CONTINUE
        DO 333 I=1,N
        DO 334 J=1,N
        A(J,I)=AA(I,J)
 334    CONTINUE
 333    CONTINUE
C
C
 888    CONTINUE
        CALL EBALAF(A,N,IA,D,K,L)
        CALL EHESSF(A,K,L,N,IA,DD)
        DO 10 I=1,N
           DO 5 J=1,N
              Z(I,J)=0.0
   5    CONTINUE
        Z(I,I)=1.0
10      CONTINUE
        CALL EHBCKF(Z,A,DD,N,MM,IA,K,L)
        CALL EQRH3F(A,N,IA,K,L,WR,WI,Z,IZ,IER)
        CALL EBBCKF(D,Z,K,L,MM,N,IZ)
C
C
        IF(ICONT.EQ.1) GO TO 226
C
        DO 2221 I=1,N
        DO 2221 J=1,N
        VV(J,I)=Z(I,J)
 2221 CONTINUE
C
C       DO 9881 I=1,N
C       WRITE (6,9882) (VV(I,J),J=1,N)
C9882 FORMAT(5F13.9)
C9881 CONTINUE
C
C       DO 2223 I=1,N
C       DO 2224 J=1,N,2
C       IF(J.GT.13) GO TO 2227
C       VRIGT(I,J)=CMPLX(VV(I,J),VV(I,J+1))
C       IF(J.EQ.13) VRIGT(I,J)=VV(I,13)
C       GO TO 2224
C2227 VRIGT(I,J)=CMPLX(VV(I,J-1),-VV(I,J))
C2224 CONTINUE
C2223 CONTINUE
C       N2=N-2
C       DO 2225 I=1,N
C       DO 2226 J=1,N2.2
C       IF(J.GT.11) GO TO 2228
C       VRIGT(I,J+1)=CMPLX(VV(I,J),-VV(I,J+1))
```

```
C     GO TO 2226
C2228 VRIGT(I,J+1)=CMPLX(VV(I,J+1),VV(I,J+2))
C2226 CONTINUE
C2225 CONTINUE
C
C
C
      DO 9999 I=1,N
      VRIGT(I,1)=VV(I,15)
 9999 CONTINUE
C
      DO 9991 I=1,N
      VRIGT(I,2)=CMPLX(VV(I,16),VV(I,17))
      VRIGT(I,3)=CMPLX(VV(I,16),-VV(I,17))
      VRIGT(I,4)=CMPLX(VV(I,18),VV(I,19))
      VRIGT(I,5)=CMPLX(VV(I,18),-VV(I,19))
      VRIGT(I,6)=CMPLX(VV(I,13),VV(I,14))
      VRIGT(I,7)=CMPLX(VV(I,13),-VV(I,14))
      VRIGT(I,8)=CMPLX(VV(I,11),VV(I,12))
      VRIGT(I,9)=CMPLX(VV(I,11 ),-VV(I,12))
      VRIGT(I,10)=CMPLX(VV(I,9),VV(I,10 ))
      VRIGT(I,11)=CMPLX(VV(I,9),-VV(I,10 ))
      VRIGT(I,12)=CMPLX(VV(I,7),VV(I,8))
      VRIGT(I,13)=CMPLX(VV(I,7),-VV(I,8))
      VRIGT(I,14)=CMPLX(VV(I,5),VV(I,6))
      VRIGT(I,15)=CMPLX(VV(I,5),-VV(I,6))
      VRIGT(I,16)=CMPLX(VV(I,3),VV(I,4))
      VRIGT(I,17)=CMPLX(VV(I,3),-VV(I,4))
      VRIGT(I,18)=CMPLX(VV(I,1),VV(I,2))
      VRIGT(I,19)=CMPLX(VV(I,1),-VV(I,2))
 9991 CONTINUE
C
C
C     WRITE(6,222)
C222  FORMAT(//,'RIGHT EIGENVECTORS OF A MATRIX',/)
      GO TO 225
 226  CONTINUE
C     WRITE(6,223)
C223  FORMAT(//,'LEFT EIGENVECTORS OF A MATRIX',/)
 225  CONTINUE
C     DO 111 I=1,N
C     WRITE(6,66) (Z(I,J),J=1,N)
C66   FORMAT(4F14.9)
C111  CONTINUE
C
C     WRITE(6,77) (WR(I),I=1,N)
C77   FORMAT(10X,3HWR=,4F16.6)
C     WRITE(6,88) (WI(I),I=1,N)
C88   FORMAT(10X,3HWI=,4F16.6,)
C
C
      IF(ICONT.EQ.1) GO TO 999
C     IF(ICONT.EQ.1) GO TO 6666
      ICONT=1
      GO TO 777
```

```
FILE: EIGVEC   FORTRAN  A                       VM/SP CONVERSATIONAL MONITOR 
```

```
C
C
C  NORMALIZE EIGENVECTORS
C     J=1
C 15  IF(J.GT.N) GO TO 35
C     IF(WI(J).NE.0.0) GO TO 25
C     ZN=Z(1,J)
C     DO 20 I=1,N
C 20  Z(I,J)=Z(I,J)/ZN
C     J=J+1
C     GO TO 15
C 25  ZZN=CMPLX(Z(1,J),Z(1,J+1))
C     DO 30 I=1,N
C        ZZ=CMPLX(Z(I,J),Z(I,J+1))/ZZN
C        Z(I,J)=REAL(ZZ)
C        Z(I,J+1)=AIMAG(ZZ)
C 30  CONTINUE
C     J=J+2
C     GO TO 15
C 35  CONTINUE

C     DO 4442 I=1,N
C     WRITE(6,4443) (Z(I,J),J=1,N)
C4443 FORMAT(4F16.9)
C4442 CONTINUE
C
C
C     IF(ICTR.EQ.0.AND.ICONT.EQ.1. ) GO TO 999
C
C
C
C     ICONT=1
C     IF(ICONT.EQ.1) GO TO 777
C
C
C
 999  CONTINUE
      DO 555 I=1,N
      DO 550 J=1,N
      UU(I,J)=Z(I,J)
 550  CONTINUE
 555  CONTINUE
C
      DO 7774 I=1,N
      DO 7774 J=1,N
      VVUU(I,J)=0.
 7774 CONTINUE
C
      DO 7771 I=1,N
      DO 7772 J=1,N
      DO 7773 K=1,N
      VVUU(I,J)=VVUU(I,J)+VV(I,K)*UU(K,J)
 7773 CONTINUE
 7772 CONTINUE
 7771 CONTINUE
C
C     DO 7775 I=1,N
```

```
C      WRITE(6,7776) (VVUU(I,J),J=1,N)
C7776 FORMAT(1X,5HVVUU=,4F16.6)
C7775 CONTINUE
C
C      IF( ICTR.EQ.0. ) GO TO 6666
C
C      DO 7779 I=1,N
C      WRITE(6,7780) (UU(I,J),J=1,N)
C7780 FORMAT(5F13.9)
C7779 CONTINUE
C
C      DO 3332 J=1,5,2
C      DO 3332 I=1,N
C      ULEFT(J,I)=CMPLX(UU(I,J),UU(I,J+1))
C      ULEFT(J+1,I)=CMPLX(UU(I,J),-UU(I,J+1))
C3332 CONTINUE
C      DO 3334 J=12,16,2
C      DO 3335 I=1,N
C      ULEFT(J-5,I)=CMPLX(UU(I,J),UU(I,J+1))
C      ULEFT(J-4,I)=CMPLX(UU(I,J),-UU(I,J+1))
C3335 CONTINUE
C3334 CONTINUE
C
      DO 9993 I=1,N
      ULEFT(1,I)=UU(I,13)
 9993 CONTINUE
C
      DO 8884 I=1,N
      ULEFT(14,I)=CMPLX(UU(I,8  ),UU(I,9  ))
      ULEFT(15,I)=CMPLX(UU(I,8  ),-UU(I,9  ))
      ULEFT(16,I)=CMPLX(UU(I,10),UU(I,11))
      ULEFT(17,I)=CMPLX(UU(I,10),-UU(I,11))
 8884 CONTINUE

      DO 9994 I=1,N
      ULEFT(2,I)=CMPLX(UU(I,14 ),UU(I,15 ))
      ULEFT(3,I)=CMPLX(UU(I,14 ),-UU(I,15 ))
      ULEFT(4,I)=CMPLX(UU(I,16),UU(I,17))
      ULEFT(5,I)=CMPLX(UU(I,16),-UU(I,17))
      ULEFT(6,I)=CMPLX(UU(I,18),UU(I,19))
      ULEFT(7,I)=CMPLX(UU(I,18),-UU(I,19))
      ULEFT(8,I)=CMPLX(UU(I,11),UU(I,12))
      ULEFT(9,I)=CMPLX(UU(I,11),-UU(I,12))
      ULEFT(10,I)=CMPLX(UU(I,9),UU(I,10 ))
      ULEFT(11,I)=CMPLX(UU(I,9),-UU(I,10 ))
      ULEFT(12,I)=CMPLX(UU(I,7),UU(I,8))
      ULEFT(13,I)=CMPLX(UU(I,7),-UU(I,8))
      ULEFT(14,I)=CMPLX(UU(I,5 ),UU(I,6 ))
      ULEFT(15,I)=CMPLX(UU(I,5 ),-UU(I,6 ))
      ULEFT(16,I)=CMPLX(UU(I,3),UU(I,4))
      ULEFT(17,I)=CMPLX(UU(I,3),-UU(I,4))
      ULEFT(18,I)=CMPLX(UU(I,1),UU(I,2))
      ULEFT(19,I)=CMPLX(UU(I,1),-UU(I,2))
 9994 CONTINUE
C
```

```
C
      DO 554 I=1,N
      DO 554 J=1,N
      VU(I,J)=0.0
 554  CONTINUE
      DO 661 I=1,N
      DO 662 J=1,N
      DO 663 K=1,N
      VU(J,I)=VU(J,I)+VRIGT(K,I)*ULEFT(J,K)
 663  CONTINUE
 662  CONTINUE
 661  CONTINUE
C
C
      DO 391 I=1,N
      DO 391 J=1,N
      IF(I.EQ.J) GO TO 391
      VU(I,J)=0.
 391  CONTINUE
C
      DO 392 I=1,N
      VU(I,I)=1./(VU(I,I))
 392  CONTINUE
C
      DO 393 I=1,N
      DO 393 J=1,N
      ULEFN(I,J)=ULEFT(I,J)*VU(J,J)
 393  CONTINUE
C
      DO 3931 I=1,N
      WRITE (6,3932) (ULEFN(I,J),J=1,N)
3932  FORMAT(10X,6HULEFN=,3(F10.6,F10.6))
3931  CONTINUE
C
C     DO 3935 I=1,N
C     DO 3935 J=1,N
C     RR(I,J)=REAL(ULEFN(I,J))
C3935 CONTINUE
C
C     WRITE(6,3936)
C3936 FORMAT(/,'REAL PARTS OF U MATRIX')
C
C     DO 3933 I=8,N
C     WRITE(6,3934) (RR(I,J),J=8,N)
C3934 FORMAT(6F10.6)
C3933 CONTINUE
C
C     WRITE(6,3937)
C3937 FORMAT(/,'IMAGINAR PARTS OF U MATRIX')
C
C     DO 3938 I=1,N
C     DO 3938 J=1,N
C     UIMAG(I,J)=AIMAG(ULEFN(I,J))
C3938 CONTINUE
C
```

```
C      DO 3939 I=8,N
C      WRITE(6,3940) (UIMAQ(I,J),J=8,N)
C3940  FORMAT(6F10.6)
C3939  CONTINUE
C
C      WRITE(6,3943)
C3943  FORMAT(/,'U21 MATRIX')
C
C      DO 3941 I=8,19
C      WRITE(6,3942) (ULEFN(I,J),J=1,7)
C3942  FORMAT(3(F10.6,F10.6))
C3941  CONTINUE
C
C      DO 558 I=1,N
C      WRITE(6,557) (VU(I,J),J=1,N)
C558   CONTINUE
C557   FORMAT(10X,3HVU=,3(F10.6,F10.6))
C
       DO 4444 I=1,N
       WRITE(6,4451)  (VRIQT(I,J),J=1,N)
 4451  FORMAT(10X,6HVRIQT=,3(F10.6,F10.6))
 4444  CONTINUE
C      DO 4445 I=1,N
C      WRITE(6,4442)  (ULEFT(I,J),J=1,N)
C4442  FORMAT(5X,6HULEFT=,3(F10.6,F10.6))
C4445  CONTINUE
C
C
       DO 133 I=1,N
       DO 133 J=1,N
       TRLEF(J,I)=ULEFN(I,J)
  133  CONTINUE
C
C      DO 177 I=1,N
C      WRITE(6,178) (TRLEF(I,J),J=1,N)
C178   FORMAT(5X,7HTULEFT=,3(F10.6,F10.6))
C177   CONTINUE
C
       DO 134 I=1,N
  134  BT(I)=0.
C
       DO 135 I=1,N
       DO 135 J=1,3
       BT(I)=BT(I)+BB(I,J)
  135  CONTINUE
C
C      WRITE(6,191) (BT(I),I=1,N)
C191   FORMAT(5X,3HBT=,5F10.6)
C
       DO 136 I=1,N
  136  BULEF(I)=0.
C
       DO 137 I=1,N
       DO 137 J=1,N
       BULEF(I)=BULEF(I)+BT(J)*TRLEF(J,I)
  137 CONTINUE
C
C     WRITE(6,192) (BULEF(I),I=1,N)
C192  FORMAT(10X,6HBULEF=,5F10.6)
C
      DO 138 I=1,N
  138 ULEFB(I)=0.
C
      DO 139 I=1,N
      DO 139 J=1,N
      ULEFB(I)=ULEFB(I)+TRLEF(I,J)*BT(J)
  139 CONTINUE
C
C     WRITE(6,193) (ULEFB(I),I=1,N)
C193  FORMAT(10X,6HULEFB=,5F10.6)
C
      DO 127 K5=1,N
      DO 127 K6=1,N
      WW(K5,K6)=0.
  127 CONTINUE
C
      DO 118 I=1,N
      DO 119 J=1,N
      DO 121 L=1,N
      DO 121 K=1,N
      GG(K,L)=(VRIGT(I,K)*VRIGT(J,L))/(RLAMDA(I)+RLAMDA(J))
  121 CONTINUE
      DO 123 K1=1,N
  123 BUVV(K1)=0.
C
      DO 124 K3=1,N
      DO 124 K4=1,N
      BUVV(K3)=BUVV(K3)+BULEF(K4)*GG(K4,K3)
C     WRITE(6,189) BUVV(K3)
C189  FORMAT(3X,5HBUVV=,2(E17.10,E17.10))
  124 CONTINUE
      DO 128 K6=1,N
  128 WW(I,J)=WW(I,J)+BUVV(K6)*ULEFB(K6)
  119 CONTINUE
C     WRITE(6,341)((GG(L1,L2),L2=1,N),L1=1,N)
C341  FORMAT(5X,3HGG=,3(F10.6,F10.6))
  118 CONTINUE
      WRITE(6,499)
  499 FORMAT(//,'VALUES OF W MATRIX',//)
      DO 129 I=1,N
      WRITE(6,141) (WW(I,J),J=1,N)
  141 FORMAT(5X,3HWW=,2(E20.10,E20.10))
  129 CONTINUE
C
C  COMPUTE ELEMENTS OF D MATRIX
C
      DO 691 I=1,N
      DO 691 J=1,N
      CTRAN(J,I)=CC(I,J)
  691 CONTINUE
```

```
C
      DO 695 I=1,N
      DO 695 J=1,N
      DT(I,J)=0.
  695 CONTINUE

      DO 692 I=1,N
      DO 693 J=1,N
      DO 694 K=1,N
      DT(I,J)=DT(I,J)+CTRAN(I,K)*CC(K,J)
  694 CONTINUE
  693 CONTINUE
  692 CONTINUE
C
      DO 699 I=1,N
      DO 699 J=1,N
      PP(I,J)=0.
  699 CONTINUE
C
      DO 696 I=1,N
      DO 697 J=1,N
      DO 698 K=1,N
      PP(I,J)=PP(I,J)+DT(I,K)*HW(K,J)
  698 CONTINUE
  697 CONTINUE
  696 CONTINUE
C
C     DO 782 I=1,N
C     WRITE(6,781) (PP(I,J),J=1,N)
C781  FORMAT(10X,3HPP=,2(E20.10,E20.10))
C782  CONTINUE
C
      WRITE(6,493)
  493 FORMAT(//, 'MAGNITUTES OF DIAGONAL ELEMENTS OF P MATRIX',//)
C
      AFAC=1.E+12
C
      DO 491 I=1,N
      CARP1=REAL(PP(I,I))*AFAC
      CARP1=CARP1*CARP1
      CARP2=AIMAG(PP(I,I))*AFAC
      CARP2=CARP2*CARP2
      CARP3=CARP1+CARP2
      AGWW(I,I)=SQRT(CARP3)
      WRITE(6,492) AGWW(I,I)
  491 CONTINUE
  492 FORMAT(//,E25.10)
C
C
 6666 STOP
      END
```

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EVALUATION OF DESIGN ALTERNATIVES FOR A FREIGHT TRAIN CONTROLLER

Appendix C

A COMPARATIVE STUDY OF SELF-TUNING CONTROLLERS FOR FREIGHT TRAINS

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17. Abstract

This project on development and optimization of control schemes for freight trains was motivated by the need to find optimal or suboptimal closed-loop control schemes for long freight trains. Several on-line and off-line freight train control strategies have been studied and reported. The relation between these control strategies is discussed in this report. The structure of an overall train operation scheme which consists of an off-line energy saving strategy and an on-line control strategy to maintain the velocity profile specified by the off-line scheme is proposed. There are five separately-bound appendices: Appendix A: AUTOMATIC CONTROL OF FREIGHT TRAINS; Appendix B: A REDUCED-ORDER MODEL FOR LONG FREIGHT TRAINS; Appendix C: A COMPARATIVE STUDY OF SELF-TUNING CONTROLLERS FOR FREIGHT TRAINS; Appendix D: MINIMUM ENERGY CONSUMPTION CONTROL OF PASSENGER/FREIGHT TRAINS WITH FIXED TERMINAL TIME; Appendix E: OPTIMAL CONTROL STRATEGY FOR MINIMUM ENERGY AND MINIMUM FUEL CONSUMPTION OF FREIGHT TRAINS

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17. Résumé

L'étude dont ce rapport rend compte a trait aux programmes visant l'automatisation et l'optimalisation de la conduite des longs convois de marchandises, étude motivée par le besoin pressant de trouver des techniques en boucle fermée qui soient optimales, ou tout proche. Le rapport présente un certain nombre de programmes de marche prédétermines et de programmes d'optimalisation de la conduite; de plus, il traite de l'interaction entre eux. Il préconise un programme du premier type axé sur l'économie d'énergie, secondé par un programme du deuxième type assurant à la fois le respect des consignes imposées et une consommation minimale d'énergie. Le rapport contient cinq annexes distinctes, intitulées comme suit: A: Automatisation de la conduite des convois de marchandises; B: Logiciel en réduction approprié aux longs convois de marchandises; C: Etude comparative des logiciels tenant compte des variations dans les paramètres de conduite; D: Consommation minimale des trains de voyageurs et de marchandises à horaires fixes; E: Stratégie d'optimalisation de la consommation d'énergie et de carburant dans le cas des trains de marchandises.

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1. INTRODUCTION

The purpose of this report is to apply self-tuning control strategies to long train operations. The problem is created by a real need to use long freight trains to carry coal uphill and downhill from the mines in the Rocky Mountains to West Coast ports. Multi-locomotive traction is required for economic reasons.

Existing train operations include pneumatic brakes on each car, a 'locotrol' (Ashman[3]) radio communication system between locomotives, and an open-loop control which results in slow train operations. Researchers were looking for a better control strategy to meet the following control objectives: achieve velocity tracking, minimize coupler forces, minimize velocity deviation, minimize the effect of noise due to grade change, and possibly minimize energy consumption. Literature in this area includes the distributed parameter approach using spectral factorization (Davis and Barry[10]), the small-scale systems approach using localized feedback control for subsystems (McLane et al.[14]), and the increased mass model approach using feedback controllers (Gruber et al.[12]; Wong and coworkers[20]). However, these controllers, based on deterministic models, do not have the ability to adapt to load changes due to removing and replacing cars at stations. The self-tuning control strategy suggested by Wong and Bayoumi[16] may be the answer to this problem.

During the past ten years, a lot of self-tuning methods have been introduced. They can be classified into two main categories based on the controller design principle. The first category is based on optimizing a control criterion. Representatives of this kind of ST control are the self-tuning minimum variance control (Astrom and Whittenmark[5]), and the self-tuning minimum combined cost controllers (Clarke & Gawthrop;[9] Bayoumi et al.[7]). The other category relies on the pole-placement controller design. Prototypes of this type are studied by Astrom et al.[6] and Wellstead et al.[15] Allidena and Hughes[2] established the relationship between Clarke's and Wellstead's self-tuning strategies.

All these methods will be discussed, implemented, and simulated here for nine-member short trains and sixty-three-member long trains.

2. TRAIN MODEL

2.1 Dynamic Model

The system under investigation is a multi-locomotive-powered train in which the locomotives and gondala cars are connected by couplers with spring and damping characteristics. Its dynamics can be modelled as a series of spring-dashpot coupler systems of point masses. A typical part of the modelled string is shown in Figure 1.

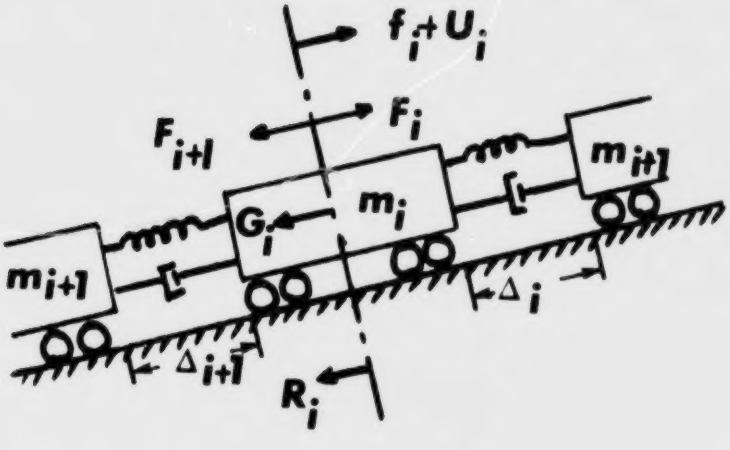

Figure 1 Modelled Train Member "i"

Using this model, we can derive a set of non-linear differential equations:

$$m_i \dot{s}_i(t) = v_i(t) \tag{1}$$

$$m_i \dot{v}_i(t) = F_i(t) - F_{i+1}(t) - R_i(t) - G_i(t) + f_i(t) + U_i(t) \tag{2}$$

$$\dot{\Delta}_i(t) = v_{i-1}(t) - v_i(t) \tag{3}$$

where: n = number of train members

s_i = distance travelled by the i^{th} train member

v_i = velocity of the i^{th} train member

m_i = mass of the i^{th} train member

Δ_i = displacement of the i^{th} coupler (between the $i-1^{th}$ and the i^{th} train members)

F_i = draft gear force of the i^{th} coupler

R_i = rolling resistance and aerodynamic drag force of the i^{th} train member

G_i = component of the gravitational force in the direction of motion.

f_i = throttle/dynamic brake force of the i^{th} train member, if it is a locomotive

U_i = braking force of the i^{th} train member if it is a gondola car. The sign U_i is non-positive to indicate that the braking force is in the opposite direction to train movement.

Equation (1) represents train movement of the i^{th} member in terms of travelling distance. Equation (2) is obtained from the above force diagram and applies Newton's third law. Equation (3) is the velocity difference between adjacent cars.

The i^{th} coupler between the $i-1^{th}$ and the i^{th} train member can be represented by a spring constant, k_i, a damping coefficient, c_i, and a dead zone, DZ. The draft gear force is given by:

$$F_i = d_1\,(k_i, \Delta_i, DZ) + d_2\,(c_i, \Delta_i, \dot{\Delta}_i, DZ) \qquad (4)$$

where: $d_1(x,y,z) = x\,(y - z \operatorname{sgn} y)$ if $|y| \geq z$

$= 0$ if $|y| < z$

$d_2(x,y,\dot{y},z) = x\,\dot{y}$ if $|y| \geq z$

$= 0$ if $|y| < z$

If the non-linear effect of the draft gear slack is not included in the model, the model of the coupler draft gear force can be simplified to:

$$\begin{aligned} F_i &= k_i\,\Delta_i + c_i\,\dot{\Delta}_i \\ &= k_i\,\Delta_i + c_i\,(v_i - V_{i+1}) \end{aligned}$$

Typical values of the coupler spring constant and damping coefficient are k_i = MN/m and c_i = 0.203 MN sec/m.

The tangential component of the gravitational force in the direction of motion is given by:

$$G_i = m_i\,g \sin \theta_i = m_i\,g\,\frac{\tan \theta_i}{\sqrt{1 + \tan^2 \theta_i}} \qquad (6)$$

where: g = the gravity constant, g = 9.80665 N/kg (m/sec.^{2})

θ_i = the angle of the grade on which the ith train member lies.

2.2 Non-Linearities

Besides the non-linear effect of the coupler dead zone, the rolling resistance and aerodynamic drag force are also non-linear functions of speed and are given by:

$$R_i = r_{0i} + r_{1i} v_i + r_{2i} v_i^2 \tag{7}$$

where r_{0i}, r_{1i} and r_{2i} are constants which depend on the surface condition of the rail, mass and number of axles of the train member, design and lubrication of its journal bearings and its cross-sectional area. They also depend on the location in the train if the i^{th} train member is a locomotive. Typical values of these constants for locomotives with mass 176,900 kg and cars with mass 99,800 kg are:

(1) front locomotive:

r_{0i} = 1,903.84 N

r_{1i} = 86.57 N sec./m

r_{2i} = 6.410 N sec.^{2}/m^2

(2) middle and end locomotives:

r_{0i} = 1,903.84 N

r_{1i} = 86.57 N sec./m

r_{2i} = 0.9037 N sec.^{2}/m^2

(3) gondola car:

r_{0i} = 1,152.09 N

r_{1i} = 49.25 N sec./m

r_{2i} = 1.0016 N sec.^{2}/m^2

The drag and resistance curves for these member categories are shown in Figure 2.

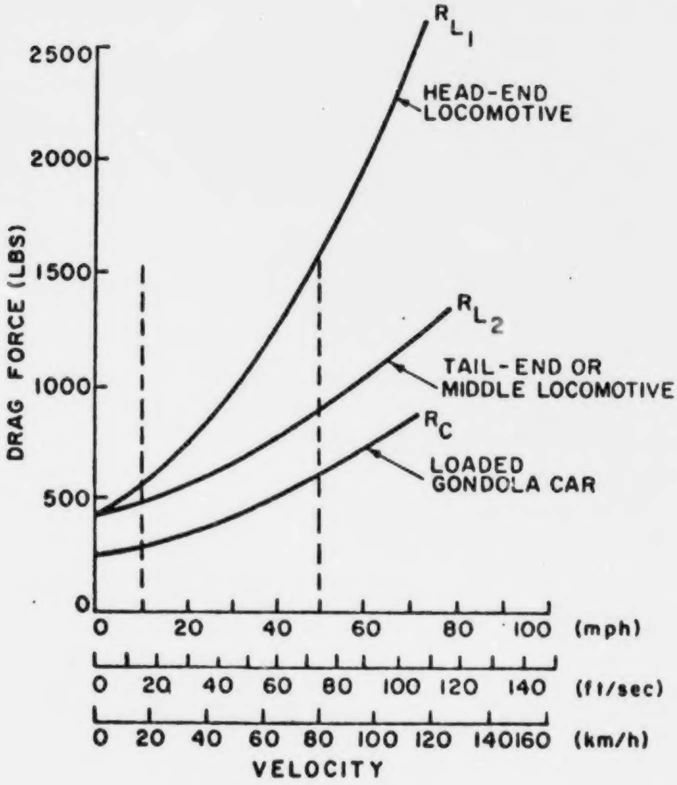

Figure 2 Aerodynamic Drag Force and Rolling Resistance Curves

2.3 Constraints

The existance constraints of the train system are travelling speed, coupler, and throttling and braking power constants:

$$0 \leq v_i(t) \leq v_{max} \quad \text{(travelling speed limit)} \qquad (8)$$

$$|\Delta_i(t)| < \Delta_{limit} \quad \text{(coupler displacement limit)} \qquad (9)$$

$$f_i \leq \mu m_L g \quad \text{(adhesion limit)} \qquad (10)$$

$f_i \; v_i \leq \eta P_{max}$ (motor power limit) (11)

$f_i \geq - \xi v_i$ (motor field current limit) (12)

$f_i \geq - Z$ (extended range dynamic brake limit) (13)

$f_i \; v_i \geq - \eta' \; P_{max}$ (motor heating limit) (14)

where: v_{max} = maximum travelling speed

Δ_{limit} = coupler displacement limit, $\Delta_{limit} = 0.159$ m.

μ = tractive adhesion coefficient $0.11 \leq \mu \leq 0.35$, dependent on track conditions

m_L = mass of a locomotive

η = tranmission efficiency of the electro-mechanical drive system, $\eta = 0.82$

P_{max} = maximum power rate of locomotive

ξ = low-speed dynamic brake build-up rate, typical value for an extended range dynamic brake is $\xi = 0.578$ MN /m

Z = maximum extended range dynamic brake power over the speed range of approximately 4 to 10 m/sec., typical value for an SD-40 unit (six axles, 3000 hp) is $Z \cong 0.267$ MN

η' = locomotive braking efficiency, $\eta' = 1.16$.

The allowable region for the tractive force constraints (equations (10) through (14)) can be illustrated by the area shown in Figure 3.

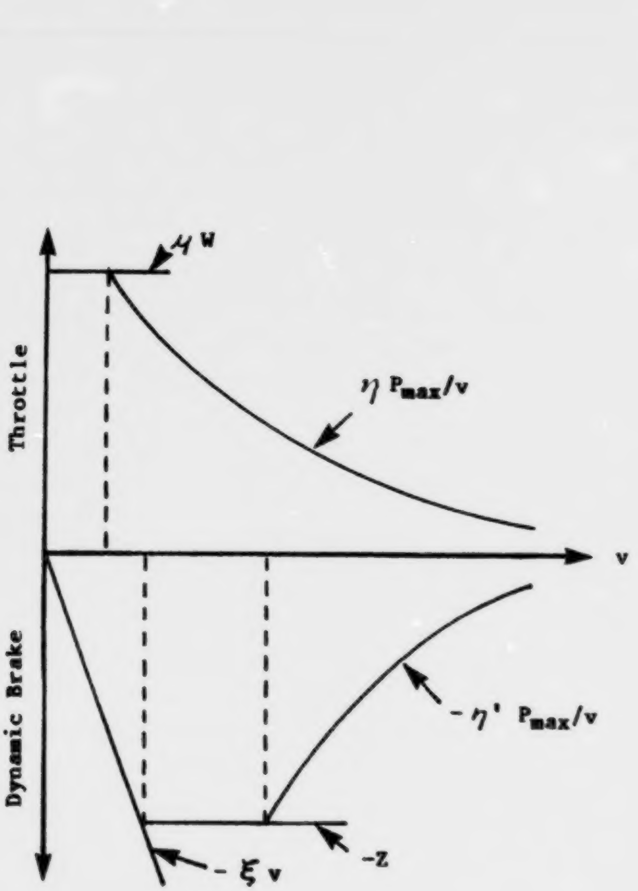

Figure 3 Tractive Force Constraints for SD-40 Locomotives

The braking constraints of each gondola car with air pressure brakes is governed by the dynamic equation and contraints:

$$U_i(t) = -\eta_2 \, p_i(t) \tag{15}$$

$$\dot{p}_i = [\eta_1 \, b_i \, (t-\tau_i) - p_i(t)]/\sigma_i \tag{16}$$

$$b_i(t) = 0 \text{ or } b_{min} \leq b_i(t) \leq b_i(t+\Delta t) \leq b_{max}, \Delta t > 0 \tag{17}$$

(air brake limit)

where: p_i = braking cylinder pressure of the i^{th} freight car

b_i = braking pipe pressure input of the i_{th} freight car

η_1 = brake cylinder constant

η_2 = brake pressure to force conversion constant

σ_i = time constant of the braking system

τ_i = time delay of the braking system

b_{min} = maximum braking input value, b_{min} = 48.2 kPa

b_{max} = maximum braking input value, b_{max} = 158.5 kPa

Equation (16) is based on a simple model of series RC connections for the brake cylinder.

2.4 Quantization

The throttle/dynamic brake range is quantized to eight levels:

$$f = L_j \ \eta P_{max}/v \quad \text{for throttles} \tag{18}$$

$$f = - L_j \ \eta' \ P_{max}/v \quad \text{for dynamic brakes} \tag{19}$$

where: $j = 0,1,2,\ldots,8$, the power level

L_j = the power ratio of throttle setting j

η = transmission efficiency, η = 0.82

η' = locomotive braking efficiency, η' = 1.16

P_{max} = maximum power of the locomotive

v = velocity of the locomotive

The power ratios were considered to be uniform ($L_j = j/8$) in the report by Ahmed and Bayoumi.[1] However, they are not uniformly distributed. Typical values of the power ratios are as follows:

Throttle Setting (j)	Power Ratio (L_j)
0	0
1	0.066
2	0.130
3	0.237
4	0.363
5	0.474
6	0.612
7	0.793
8	1

2.5 Simplifying Assumption

Major difficulties of the controller design of an unsimplified train system are:

(a) The number of inputs and state variables is large. A typical three-locomotive, sixty-car train with individual air pressure brakes on each car can be modelled by a dynamic system of 63 inputs and 185 states.

(b) There are constraints on the inputs and states, namely on travelling speed, coupler displacements, throttling power and braking power.

(c) There are non-linearities in the model such as the slacks of the couplers and rolling resistance and aerodynamic design.

(d) Not all states are measurable. The coupler displacements are hard to measure on line due to technical problems and a minimum number of car velocities should be measured for econmic reasons.

Hence, there is a need to simplify the train model for the controller design. These simplications are as follows:

(a) The model is linearlized to a linear time invariant system around the nominal velocity.

(b) The pure time delay of the braking system is neglected.

(c) The dead zones of the couplers are neglected.

(d) The configuration of the train is symmetric with the locomotives equally spaced along the train.

(e) The whole train can be equally partitioned according to the locomotives. Parallel braking inputs are applied to each car of each partitioned block. Hence, the large number of brake inputs can be reduced significantly.

Note that the reason for assuming a symmetric train configuration instead of placing all locomotives at both ends is to reduce the maximum steady-state coupler force. Partitioning of a typical three-locomotive train based on the steady-state behaviour of the train is shown in Figure 4.

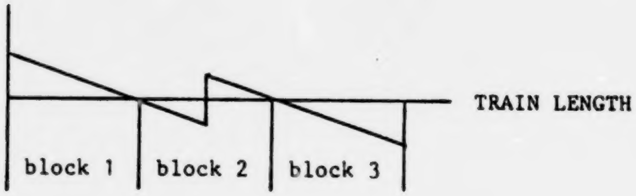

Figure 4 Partitioning of a Symetric Train

2.6 Off-Line Schedule

The linearization step is carried out by a primary control scheme which is referred to as the off-line schedule. The objective of an off-line schedule is to provide the target velocity profile and off-line brake and throttle inputs on each terrain section based on roughly measured information on the terrain.

Basically, the off-line schedule is chosen by using a simple model for the train, such as considering the whole train as a unit body. By looking at the force diagram of the body mass, one knows approximately how much external force should be applied to the train on a given grade. A more sophisticated off-line schedule may involve minimizing the travelling time subject to the limitations, constraints, curvature and grades of terrain sections.

The next step is to determine the approximate time instants to apply the off-line throttle and brake. The particular scheme chosen here is as follows:

(a) For velocity change, the schedule is applied instantaneously for each active member at the instant when the first locomotive reaches the velocity change point.

(b) For grade change, the schedule is applied to the active block members at the moment when the centre of mass of the block reaches the grade change point. The reason for such application is to reduce the total net force acting on the trains.

Details of this off-line schedule are found in the report by Wong and Bayoumi.[16]

2.7 Linearization and the Simplified Model

To simplify controller design, the train model can be linearized around the nominal values of speed, brake pressure and off-line inputs. The large number of states can be reduced by choosing a reduced-order model approach. The one discussed in this section is based on the decentralized lumped mass subsystems approach (see Wong and Bayoumi[16]). Decentralization is carried out based on the fact that the coupling force between any two adjacent blocks is negated by the steady-state behaviour of the train. The order reduction step is carried out by lumping the adjacent cars in each block together as individual increased mass cars. The simplified subsystems of a three-locomotive train are shown in Figures 5, 6 and 7.

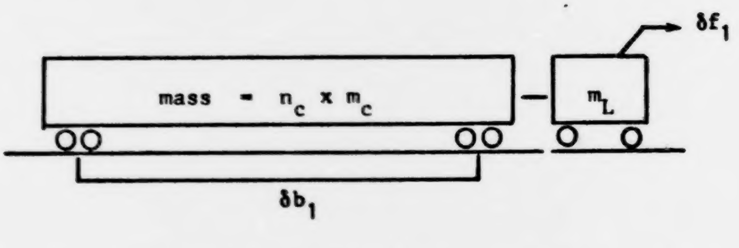

Figure 5 Decentralized Two-Member Subsystem of the Front Block

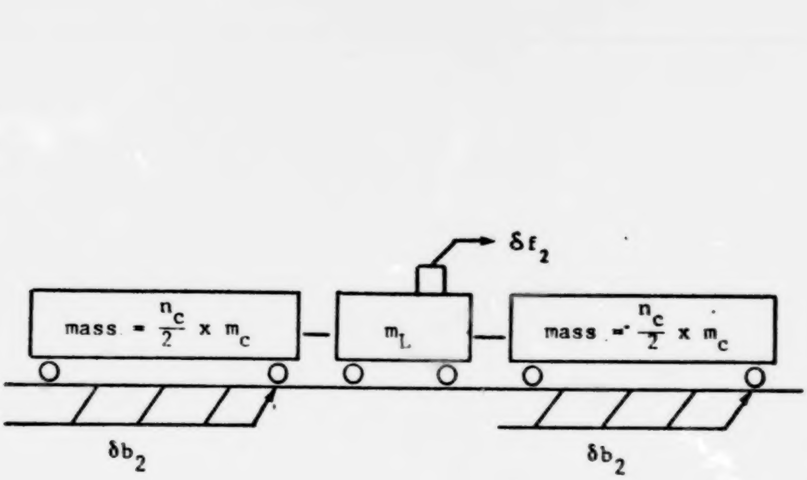
δf₂
mass = nc/2 x mc
mL
mass = nc/2 x mc
δb₂
δb₂

Figure 6 Decentralized Three-Member Subsystem of the Middle Block

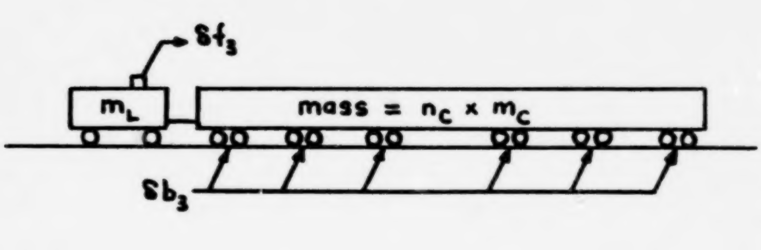
δf₃
mL
mass = nc x mc
δb₃

Figure 7 Decentralized Two-Member Subsystem of the Last Block

The transfer function of the off-line subsystems are found to be:

$$\frac{Y_1(s)}{U_1(s)} = \frac{Y_3(s)}{U_3(s)} = \frac{s^2 + (c/n_c m_c)\, s + (k/n_c m_c)}{s^3 + (c/m_L + c/n_c m_c)s^2 + (k/m_L + k/n_c m_c)s} \tag{20}$$

$$\frac{Y_1(s)}{B_1(s)} = \frac{Y_3(s)}{B_3(s)} = \frac{n_1\ n_2/m_c\ (c/m_c\ s + k/m_L)}{s\ (\sigma s+1)\ [s^2 + (c/m_L + c/n_c m_c)\ s + k/m_L + k/n_c m_c]} \tag{21}$$

$$\frac{Y_2(s)}{U_2(s)} = \frac{[s^2 + (2c/n_c m_c)\ s + (2\ k/n_c m_c)]^2\ [s + 1/\sigma]}{\det\ (sI - A_2)} \tag{22}$$

$$\frac{Y_2(s)}{Y_2(s)} = \frac{4\ \eta_1 \eta_2/m_c\ \sigma[(c/m_L)s + k/m_L]\ [s^2 + (2c/n_c m_c)s + 2k/n_c m_c]}{\det\ (sI - A_2)} \tag{23}$$

where Y_i, U_i and B_i are the Laplace transforms of the velocity derivation, the throtting input derivation in per-unit mass of the locomotive mass, and the braking input derivation respectively. $n_c = n/3-1$ is the number of cars in each block.

$$A = \begin{bmatrix} -2c/n_c m_c & -k/n_c m_c & 2c/n_c m_c & 0 & 0 & 2\;{}_2 m_c \\ 1 & 0 & -1 & 0 & 0 & 0 \\ c/m_L & k/m_L & -2c/m_L & -k/m_L & c/m_L & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 2c/n_c m_c & 2k/n_c m_c & -2c/n_c m_c & 2\eta_2/m_c \\ 0 & 0 & 0 & 0 & 0 & -1/\sigma \end{bmatrix}$$

Note that the system parameters are functions of n, m_L, m_c, k, c, σ, η_1 and η_2.

By introducing the A/D sampler and D/A zero order hold, each of the above subsystems can be discretized to a discrete time ARMA model of the form:

$$A(z^{-1})\, y(t) = B(z^{-1})\, u(t-1) \tag{24}$$

where: z^{-1} = backward shift operator

t = number of sampling periods

$A(z^{-1}) = 1 + A_1 z^{-1} + \dots + A_n z^{-n}$

$B(z^{-1}) = B_0 + B_1 z^{-1} + \dots + B_m z^{-m}$

Or in vector notation,

$$y(t) = P(t)\, x\,(t) \tag{25}$$

where: $P(t) = [B_0, B_1, \dots, B_m, -A_1, \dots, -A_n]$

$x(t) = [u(t-1), \dots, u(t-m-1), y(t-1), \dots, j(t-n)]^T$

(m,n) = model order.

3. SELF-TUNING METHODS

3.1 Self-Tuning Minimum Variance Control (ST-MV)

Self-tuning minimum variance was studied by Astrom et al.[4] The aim of ST-MV is to minimize the variance of the expected output. The criterion is to minimize:

$$J_0 = E\ (y^2(t+d)) \tag{26}$$

in steady state. It is well known that if the polynomial B has all its zeros inside the unit disc, the solution is stable. In the train control application, since no pure time delay is assumed, the best one can do is to minimize $E\ y^2(t+1)$, i.e. $d=1$. Hence, the implicit model used for ST-MV is precisely the explicit model given by equations (24) or (25).

The overall ST-MV algorithm can be expressed as:

(i) Estimate α's and β's from the estimation model:

$$\begin{aligned} y(t) = {} & \beta_0\ u(t-1) + \ldots + \beta_m\ u(t-m-1) \\ & + \alpha_1\ y(t-1) + \ldots + \alpha_n\ y(t-n) + \varepsilon(t) \end{aligned} \tag{27}$$

by minimizing the sum of squares of the estimation error $\varepsilon(t)$.

(ii) Apply the control:

$$\begin{aligned} u(t) = {} & -[\beta_1\ u(t-1) + \ldots + \beta_m\ u(t-m) \\ & + \alpha_1\ y(t) + \ldots + \alpha_n\ y(t-n+1)]/\beta_0 \end{aligned} \tag{28}$$

(iii) Increment the time counter, t, and go to (i) for next interation.

Note that there are many versions of recursive algorithms for updating parameters α and β. The conventional recursive least squares (RLS) is the most popular. Other recursive estimation methods have been introduced to overcome the numerical ill-conditioned problem. The recursive U-D updating method based on the Chelosky factorization will be discussed in the next section. This method will be used to replace the conventional RLS method to update the model parameters.

3.2 Self-Tuning Minimum Cost Control (ST-MC) (Clarke & Garthrop)

This class of control laws was developed by Clarke and Gawthrop.[9] The aim of this control is to minimize a cost function given by:

$$J_1 = E\left([(Py(t+d) - Rw(t)]^2 + [Q'\ u(t)]^2\right) \tag{28}$$

where P, R and Q' are polynomials in z^{-1} and w is the control setpoint.

It has been shown that an equivalent problem is to minimize the criterion:

$$J_2 = E(\Phi(t+d)^2) \tag{30}$$

where: $\Phi(t+d) = Py(t+d) + Q''\ u(t) - R\ w(t)$

$Q''(z^{-1}) = Q'(0)\ Q'(z^{-1})/B_0$

The introduction of Φ (which is usually called the generalized output) gives us a simple model structure of the form:

$$\Phi(t+d) = H\ u(t) + G\ y(t) + E\ w(t) + \varepsilon(t) \tag{31}$$

where: H, G and E are polynomials in z^{-1}.

A model of this type is referred to as an implicit model. In the case of d=1, the polynomials H, G and E have the following forms:

$$H(z^{-1}) = P(z^{-1})\ B(z^{-1}) + Q''(z^{-1}) \tag{32}$$

$$G(z^{-1}) = P(z^{-1})\ [A_1 + A_2\ z^{-1} + \ldots + A_n\ z^{-n+1}] \tag{33}$$

$$E(z^{-1}) = -\ R(z^{-1}) \tag{34}$$

Hence, the overall ST-MC algorithm can be summarized as:

(i) From the generalized output:

$$\Phi(t) = P(z^{-1})\ y(t) + Q''(z^{-1})\ u(t-1) - R((z^{-1})\ w(t-1) \qquad (35)$$

(ii) Estimate the model parameters from the model:

$$\begin{aligned}\Phi(t) = {} & h_0\ u(t-1) + \ldots + h_{m'}\ u\ (t-m'-1) + g_0\ y(t-1) + \ldots + \\ & g_{n'}\ y(t-n'-1) + e_0\ w(t-1) + \ldots + e_{k'}\ w(t-k'-1) + \mathcal{E}(t)\end{aligned} \qquad (36)$$

(iii) Apply the control law:

$$\begin{aligned}u(t) = {} & -\ [h_1\ u(t-1) + \ldots + k_{m'}\ u(t-m') + g_0\ y(t) + \ldots \\ & + g_{n'}\ y(t-n') + \ell_0\ w(t) + \ldots + \ell_{k'}\ w(t-k')]/h_0\end{aligned} \qquad (37)$$

(iv) Increment the value of t and go to (i) for next iteration.

The model order (m', n', k') can be deduced from equations (32), (33) and (34) and are given by:

$$m' = \max\ (n_P + m,\ n_{Q''}) \qquad (38)$$

$$n' = n_P + n-1 \qquad (39)$$

$$k' = n_R \qquad (40)$$

where n_P, $n_{Q''}$ and n_R are the degrees of the polynomials P, Q'' and R, respectively.

For simplicity, polynomials P and R are often chosen to be the unity and Q'' is a first-order polynomial with zero at the unity.

The main advantage of this kind of control over ST-MV is that ST-MC can be applied to non-minimum phase systems (the polynomial B may have zeros outside the unit disc) with suitable weighting polynomials, P, Q'' and R. Another advantage is that the magnitude of the control signal can be reduced to acceptable levels since the control signal is also penalized in the criteria.

3.3 Self-Tuning Minimum Cost Control (ST-MC) (Bayoumi, Wong and El-Bagoury[7])

This controller was originally introduced by Bayoumi et al. for the MIMO systems. This approach is different to that of Clarke and Gawthrop, introducing successive estimations in order to reduce computation time and storage. Like Clarke and Gawthorp's criterion, the aim of the control is to minimize the cost function:

$$J_3 = E\left([y(t+d) - y_r(t+d)]^2 + Q\,[T\,u(t)]^2\right) \qquad (41)$$

where T is a polynomial in z^{-1} of degree k and Q is a non-negative number.

An explicit model identification scheme is needed for estimation. Since d=1, the successive estimation is not required and the overall control scheme is summarized as:

(i) Estimate α and β from the explicit model:

$$\begin{aligned} y(t) = \beta_0\,u(t-1) + \ldots + \beta_m\,u(t-m-1) + \alpha_1\,y(t-1) \\ + \ldots + \alpha_n\,y(t-n) + \varepsilon(t) \end{aligned} \qquad (42)$$

(ii) Compute the predicted output $y\,(t+1|t,\ u=0)$ from:

$$\begin{aligned} y(t+1|t,\ u=0) = \beta_1\,u(t-1) + \ldots + \beta_m\,u(t-m) \\ + \alpha_1\,y(t) + \ldots + \alpha_n\,y(t-n+1) \end{aligned} \qquad (43)$$

(iii) Apply the control:

$$\begin{aligned} u(t) = (\beta_0[y(t+1|t,\ u=0) - y_r(t+1)] - Q\,[T_1\,u(t-1) \\ + \ldots + T_k\,u(t-k)])/(\beta_0^2 + Q) \end{aligned} \qquad (44)$$

(iv) Increment the value of t and go to (i) for next iteration.

Note that the control law is identical to that of Clarke and Gawthrop for an appropriate choice of Q and T. The major difference in the two is the model structure. Also note that the computation time and storage of this algorithm are both less because the number of parameters to be estimated is less.

3.4 Self-Tuning Pole-Placement Control (ST-PP) (Astrom)

Astrom et al.[6] developed the self-tuning pole-placement control for the servo control of deterministic SISO systems. The objective of the pole-placement design is to find a controller where the closed-loop system is pre-determined with the given transfer function:

$$G(z^{-1}) = \frac{Q(z^{-1})}{P(z^{-1})} \tag{45}$$

The controller considered is of the form:

$$R\ u(t) = T\ y_r(t+d) - S\ y(t) \tag{46}$$

where R, S and T are polynomial functions of z^{-1}. The controller consists of a feed-forward as well as a feed-back portion. A block diagram of the closed-loop system is shown in Figure 8.

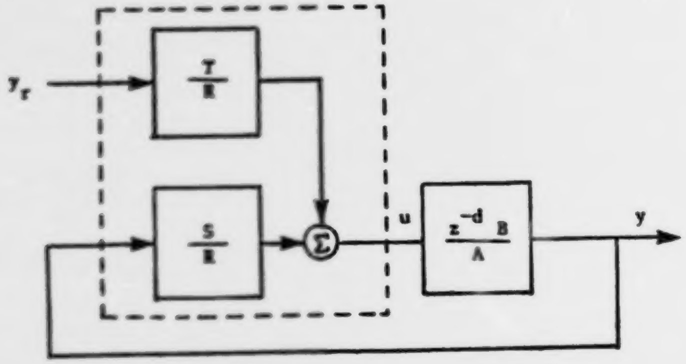

Figure 8 Block Diagram of the Closed-Loop System

The closed-loop transfer function is given by:

$$G = \frac{TB}{AR + z^{-d}BS} \tag{47}$$

The pole placement design problem is thus to find the polynomials R, S and T, such that:

$$\frac{TB}{AR + z^{-d}\ BS} = \frac{Q}{P} \tag{48}$$

With Q = 1 (all process zeros cancelled), the design involves solving the equation:

$$AR + z^{-d}\ BS = PTB \tag{49}$$

Since B does not divide A, it must divide R, i.e. R = BR', and equation (49) can then be written as:

$$AR' + z^{-d}\ S = PT \tag{50}$$

Two types of self-tuning strategies can be applied. The first is the use of an explicit identification scheme together with the solution of the design equation (equation (50)). However, the design calculation involved in solving equation (50) is time consuming. The second type of algorithm is based on implicit identification of the parameters of a model which is chosen properly to simplify the design calculation above. The model is obtained as follows:

$$PT\ y(t) = AR'\ y(t) + S\ y(t-d) = R\ u(t-d) + S\ y(t-d) \tag{51}$$

The implicit algorithm is summarized as follows:

(i) Form the generalized output:

$$\Phi(t) = P(z^{-1})\ T(z^{-1})\ y(t) \tag{52}$$

(ii) Estimate R and S from:

$$\Phi(t) = r_0\ u(t-d) + \ldots + r_m\ u(t-d-m) + s_0\ y(t-d) + \ldots + s_n\ y(t-d-n)$$

(iii) Apply the control:

$$u(t) = [T(z^{-1})\ y_n(t+d) - s_0\ y(t) - \ldots - s_n\ y(t-n) - r_1\ u(t-1) - \ldots - r_m\ u(t-m)]/r_0 \tag{53}$$

(iv) Increment the time counter, t, and go to (i) for next iteration.

3.5 Self-Tuning Pole-Placement Control (ST-PP) (Wellstead)

This technique has been developed by Wellstead[15] and adopts a classical control approach where the objective is to locate the closed-loop system poles at prespecified locations for noise regulation. It is assumed that the system is represented by the model:

$$A(z^{-1})\ y(t) = B(z^{-1}) + C(z^{-1})\ e(t) \tag{54}$$

where A, B and C are polynomial functions of z^{-1} and e is the random noise with Gaussian distribution.

The controller under investigation has the following structure:

$$H\ u(t) + G\ y(t) = 0 \tag{55}$$

where H and G are polynomial functions of z^{-1}.

When this control law is incorporated into the system, the following closed-loop description is obtained:

$$(AH + z^{-d}\ BG)\ y(t) = HC\ e(t) \tag{56}$$

Hence, the pole placement design problem is to find the polynomials H and G such that:

$$AH + z^{-d}\ BG = CT \tag{57}$$

where T denotes the closed-loop characteristic polynomial, and the polynomial H becomes the closed-loop zero polynomial.

The Wellstead ST-PP algorithm can then be summarized as:

(i) Estimate A, B and C explicitly from:

$$y(t) = \beta_0\, u(t-d) + \ldots + \beta_m\, u(t-d-m) + \alpha_1\, y(t-1) + \ldots + d_n\, y(t-n) + C_1\, \varepsilon(t-1) + \ldots + C_k\, \varepsilon(t-k) + \varepsilon(t) \qquad (58)$$

(ii) Solve for H and G from the linear equation:

$$AH + z^{-d}\, BG = CT \qquad (59)$$

(iii) Apply control law:

$$H\, u(t) + G\, y(t) = 0 \qquad (60)$$

(iv) Increment the value of t and go to (i) for the next iteration.

Note that in order for equation (59) to have a solution, the order of the regulation polynomials must be:

$$n_G = n_A - 1 \qquad (61)$$

$$n_H = n_B + d - 1 \qquad (62)$$

The number of closed-loop poles must satisfy:

$$n_T \leq n_A + n_B + d - n_c - 1 \qquad (63)$$

The attractive features of this ST-PP are its generality to deal with non-minimum phase systems together with the design freedom which it gives the engineer. However, the flexibility is exchanged for an increase in computation which may make it unsuitable for certain applications. Computation of the controller parameters requires more computation time and storage than any other self-tuning algorithm, since a set of linear equations (equation (59)) has to be solved in each iteration.

3.6 Generalized Self-Tuning Pole-Placement Control (ST-PP-G) (Allidina and Hughes)

The linking of Clarke's and Wellstead's self-tuning control algorthims was established by Allidina and Hughes[2] and is called the ST-PP-G. This controller aims to minimize a cost function and adopt it in such a way that the closed-loop poles are located at prespecified locations.

Recall that Clarke's controller is of the form:

$$H\,u(t) + G\,y(t) + E\,w(t) = 0 \tag{64}$$

which gives the closed-loop description:

$$y(t) = \frac{BR}{BP + AQ''}\,w(t-d) + \frac{H}{BP + AQ''}\,e(t) \tag{65}$$

If P and Q are chosen such that:

$$BP + AQ'' = T \tag{66}$$

where T is the prespecified polynomial in z^{-1}, then equation (65) can be rewritten as:

$$y(t) = \frac{BR}{T}\,w(t-d) + \frac{H}{T}\,e(t) \tag{67}$$

If $w(H) = 0$, i.e. reference following is not required, and the controller only regulates for random distrubances, equation (67) reduces to:

$$y(t) = \frac{H}{T}\,e(t) \tag{68}$$

The appropriate P and Q'' of the performance cost function can be obtained from the solution of equation (66). However, it is possible to obtain P and Q'' from the implicit parameters. Recall, from Wellstead's pole-placement design, that if we only consider white noise,

$$AH + z^{-d} BG = T \tag{69}$$

and from Clarke's minimum cost design, we have

$$H = PB + Q'' \tag{70}$$

Eliminating A and B using equations (69) and (70), the PP design equation (equation (66)), can be reduced to:

$$PH - z^{-d} Q'' G = T \tag{71}$$

Hence, the generalized ST-PP algorithm can be summarized as follows:

(i) Form the generalized output:

$$\Phi(t) = P y(t) + Q'' n(t-1) - R w(t-1) \tag{72}$$

(ii) Estimate the model parameters H, G and E from:

$$\Phi(t) = H u(t-d) + G y(t-d) + E w(t-d) + \varepsilon(t) \tag{73}$$

(iii) Apply the control law:

$$H u(t) + G y(t) + E w(t) = 0 \tag{74}$$

(iv) Solve for P and Q" from the design rule:

$$PH - z^{-d} Q'' G = T \tag{75}$$

(v) Increment the value to t and go to (i) for the next iteration.

Note that the routine to solve for P and Q" (in step (iv)) need not be continuously on line. P and Q may be tuned periodically. The advantage of such a controller is that it combines the robustness of pole assignment with the ease of reference following provided by the minimum cost control.

4. ESTIMATION METHODS

4.1 Conventional RLS Method

The conventional RLS method has become widely used for on-line identification since reported by Kalman.[13]. The aim of this method is to obtain the best parameter estimates of a linear model strucure by minimizing the sum of squares of the estimation error. By introducing the forgetting factor, w, to eliminate the effect of past data, the weighted least square criterion to be minimized is:

$$S = w^{t-1}\,\mathcal{E}(1)^2 + \ldots + w\,\mathcal{E}(t+1)^2 + \mathcal{E}(t)^2 \qquad (76)$$

where the model error is of the form:

$$\mathcal{E}(k) = y(k) - P(t)\,x(k) \qquad k=1,2,\ldots,t \qquad (77)$$

The conventional RLS estimating method requires computing the Kalman gain vector, K, and updating the inverse information matrix, R, at each step. The RLS algorithm, starting with $R(0) = I_N/\delta$; I_N the identify matrix, and $\delta > 0$ the small value, is updated as follows:

(a) Gain $$K(t) = \frac{R(t-1)\,x(t)}{w + x^T(t)\,R(t-1)\,x(t)}$$

(b) Matrix $$R(t) = [R(t+1) - K(t)\,x^T(t)\,R(t-1)]/w$$

(c) Error $$\mathcal{E}(t) = y(t) - P(t-1)\,x(t)$$

(d) Estimates $$P(t) = P(t-1) + \mathcal{E}(t)\,K^T(t)$$

This method does not guarantee that the maxtrix R(t) is positive definite at all iterations and hence may cause parameter "blow up" problems for long-term identification.

4.2 Recursive U-D Factorization

This algorithm is suggested by Bierman[8] for updating the parameters, and is one method introduced to avoid the possibility of the positive definite matrix, R(t), losing this property due to rounding errors. The method involves factorizing:

$$R(t) = U(t)\ D(t)\ U^T(t)$$

where U(t) is an upper triangular matrix with unit diagonal element and D(t) is a diagonal matrix. The recursive U-D factorization algorithm with forgetting factor is (start with U(0) = zero matrix, D(0) = I_N/δ):

(a) U-D update: $f = U^T(t-1)\ x(t)$

$$v = D(t-1)\ f$$

$$\alpha_j = \alpha_{j-1} + v_j\ f_j \qquad j=1,\dots,N,(N=\text{dimension of } x)$$
$$(\alpha_0 = w)$$

$$d_j(t) = d_j(t-1)\ \alpha_{j-1}/w\ \alpha_j$$

$$a_j = -\ f_j/\alpha_{j-1}$$

$$\underline{u}_j(t) = \underline{u}_j(t-1) + a_j\ K_j;\ \underline{u}_j \text{ is the } j^{th} \text{ column of } U$$

$$K_{j+1} = K_j + v_j\ \underline{u}_j(t-1);\ K_2 = [v_1,0,\dots,0]^T$$

(b) Gain: $K(t) = K_{N+1}/\alpha_{N+1}$

(c) Error: $\varepsilon(t) = y(t) - P(t-1)\ x\ (t)$

(d) Estimates: $P(t) = P(t-1) + \varepsilon(t)\ K^T(t)$

Note that $\alpha_j \geq \alpha_{j-1} \geq \alpha_0 = w$ and $d_j(0)$ is positive for all j=1,...,N. This implies that all $d_j(t)$ are positive at any time, t. Hence, $R(t) = U(t)\ D(t)\ U^T(t)$ is always positive definite. Comparison of the recursive U-D method, the conventional RLS method and other estimation algorithms can be found in Wong, Bayoumi and Nuyan.[21]

5. SIMULATION

5.1 Simulation Results of a Nine-Member Train

A three-locomotive, six-car train was simulated over a flat/uphill section and a flat/downhill section with two per cent slope for fifty seconds. The data used for simulation, together with the precomputed steady-state values, are shown in Tables 1, 2 and 3.

TABLE 1

DATA FOR THE THREE-LOCOMTIVE, SIX-CAR TRAIN

	Each Locomotive	Each Car	Total
Length (m)	21.34	19.82	182.88
Mass (km)	176,900	99,800	1,120,000

$k = 7$ MN/m, $C = 0.203$ MN sec/m, $\eta_1 = 2.0$, $\eta_2 = 0.929$ m^2, $\sigma = 2$ sec.

TABLE 2

DATA AND SCHEDULE VALUES FOR NINE-MEMBER TRAIN, UPHILL SIMULATION

	Section		
	1	2	3
Distance of sections (m)	304.8	609.6	1,524
Steady-state velocities (km/h)	64.37	64.37-56.33	56.33
Acceleration profile (m/sec^2)	0	-0.1229	0
Grade of sections (deg.)	0	2	2
Total s.s. rolling resistance (MN)	0.0273	0.0249	0.0249
Total gravitational forces (MN)	0	0.2215	0.2215
Total external forces (MN)	0.0273	0.1076	0.2464
s.s. braking forces (N/kg)	0	0	0
s.s. braking inputs (kPa)	0	0	0
s.s. throttle (N/kg)	0.0514	0.2027	0.4643

TABLE 3

DATA AND SCHEDULE VALUES FOR NINE-MEMBER TRAIN, DOWNHILL SIMULATION

	Section		
	1	2	3
Distance of section (m)	304.8	609.6	1,524
Steady-state velocities (km/h)	64.37	64.37-56.33	56.33
Acceleration profile (m/sec^2)	0	-0.1229	0
Grade of section (deg.)	0	-2	-2
Total s.s. rolling resistance (MN)	0.0273	0.0249	0.0249
Total gravitational forces (MN)	0	0.2215	0.2215
Total external forces (MN)	0.0273	-0.3354	-0.1965
s.s. braking forces (N/kg)	0	-0.2969	-0.1740
s.s. braking intpus (kPa)	0	-15.9371	-9.3391
s.s. throttle (N/kg)	0.0514	-0.2969	-0.1740

The train is required to decelerate from 64.37 to 56.33 km/h after it passes the grade change point (304.8 m from the starting point) for both the uphill and downhill simulations. The off-line open-loop schedule and the decentralized self-tuning throttle control schemes were simulated. The simulated self-tuning throttle control schemes include all the ST algorithms described in Section 3. Various penalty functions for the ST optimal control and several pole locations were studied to illustrate the effect of the performance with various control objectives. The range of $Q = 0$, 0.1 and 1 and the closed-loop pole location of $z = 0.5$, $= -.1$ and a two-pole situation have been tested. A forgetting factor, $w=0.95$, was arbitrarily chosen for the simulations.

The performance of the off-line schedule and the ST adaptive control schemes for simulating a nine-member train moving uphill/downhill are listed in Tables 4 and 5. These tables include various sample variances, maximal displacements, maximal coupler forces and the position and time of occurrences.

TABLE 4

PERFORMANCE OF THE SELF-TUNING CONTROL SCHEMES FOR UPHILL EVALUATION OF A NINE-MEMBER TRAIN

	Performance						
Methods	Variance of Velocity (m^2/sec^2)	Variance of Coupler Displacement (mm^2)	Maximum Coupler Displacement (mm)	Maximum Coupler Force (MN)	Position of Occurrence (car #)	Time of Occurrence (sec.)	Results (Annex)
Open-loop off-line schedule	0.00348	9.2	9.75	0.0688	8-9	16.0	A.1
Self-Tuning Minimum Cost Control:							
Astrom's MV	0.00002	9.2	6.60	0.0465	8-9	25.6	A.2
Bayoumi's Q=0.1	0.00010	9.2	8.26	0.0583	8-9	16.1	A.3
Bayoumi's Q=1	0.00034	9.2	9.17	0.0648	8-9	16.0	A.4
Clarke's Q"=0.1	0.00002	9.3	6.39	0.0450	8-9	26.0	A.5
Clarke's Q=1	0.00012	9.8	7.23	0.0510	8-9	25.4	A.6
Self-Tuning Pole-Placement Control:							
Astrom (0.5)	0.00039	27.8	28.50	0.2009	5-6	0.4	A.7
Astrom (-0.1)	0.00015	102.1	20.08	0.1416	2-3	0.5	A.8
Astrom (0.2,0.3)	0.00038	120.7	29.77	0.2099	4-5	37.6	A.9
Wellstead (0.5)	0.00127	27.8	45.81	0.3230	4-5	35.6	A.10
Wellstead (-0.1)	0.00232	176.5	87.24	0.6150	2-3	4.8	A.11
Wellstead (0.2,0.3)	0.00049	18.5	40.00	0.2820	4-5	8.6	A.12
Allidina (0.5)	0.00008	9.2	6.91	0.0487	8-9	25.4	A.13
Allidina (-0.1)	0.00008	9.2	6.89	0.0486	8-9	25.4	A.14
Allidina (0.2,0.3)	0.00013	9.2	23.60	0.1664	8-9	22.6	A.15
Limits	-	-	159.00	1.113	-	-	

TABLE 5

PERFORMANCE OF THE SELF-TUNING CONTROL SCHEMES FOR DOWNHILL SIMULATION OF A NINE-MEMBER TRAIN

	Performance						
Methods	Variance of Velocity (m^2/sec^2)	Variance of Coupler Displacement (mm^2)	Maximum Coupler Displacement (mm)	Maximum Coupler Force (MN)	Position of Occurrence (car #)	Time of Occurrence (sec.)	Results (Annex)
Open-loop off-line schedule	0.02974	9.2	9.75	0.0408	8-9	15.9	A.15
Self-Tuning Minimum Cost Control:							
Astrom's MV	0.00001	9.2	6.02	0.0424	8-9	14.2	A.17
Bayoumi's Q=0.1	0.00146	9.2	5.46	0.0385	2-3	9.2	A.18
Bayoumi's Q=1	0.01980	9.2	8.17	0.0577	4-5	7.7	A.19
Clarke's Q"=0.1	0.00001	0.3	3.25	0.0229	4-5	8.6	A.20
Clarke's Q"=1	0.00042	0.6	4.84	0.0341	2-3	8.2	A.21
Self-Tuning Pole-Placement Control:							
Astrom (0.5)	0.00040	9.2	28.50	0.2009	5-6	0.4	A.22
Astrom (-0.1)	0.00015	1 9.2	20.08	0.1416	2-3	0.5	A.23
Astrom (0.2,0.3)	0.00018	83.6	28.38	0.2001	6-7	0.5	A.24
Wellstead (0.5)	0.00127	27.8	45.81	0.3230	1-2	49.9	A.25
Wellstead (-0.1)	0.00492	455.2	118.64	0.8364	6-7	29.9	A.26
Wellstead (0.2,0.3)	0.00029	0.0	29.72	0.2095	4-5	14.8	A.27
Allidina (0.5)	0.00017	0.0	16.98	0.1197	8-9	41.6	A.28
Allidina (-0.1)	0.00015	0.0	4.29	0.0302	2-3	8.2	A.29
Allidina (0.2,0.3)	0.00020	0.0	24.82	0.1750	1-2	27.4	A.30
Limits	-	-	159.00	1.113	-	-	

Tables 4 and 5 show that the variance of velocity has been reduced significantly for both uphill and downhill simulations. It is also shown that the minimum variance controller is the most effective one to reduce the variance of velocity deviation. For the results of the pole-placement control algorithms, Allidina's is the most effective method.

The maximum coupler displacements are compared with the Δ_{limit} = 0.159 m (coupler force limit = 1.113 MN) as a check of the control performance. It is shown that none of the control schemes has a coupler that exceeds the limit in the simulations.

The simulation results are presented in graph form in Annex A. Figures A.1 through A.15 show the uphill simulation results and Figures A.16 through A.30 show the downhill simulation results.

5.2 Simulation Results of a Sixty-Three-Member Train

A three-locomotive, sixty-car train was simulated next, along flat/uphill and flat/downhill sections with two per cent grade for 200 seconds. The data used for the long train simulations are shown in Table 6.

TABLE 6

DATA FOR THE THREE-LOCOMOTIVE, SIXTY-CAR TRAIN

	Each Locomotive	Each Car	Total
Length (m)	21.34	19.81	1,252.7
Mass (km)	176,900	99,800	6,518,000

k = 7 MN/m, C = 0.203 MN sec/m, η_1 = 2.0, η_2 = 0.929 m^2, σ = 12 sec.

Descriptions of the sections and the precomputed steady-state values are shown in Tables 7 and 8.

TABLE 7

DATA AND SCHEDULE VALUES FOR SIXTY-THREE-MEMBER TRAIN, UPHILL SIMULATION

	Section		
	1	2	3
Distance of sections (m)	1,371.6	2,743.2	5,486.3
Steady-state velocities (km/h)	64.37	64.37-56.33	56.33
Acceleration profile (m/sec^2)	0	0.273	0
Grade of sections (deg.)	0	2	2
Total s.s. rolling resistance (MN)	0.1550	1.426	0.1426
Total gravitational forces (MN)	0	1.2782	1.2782
Total external forces (MN)	0.1550	1.2427	1.4207
s.s. braking forces (N/kg)	0	0	0
s.s. braking inputs (kPa)	0	0	0
s.s. throttle (N/kg)	0.2920	2.3415	2.6771

TABLE 8

DATA AND SCHEDULE VALUES FOR SIXTY-THREE-MEMBER TRAIN, DOWNHILL SIMULATION

	Section		
	1	2	3
Distance of section (m)	1,371.6	2,743.2	5,486.4
Steady-state velocities (km/h)	64.37	64.37-56.33	56.33
Acceleration profile (m/sec^2)	0	-0.0273	0
Grade of section (deg.)	0	-2	-2
Total s.s. rolling resistance (MN)	0.1550	0.1426	0.1426
Total gravitational forces (MN)	0	1.2782	1.2782
Total external forces (MN)	0.1550	-1.3137	-1.1356
s.s. braking forces (N/kg)	0	-0.2015	-0.1742
s.s. braking intpus (kPa)	0	-10.8166	-9.3504
s.s. throttle (N/kg)	0.2920	-0.2015	-0.1742

The off-line open-loop schedule and some selected decentralized self-tuning throttling control schemes were simulated. The performances of the off-line schedules and the ST-PP control schemes for the uphill/downhill simulation of a sixty-three-member train are listed in Tables 9 and 10. It has been shown in the long train simulation study that Clarke's control scheme has better performance than Allidina's and both ST controls have reduced the velocity variation by more than thirty times.

The simulation results are presented in graph form in Annex B.

TABLE 9

PERFORMANCE OF THE SELF-TUNING CONTROL SCHEMES FOR UPHILL SIMULATION OF A SIXTY-THREE-MEMBER TRAIN

	Performance						
Methods	Variance of Velocity (m^2/sec^2)	Variance of Coupler Displacement (mm^2)	Maximum Coupler Displacement (mm)	Maximum Coupler Force (MN)	Position of Occurrence (car #)	Time of Occurrence (sec.)	Results (Annex)
Open-loop off-line schedule	0.10383	0.0	100.07	0.7055	31-32	81.1	B.1
Self-Tuning Minimum Cost Control:							
Clarke's $Q''=0.1$	0.00036	894.5	64.39	0.4539	62-63	91.8	B.2
Clarke's $Q''=1$	0.00410	836.6	64.82	0.4570	62-63	95.3	B.3
Self-Tuning Pole-Placement Control:							
Astrom (0.5)	0.00246	603.8	98.87	0.6970	1-2	191.0	B.4
Astrom (-0.1)	0.00031	566.6	56.76	0.4002	31-32	92.4	B.5
Astrom (0.2,0.3)	0.00142	733.9	104.99	0.7402	1-2	198.5	B.6
Allidina (0.5)	0.00288	780.3	68.53	0.4831	1-2	146.7	B.7
Allidina (-0.1)	0.00269	780.3	64.02	0.4513	62-63	95.8	B.8
Allidina (0.2,0.3)	0.00277	780.3	64.16	0.4523	62-63	95.9	B.9
Allidina (0.5,0.5)	0.00256	771.0	64.59	0.4554	1-2	74.10	B.10
Limits	-	-	159.00	1.113	-	-	

TABLE 10

PERFORMANCE OF THE SELF-TUNING CONTROL SCHEMES FOR DOWNHILL SIMULATION OF A SIXTY-THREE-MEMBER TRAIN

	Performance						
Methods	Variance of Velocity (m^2/sec^2)	Variance of Coupler Displacement (mm^2)	Maximum Coupler Displacement (mm)	Maximum Coupler Force (MN)	Position of Occurrence (car #)	Time of Occurrence (sec.)	Results (Annex)
Open-loop off-line schedule	0.01384	18.5	20.76	0.1464	50-51	63.7	B.11
Self-Tuning Minimum Cost Control:							
Clarke's Q"=0.1	0.00005	6.8	16.83		32-33	42.7	B.12
Clarke's Q"=1	0.00079	8.3	20.12		54-55	69.4	B.13
Self-Tuning Pole-Placement Control:							
Astrom (0.5)	0.00153	9.2	55.64	0.3923	62-63	107.4	B.14
Astrom (-0.1)	0.00026	9.2	31.02	0.2187	1-2	0.3	B.15
Astrom (0.2,0.3)	0.00073	37.1	51.38	0.3622	32-33	116.0	B.16
Allidina (0.5)	0.00038	9.2	19.12	0.1348	54-55	69.4	B.17
Allidina (-0.1)	0.00036	9.2	19.05	0.1343	54-55	69.4	B.18
Allidina (0.2,0.3)	0.00034	9.2	18.82	0.1327	54-55	69.4	B.19
Allidina (0.5,0.5)	0.00041	9.2	19.28	0.1359	54-55	69.4	B.20
Limits	-	-	159.00	1.113	-	-	

6. CONCLUSION

A comparative study of self-tuning controllers has been carried out by decentralizing train operation. Some of the proposed controllers proved encouraging for further consideration. By introducing the recursive U-D parameter estimation method, the parameter blow-up problem due to numerical rounding-off of floating point numbers has been avoided. It has been shown in the simulation that the self-tuning control is robust to the effect of disturbances.

Further development of the work in the self-tuning control of freight trains may involve other features of STC, e.g. temporary missing output data between locomotives, and considering the time delay for the throttling input application as another unknown parameter, based on the work by Wong and Bayoumi.[17,18]

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ANNEX A

GRAPHIC DISPLAY OF NINE-MEMBER TRAIN SIMULATION RESULTS

Description of the curves in Figures A.1 through A.28:

Figure (a): ———————— v_1
——— – – – —— v_5
——— – ——— v_9
——— · ——— v_{ref}

Figure (b): ———————— 2
——— – – ——— 5
——— – ——— 9

Figure (c): ———————— f_1
——— – – ——— f_2
——— – ——— f_3

Figure (d): ———————— b_1
——— – – ——— b_2
——— – ——— b_3

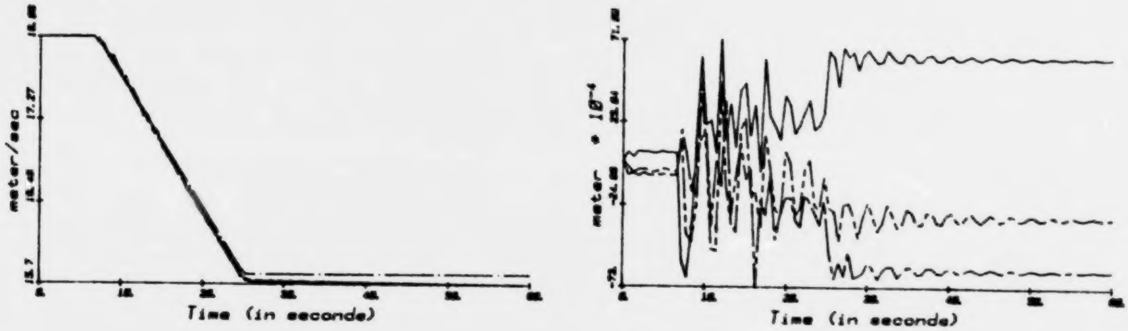


Fig. (a): Velocity

Fig. (b): Coupling displacement

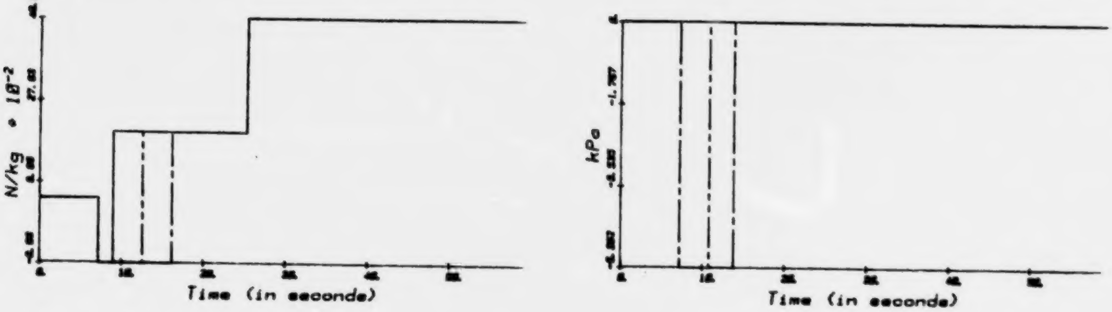


Fig. (c): Throttling force

Fig. (d): Brake pipe pressure

A.1 Nine-Car Train, Uphill, Two Per Cent Grade, Off-Line Schedule

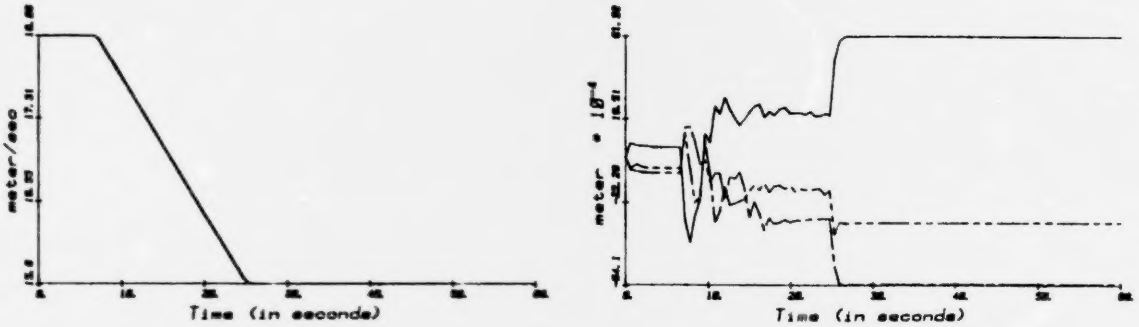


Fig. (a): Velocity

Fig. (b): Coupling displacement

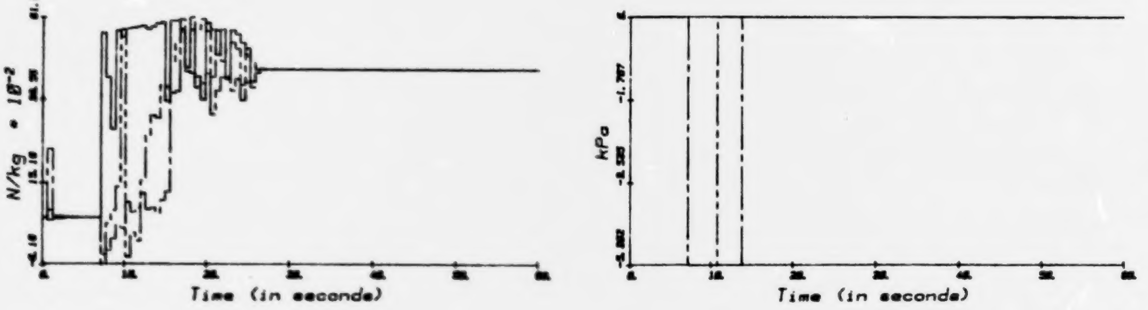


Fig. (c): Throttling force

Fig. (d): Brake pipe pressure

A.2 Nine-Car Train, Uphill, Two Per Cent Grade,
Off-Line + 3 Astrom's STR-UD (Q=0.0)

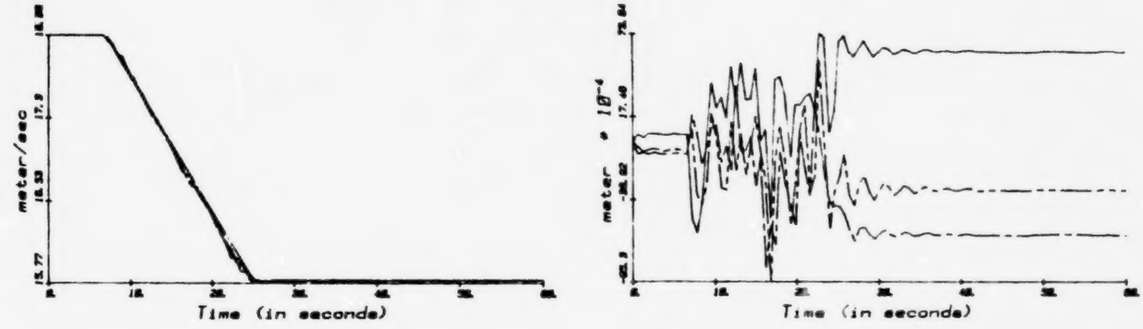


Fig. (a): Velocity

Fig. (b): Coupling displacement

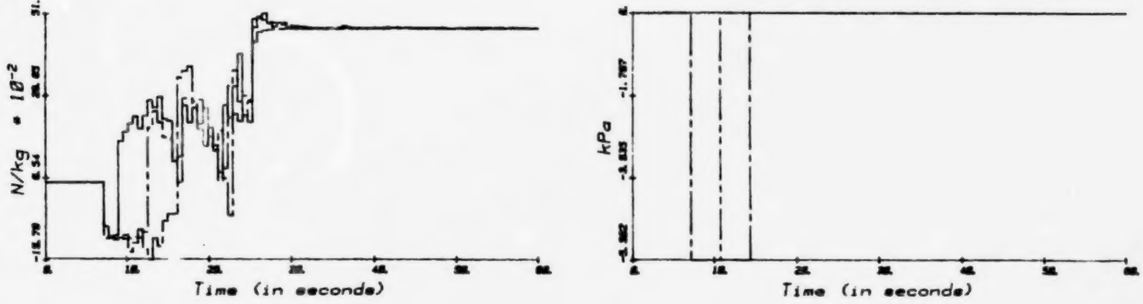


Fig. (c): Throttling force

Fig. (d): Brake pipe pressure

A.3 Nine-Car Train, Uphill, Two Per Cent Grade
Off-Line + 3 Bayoumi's STR-UD (Q=0.1)

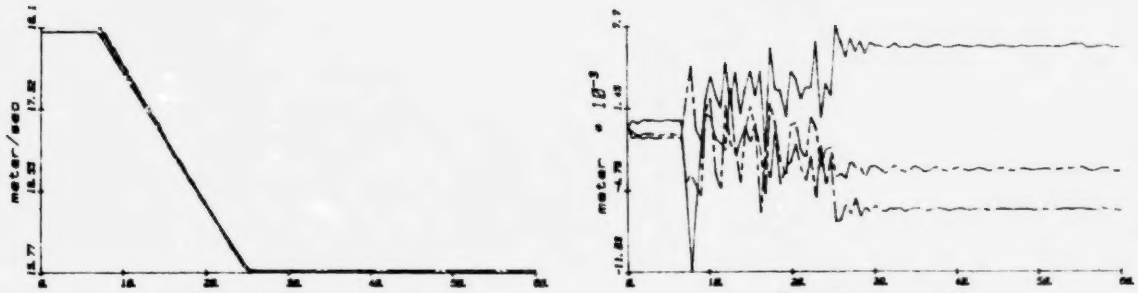


Fig. (a): Velocity

Fig. (b): Coupling displacement

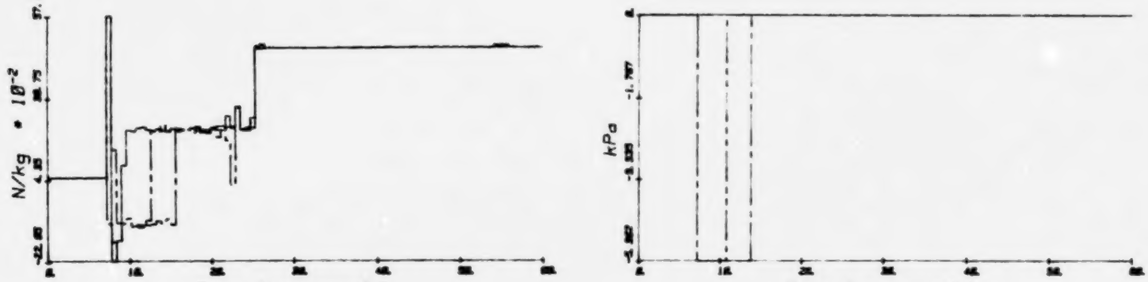


Fig. (c): Throttling force

Fig. (d): Brake pipe pressure

A.4 Nine-Car Train, Uphill, Two Per Cent Grade
Off-Line + 3 Bayoumi's STR (Q=1.0)

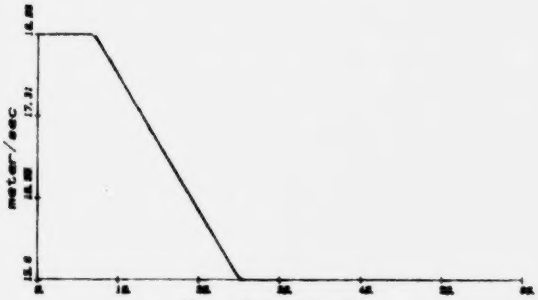


Fig. (a): Velocity

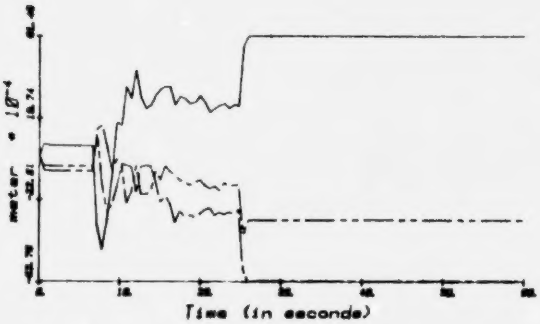


Fig. (b): Coupling displacement

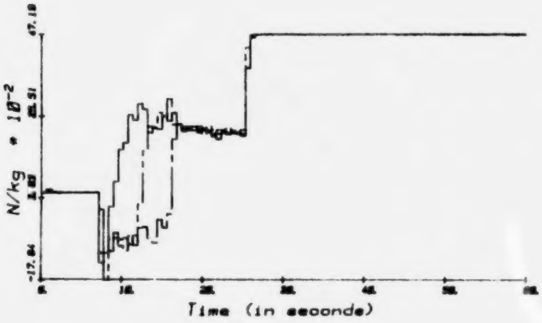


Fig. (c): Throttling force

A.5 Nine-Car Train, Uphill, Two Per Cent Grade, Using 3 Clarke-Gawthrop STR (Q"=0.1)

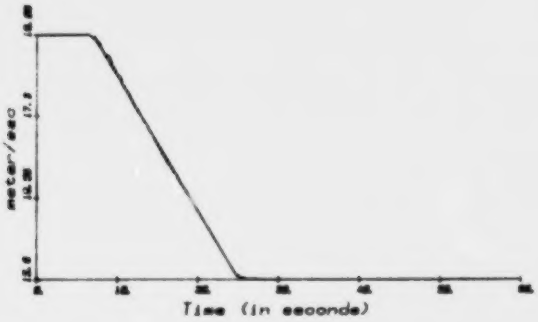


Fig. (a): Velocity

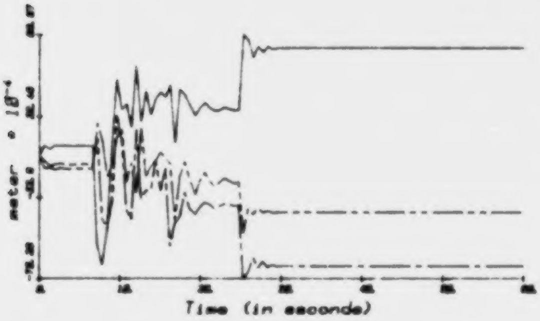


Fig. (b): Coupling displacement

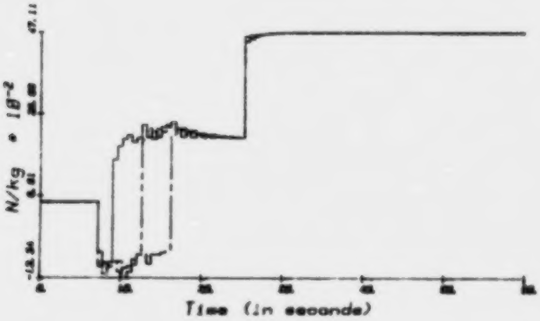


Fig. (c): Throttling force

A.6 Nine-Car Train Uphill, Two Per Cent Grade, Using 3 Clarke-Gawthrop STR (Q"=1.0)

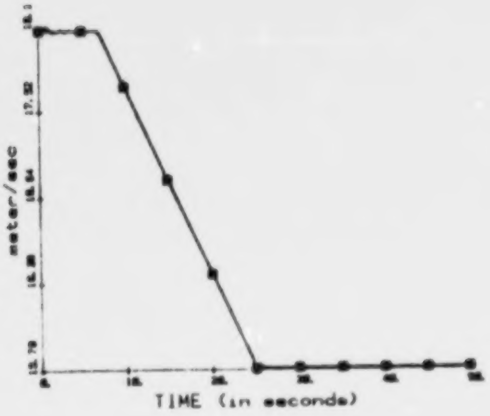


FIG (A): VELOCITY

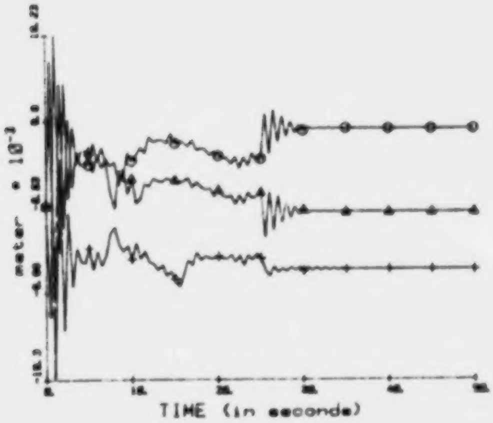


FIG (B): COUPLING DISPLACEMENT

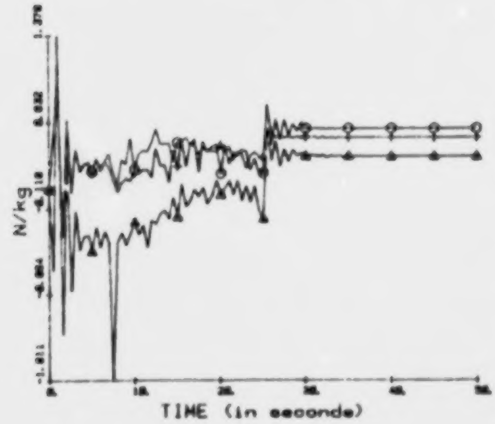


FIG (C): THROTTLING FORCE

A.7 Uphill Simulation Results of a Nine-Car Train Using 3 Astrom's STPP Throttle Control (Pole=0.5)

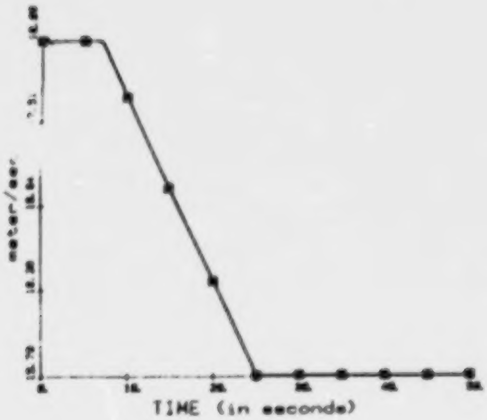


FIG (A): VELOCITY

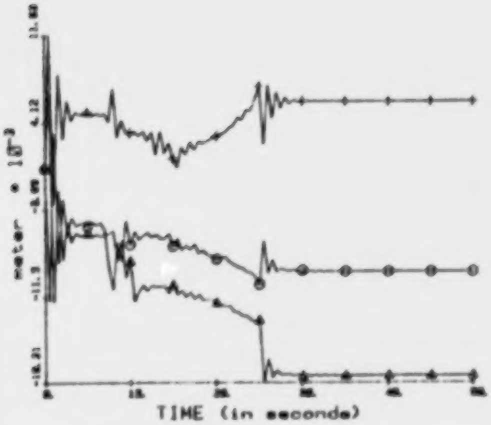


FIG (B): COUPLING DISPLACEMENT

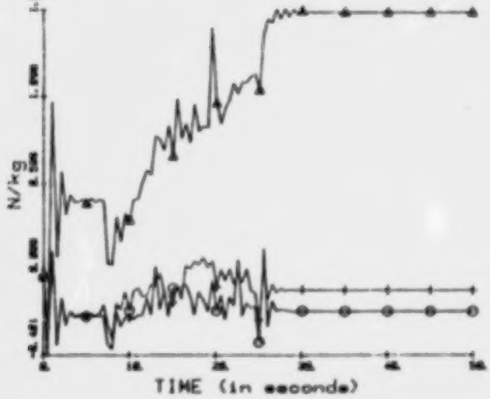


FIG (C): THROTTLING FORCE

A.8 Uphill Simulation Results of a Nine-Car Train Using 3 Astrom's STPP Throttle Control (Pole=-0.1)

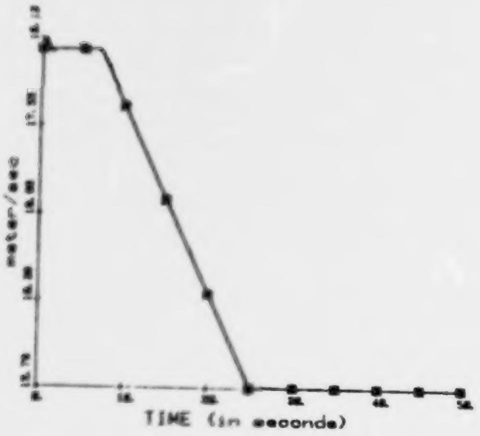


FIG (A): VELOCITY

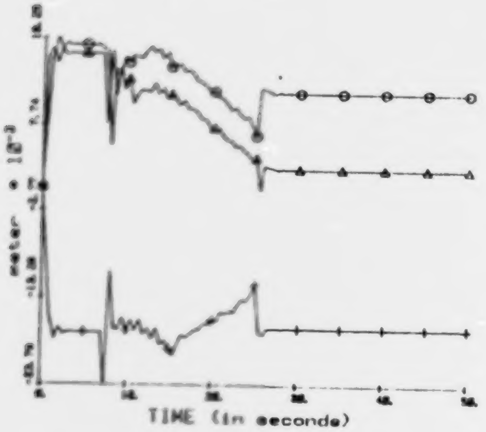


FIG (B): COUPLING DISPLACEMENT

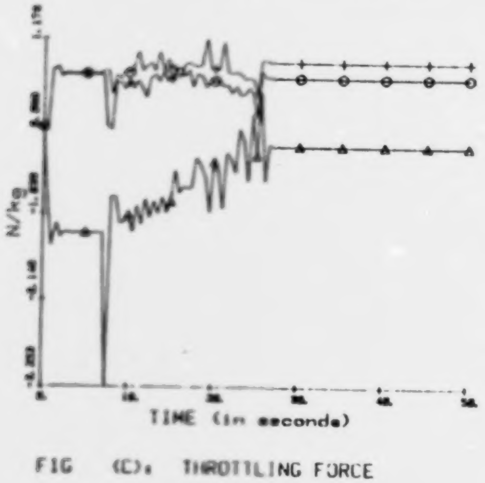


FIG (C): THROTTLING FORCE

A.9 Uphill Simulation Results of a Nine-Car Train
Using 3 Astrom's STPP Throttle Control (Pole=0.2;0.3)

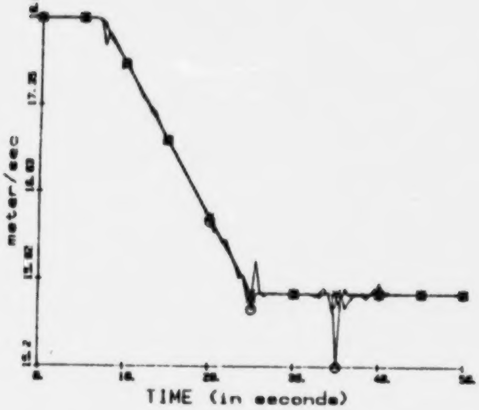


FIG (A): VELOCITY

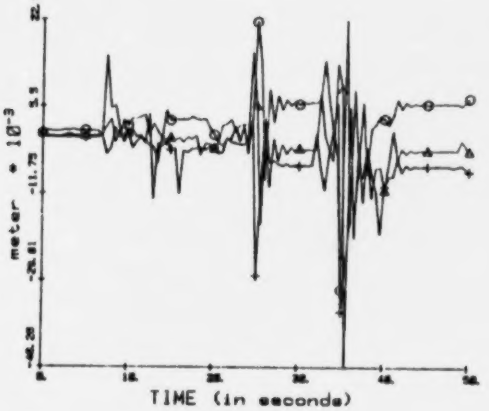


FIG (B): COUPLING DISPLACEMENT

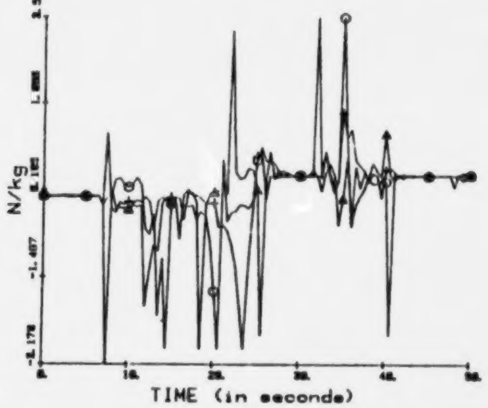


FIG (C): THROTTLING FORCE

A.10 Uphill Simulation Results of a Nine-Car Train Using 3 Wellstead's STPP Throttle Control (Pole=0.5)

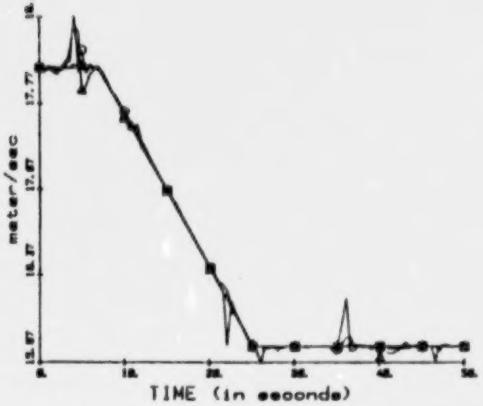


FIG (A): VELOCITY

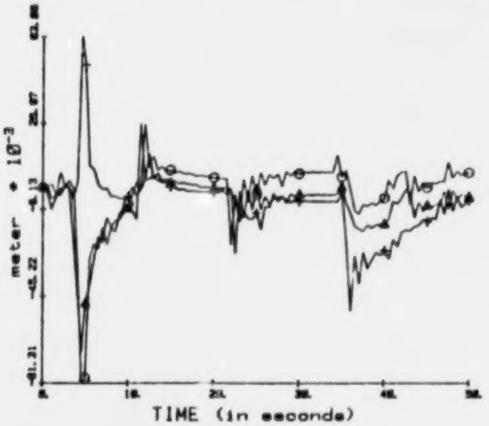


FIG (B): COUPLING DISPLACEMENT

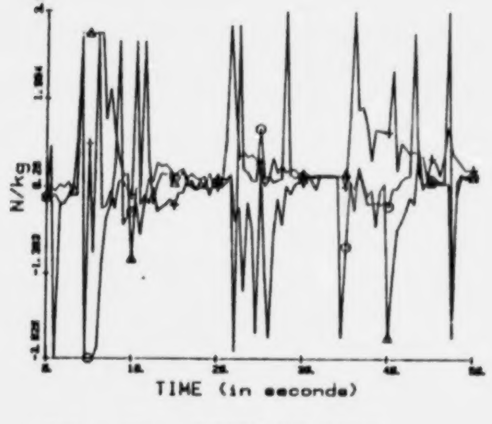


FIG (C): THROTTLING FORCE

A.11 Uphill Simulation Results of a Nine-Car Train Using 3 Wellstead's STPP Throttle Control (Pole=0.1)

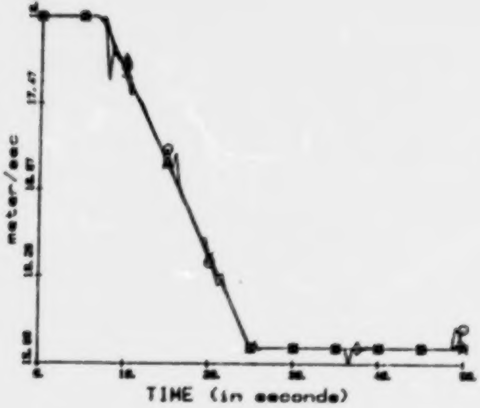


FIG (A): VELOCITY

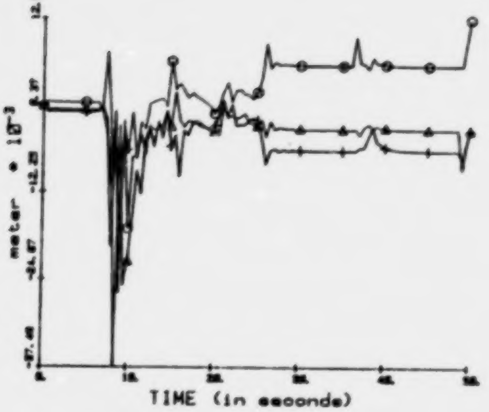


FIG (B): COUPLING DISPLACEMENT

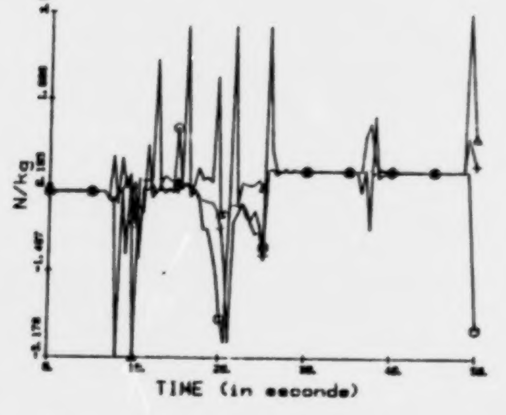


FIG (C): THROTTLING FORCE

A.12 Uphill Simulation Results of a Nine-Car Train
Using 3 Wellstead's STPP Throttle Control (Pole=0.2.0.3)

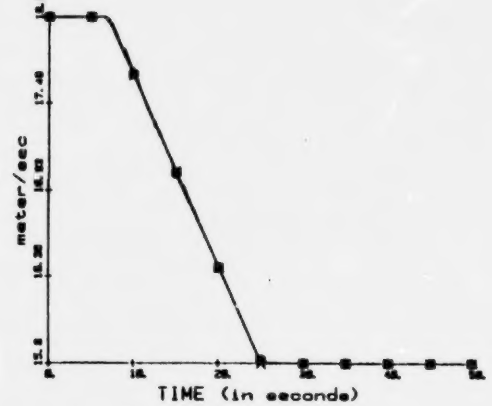


FIG (A): VELOCITY

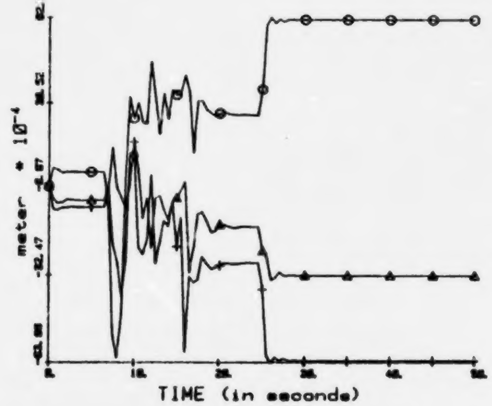


FIG (B): COUPLING DISPLACEMENT

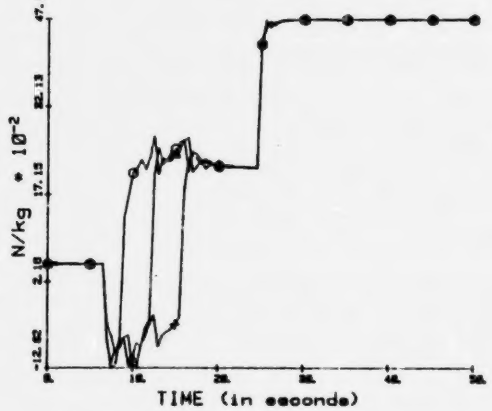


FIG (C): THROTTLING FORCE

A.13 Uphill Simulation Results of a Nine-Car Train Using 3 Allidina's STPPG Throttle Control (Pole=0.5)

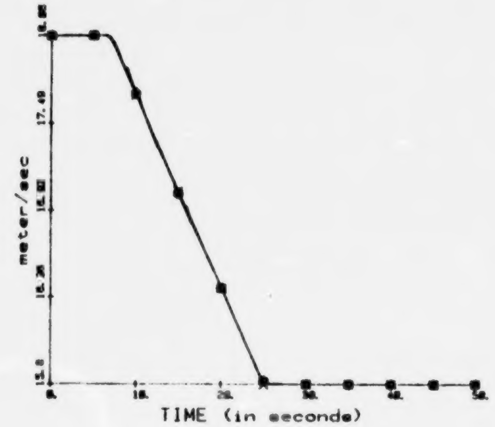


FIG (A): VELOCITY

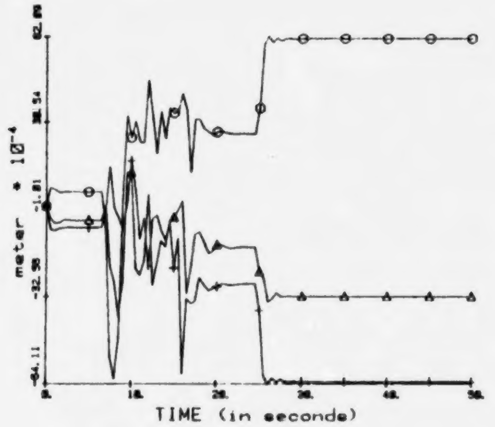


FIG (B): COUPLING DISPLACEMENT

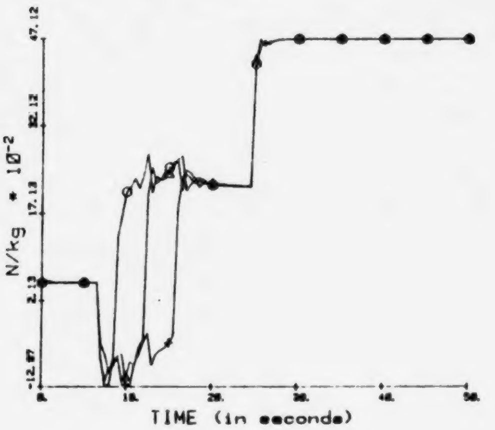


FIG (C): THROTTLING FORCE

A.14 Uphill Simulation Results of a Nine-Car Train Using 3 Allidina's STPPG Throttle Control (Pole=-0.1)

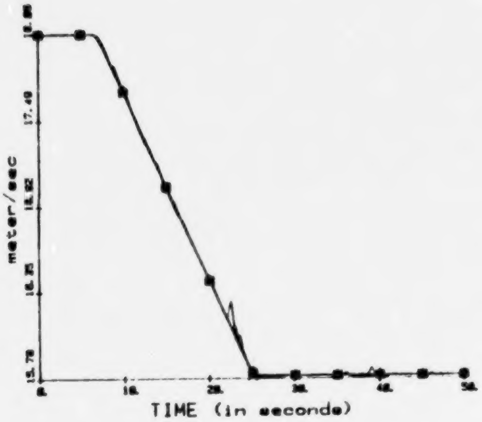


FIG (A): VELOCITY

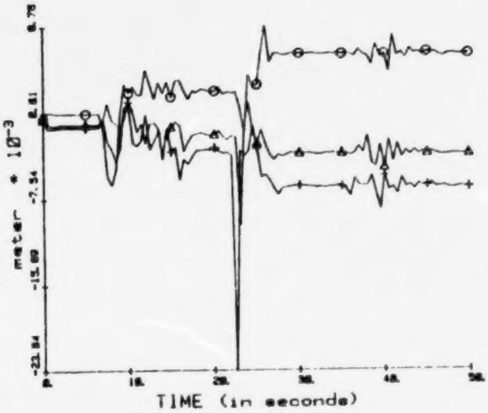


FIG (B): COUPLING DISPLACEMENT

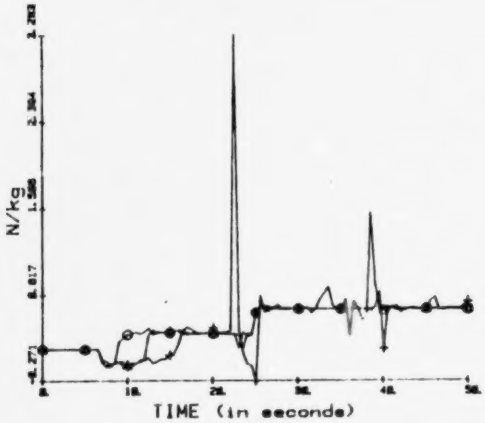


FIG (C): THROTTLING FORCE

A.15 Uphill Simulation Results of a Nine-Car Train
Using 3 Allidina's SISO-STPPG Throttle Control (Pole=0.2;0.3)

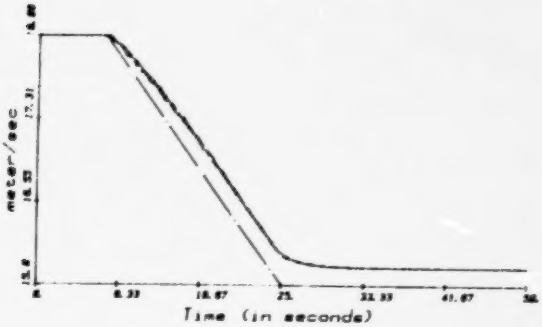


Fig. (a): Velocity

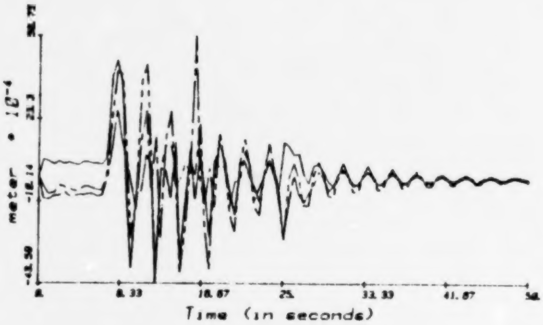


Fig. (b): Coupling displacement

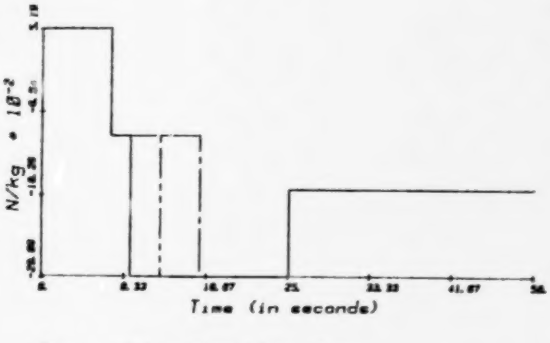


Fig. (c): Throttling force

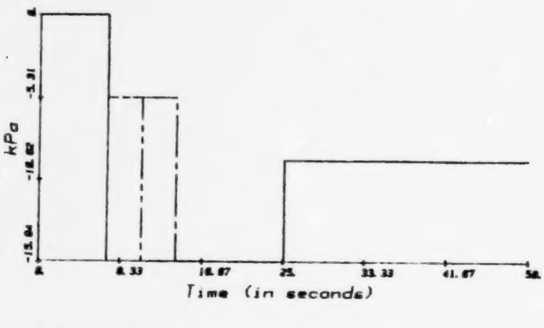


Fig. (d): Brake pipe pressure

A.16 Nine-Car Train, Downhill, Two Per Cent Grade, Off-Line Schedule

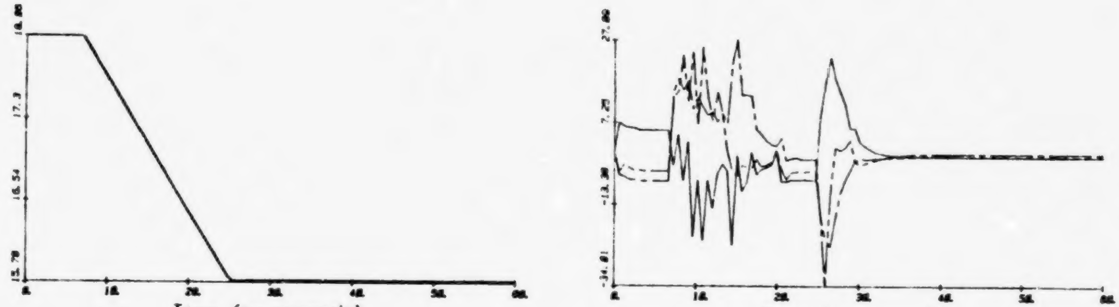


Fig. (a): Velocity

Fig. (b): Coupling displacement

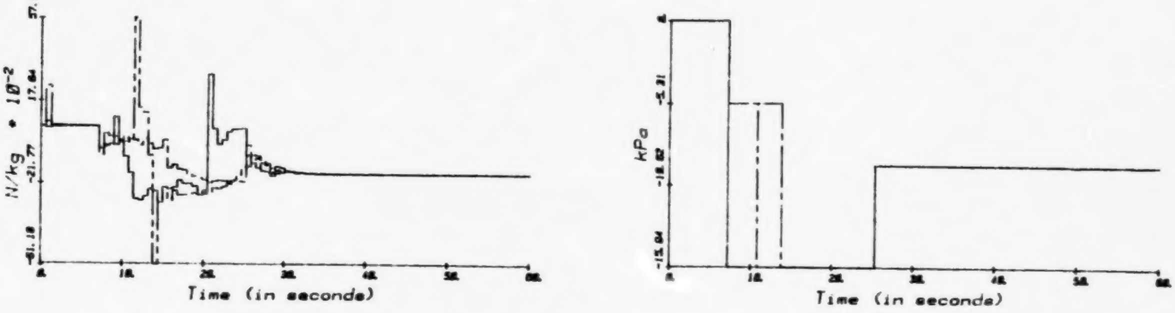


Fig. (c): Throttling force

Fig. (d): Brake pipe pressure

A.17 Nine-Car Train, Downhill, Two Per Cent Grade,
Off-Line + 3 Astrom's STR-UD (Q=0.0)

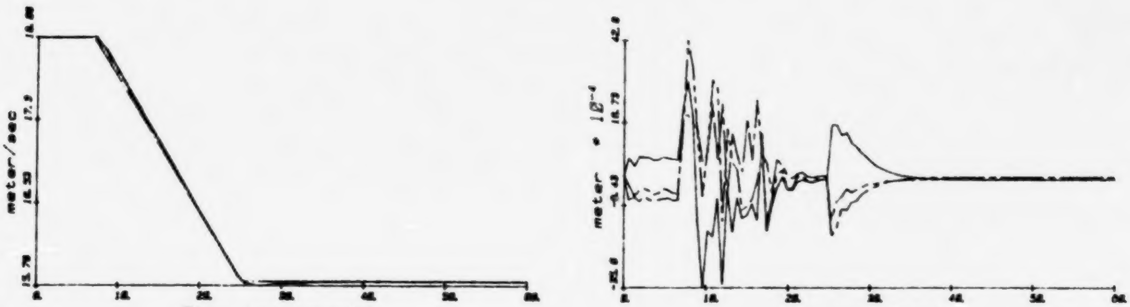


Fig. (a): Velocity

Fig. (b): Coupling displacement

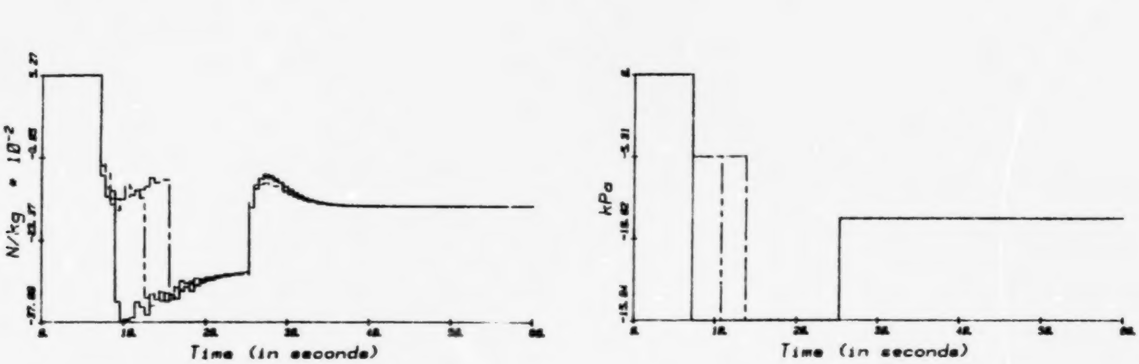


Fig. (c): Throttling force

Fig. (d): Brake pipe pressure

A.18 Nine-Car Train, Downhill, Two Per Cent Grade, Off-Line + 3 Baymoumi's STR-UD (Q=0.1)

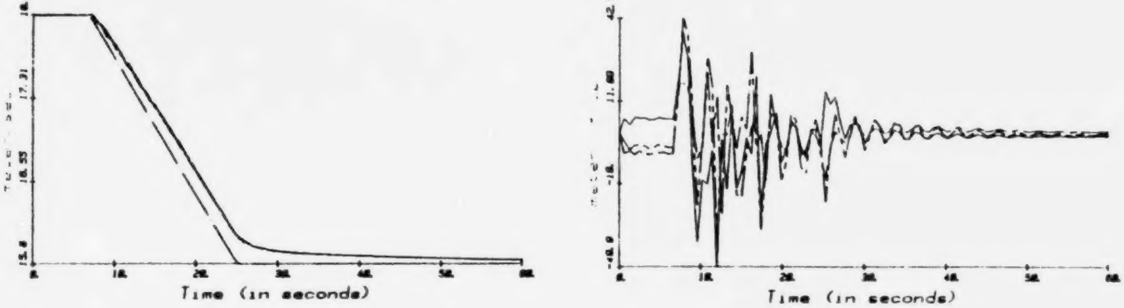


Fig. (a): Velocity

Fig. (b): Coupling displacement

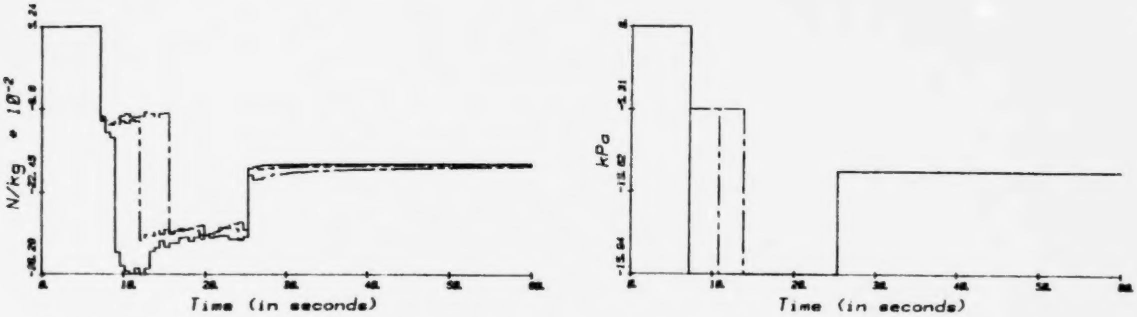


Fig. (c): Throttling force

Fig. (d): Brake pipe pressure

A.19 Nine-Car Train, Downhill, Two Per Cent Grade,
Off-Line + 3 Bayoumi's STR-UD (Q=1.0)

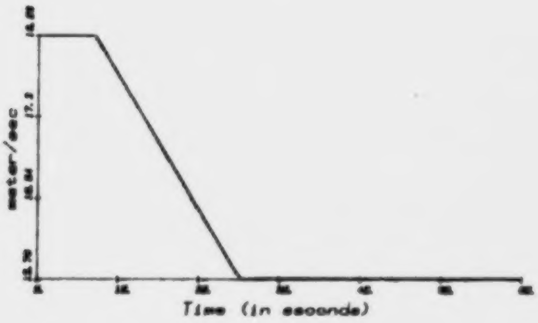


Fig. (a): Velocity

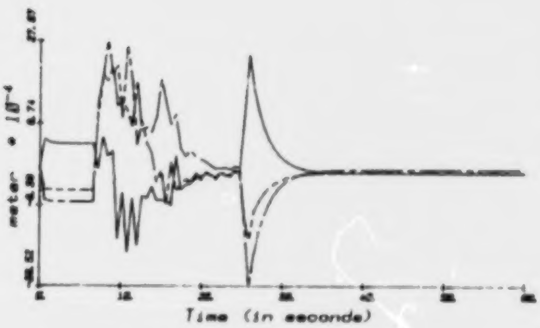


Fig. (b): Coupling displacement

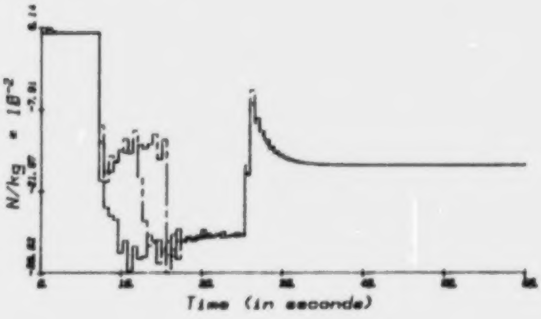


Fig. (c): Throttling force

A.20 Nine-Car Train, Downhill, Two Per Cent Grade, Using 3 Clarke-Gawthrope STR (Q"=0.1)

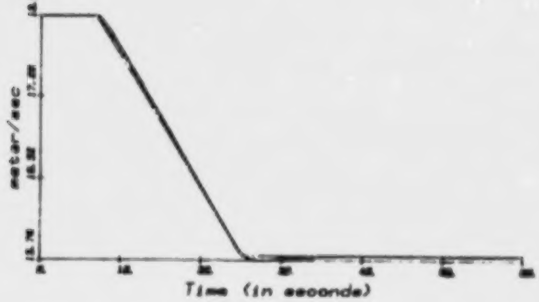


Fig. (a): Velocity

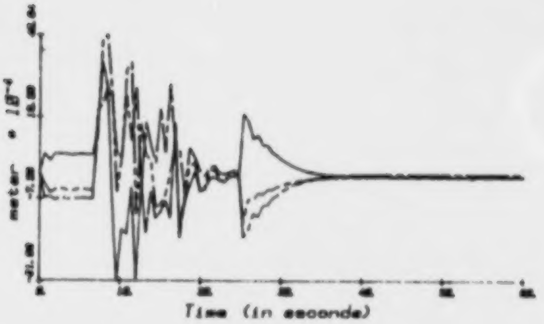


Fig. (b): Coupling displacement

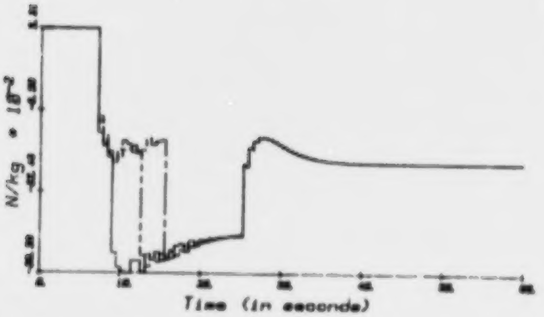


Fig. (c): Throttling force

A.21 Nine-Car Train, Downhill, Two Per Cent Grade, Using 3 Clarke-Gawthrop STR (Q"=1.0)

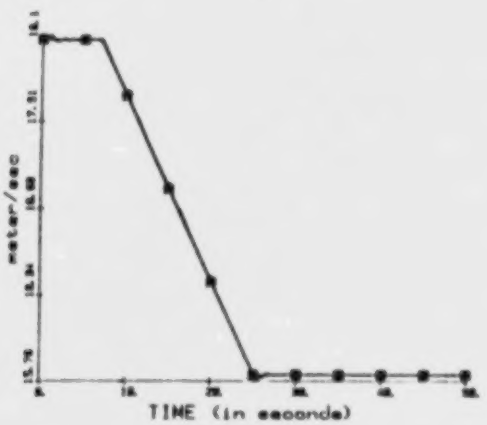


FIG (A): VELOCITY

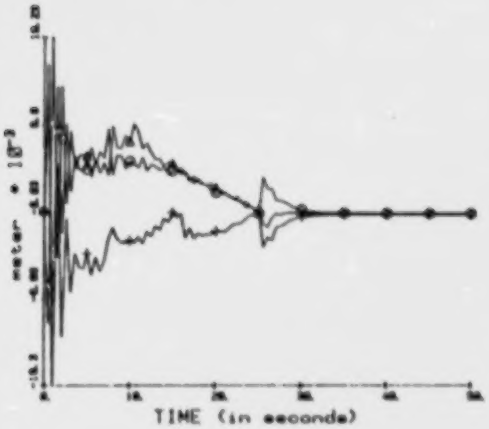


FIG (B): COUPLING DISPLACEMENT

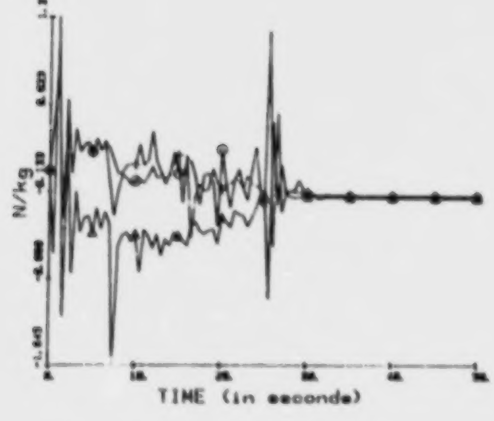


FIG (C): THROTTLING FORCE

A.22 Downhill Simulation Results of a Nine-Car Train Using 3 Astrom's STPP Throttle Control (Pole=0.5)

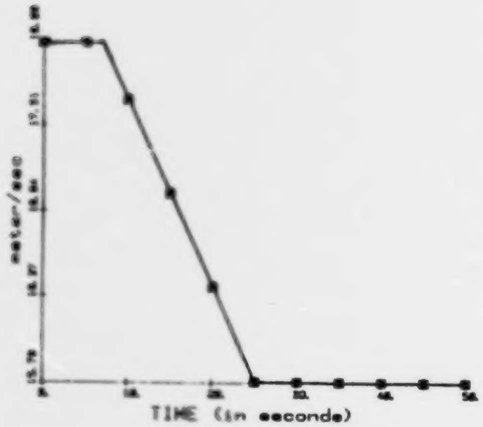


FIG (A): VELOCITY

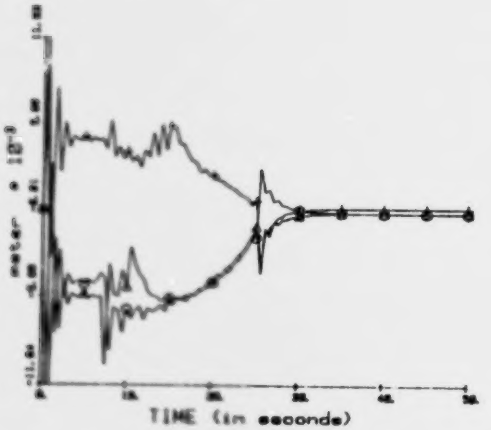


FIG (B): COUPLING DISPLACEMENT

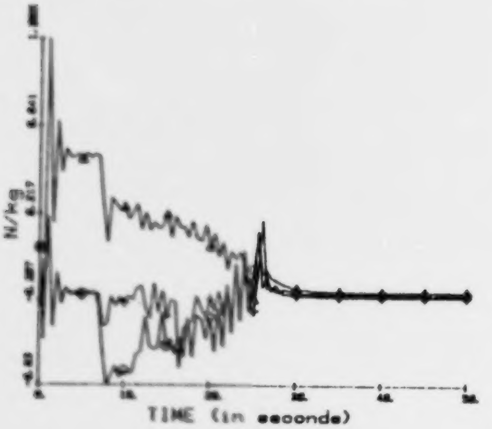


FIG (C): THROTTLING FORCE

A.23 Downhill Simulation Results of a Nine-Car Train Using 3 Astrom's STPP Throttle Control (Pole=-0.1)

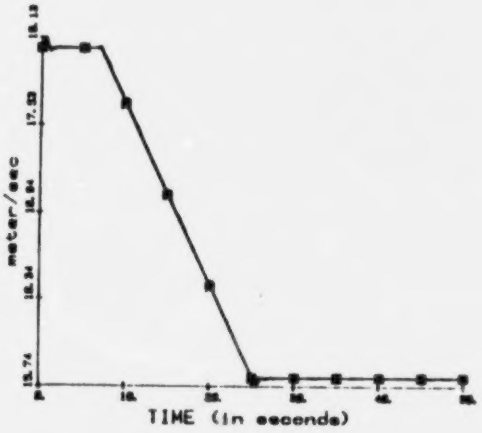


FIG (A): VELOCITY

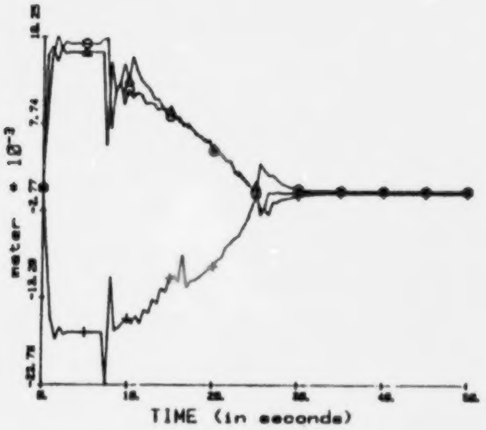


FIG (B): COUPLING DISPLACEMENT

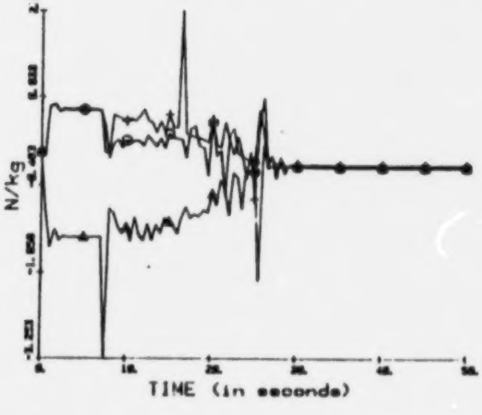


FIG (C): THROTTLING FORCE

A.24 Downhill Simulation Results of a Nine-Car Train Using 3 Astrom's STPP Throttle Control (Pole=0.2;0.3)

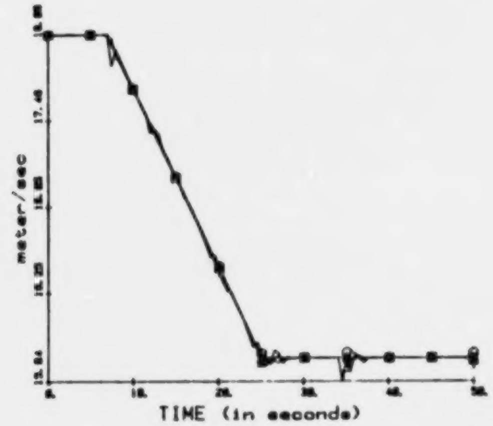


FIG (A): VELOCITY

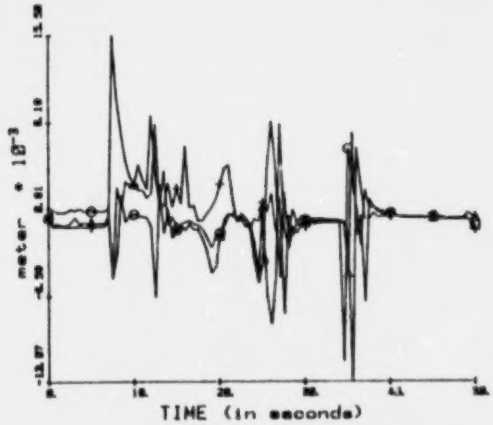


FIG (B): COUPLING DISPLACEMENT

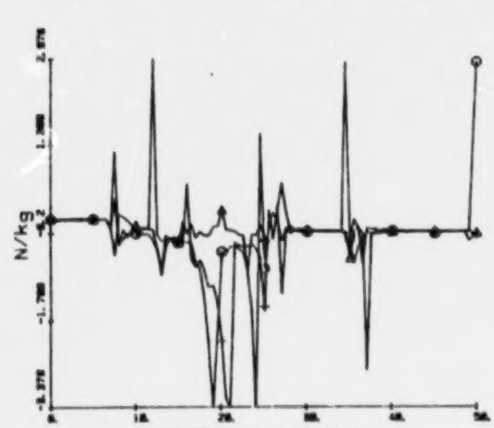


FIG (C): THROTTLING FORCE

A.25 Downhill Simulation Results of a Nine-Car Train Using 3 Wellstead's STPP Throttle Control (Pole=0.5)

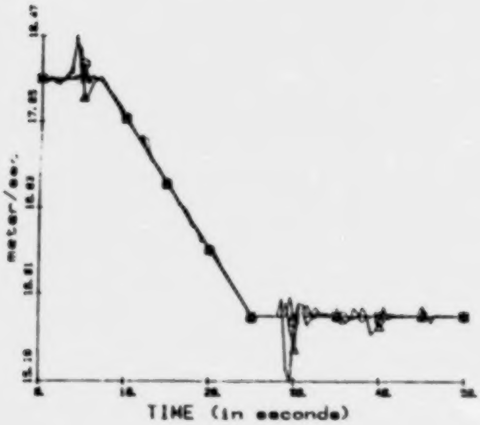


FIG (A): VELOCITY

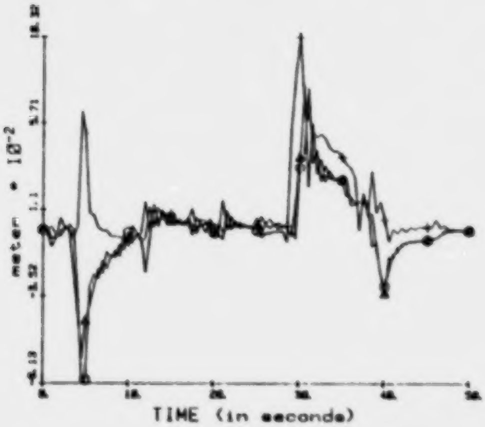


FIG (B): COUPLING DISPLACEMENT

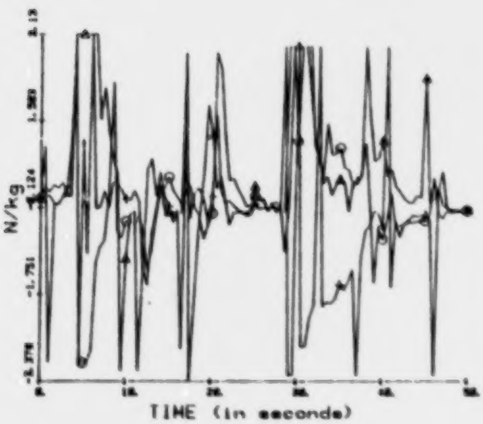


FIG (C): THROTTLING FORCE

A.26 Downhill Simulation Results of a Nine-Car Train Using 3 Wellstead's STPP Throttle Control (Pole=-0.1)

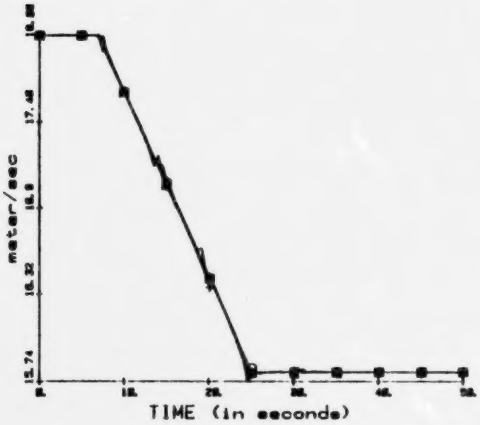


FIG (A): VELOCITY

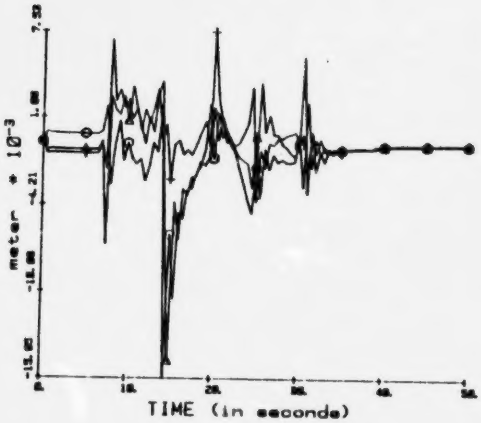


FIG (B): COUPLING DISPLACEMENT

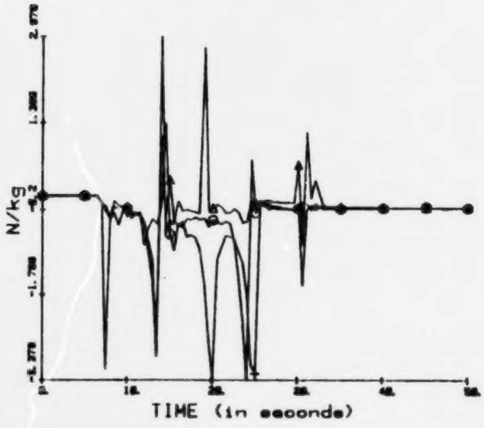


FIG (C): THROTTLING FORCE

A.27 Downhill Simulation Results of a Nine-Car Train
Using 3 Wellstead's STPP Throttle Control (Pole=0.2;0.3)

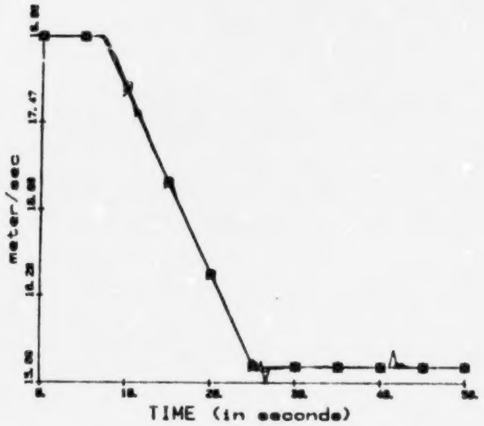


FIG (A): VELOCITY

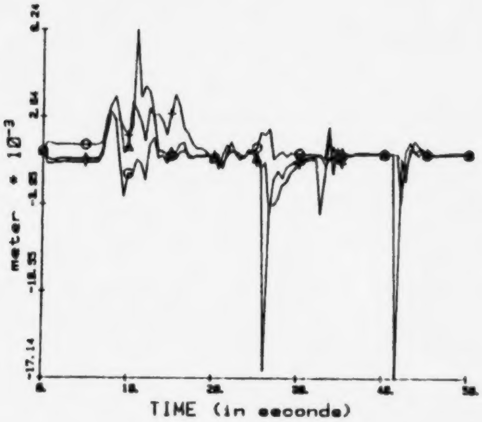


FIG (B): COUPLING DISPLACEMENT

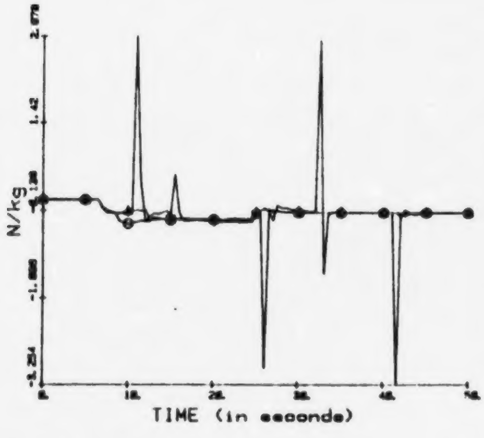


FIG (C): THROTTLING FORCE

A.28 Downhill Simulation Results of a Nine-Car Train
Using 3 Allidina's STPP Throttle Control (Pole=0.5)

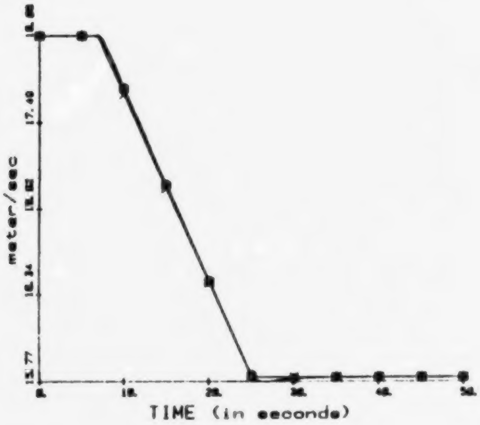


FIG (A): VELOCITY

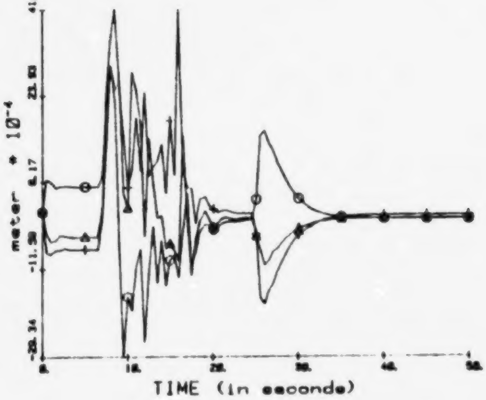


FIG (B): COUPLING DISPLACEMENT

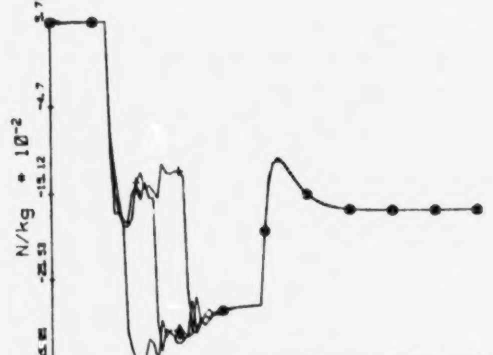


FIG (C): THROTTLING FORCE

A.29 Downhill Simulation Results of a Nine-Car Train Using 3 Allidina's STPP Throttle Control (Pole=-0.1)

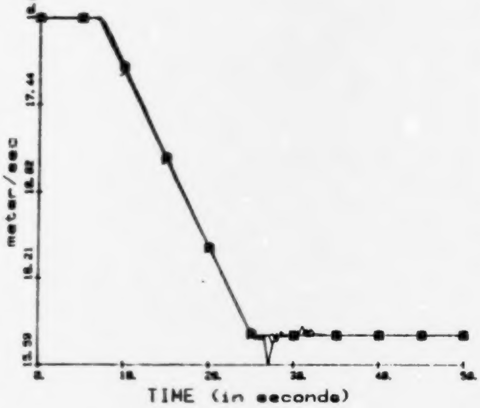


FIG (A): VELOCITY

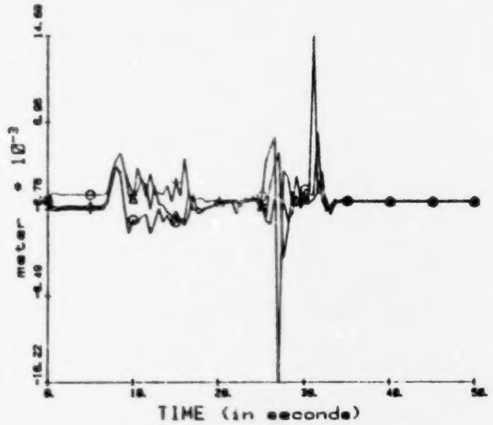


FIG (B): COUPLING DISPLACEMENT

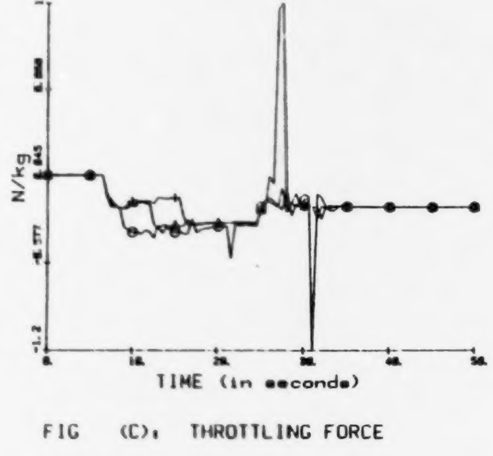


FIG (C): THROTTLING FORCE

A.30 Downhill Simulation Results of a Nine-Car Train
Using 3 Allidina's STPP Throttle Control (Pole=0.2,0.3)

ANNEX B

GRAPHIC DISPLAY OF SIXTY-THREE-MEMBER TRAIN SIMULATION RESULTS

Description of the curves in Figures B.1 through B.28:

Figure (a): ———————— v_1

—— – – —— v_{32}

—— – —— v_{63}

—— · —— v_{ref}

Figure (b): ———————— 2

—— – – —— 32

—— – —— 63

Figure (c): ———————— f_1

—— – – —— f_2

—— – —— f_3

Figure (d): ———————— b_1

—— – – —— b_2

—— – —— b_3

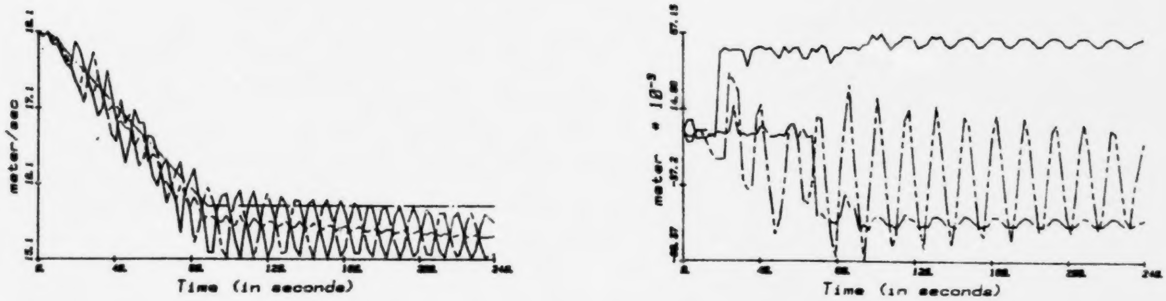


Fig. (a): Velocity

Fig. (b): Coupling displacement

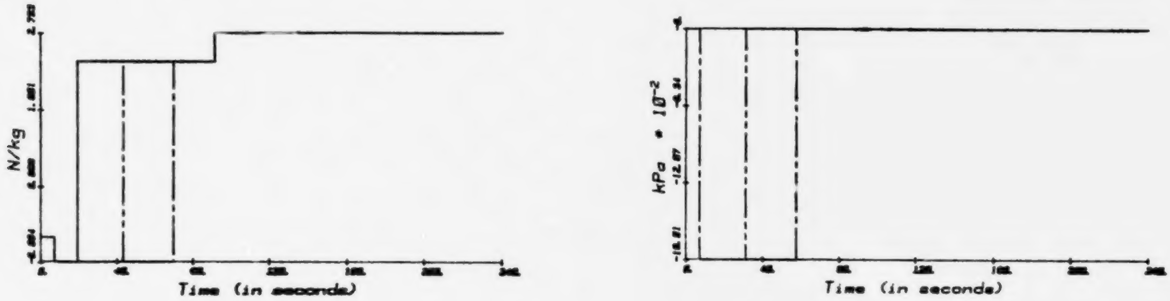


Fig. (c): Throttling force

Fig. (d): Brake pipe pressure

B.1 Six-Three-Car Train, Uphill, Two Per Cent Grade,
Off-Line Schedule Only

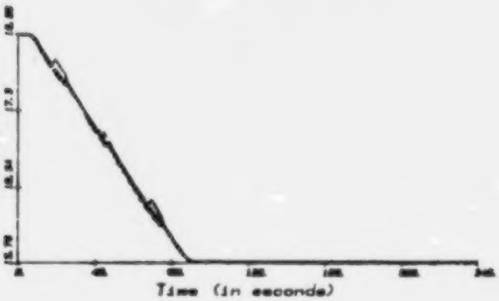


Fig. (a): Velocity

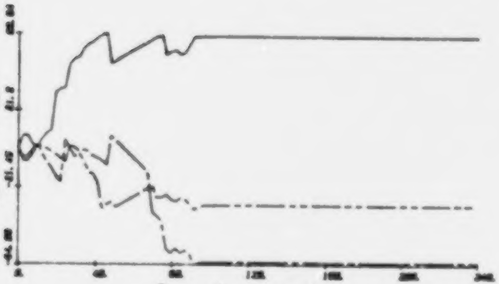


Fig. (b): Coupling displacement

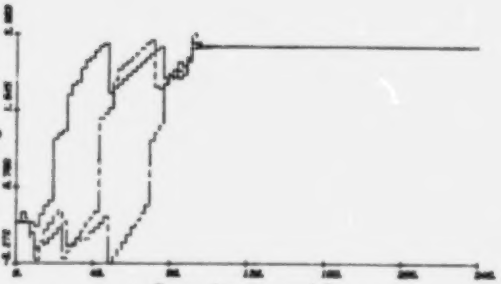


Fig. (c): Throttling force

B.2 Six-Three-Car Train, Uphill, Two Per Cent Grade, Using 3 Clarke-Gawthrop STR (Q'=0.1)

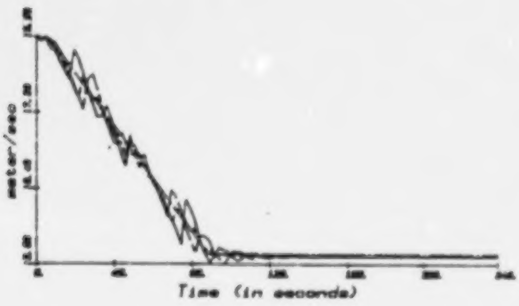


Fig. (a): Velocity

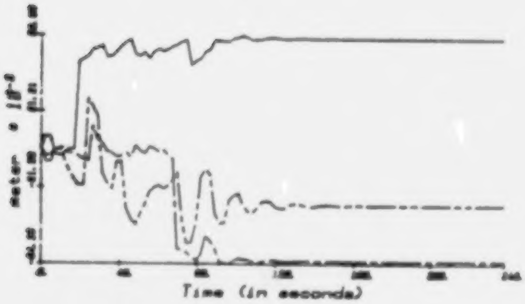


Fig. (b): Coupling displacement

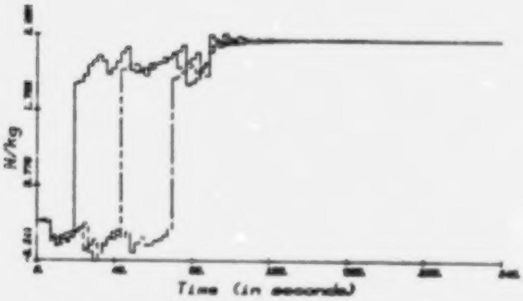


Fig. (c): Throttling force

B.3 Six-Three-Car Train, Uphill, Two Per Cent Grade, Using 3 Clarke-Gawthrop STR (Q'=1.0)

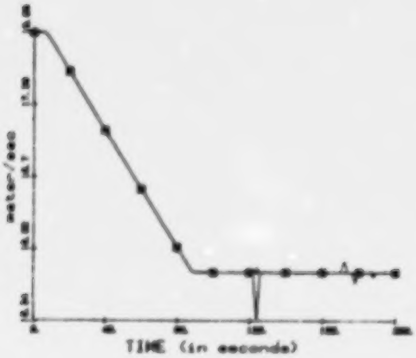


FIG (A): VELOCITY

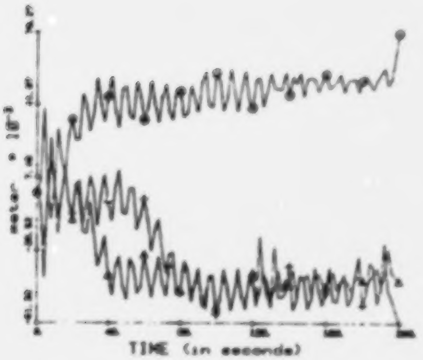


FIG (B): COUPLING DISPLACEMENT

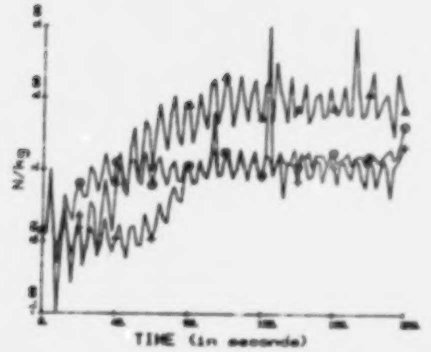


FIG (C): THROTTLING FORCE

B.4 Uphill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP2 Throttle Control (Pole=0.5)

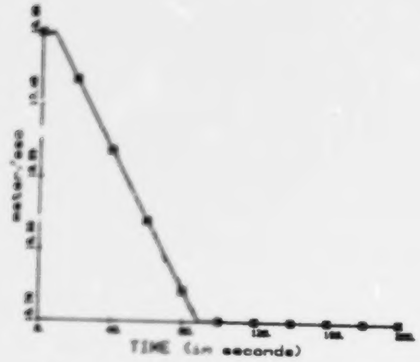


FIG (A): VELOCITY

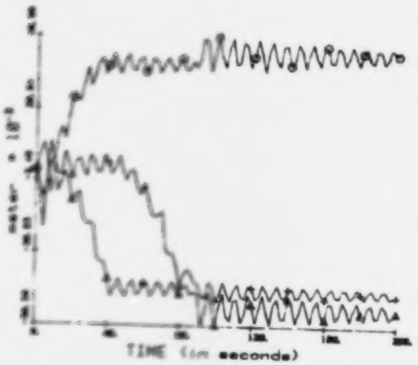


FIG (B): COUPLING DISPLACEMENT

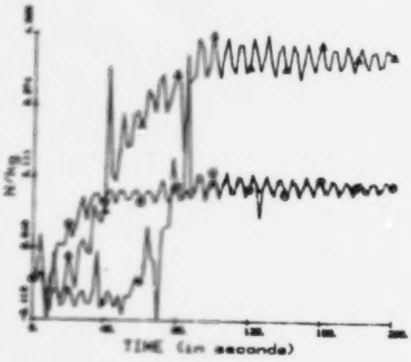


FIG (C): THROTTLING FORCE

B.5 Uphill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP2 Throttle Control (Pole=0.1)

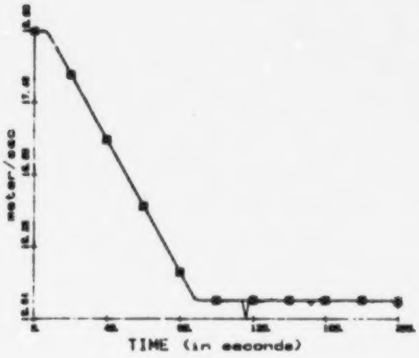


FIG (A): VELOCITY

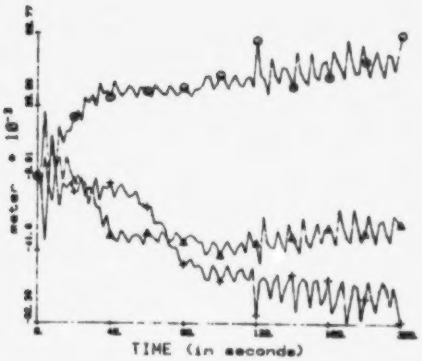


FIG (B): COUPLING DISPLACEMENT

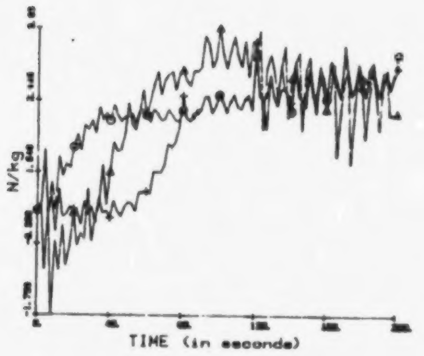


FIG (C): THROTTLING FORCE

B.6 Uphill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP2 Throttle Control (Pole=0.2,0.3)

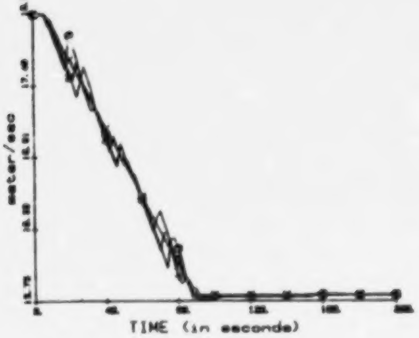


FIG (A): VELOCITY

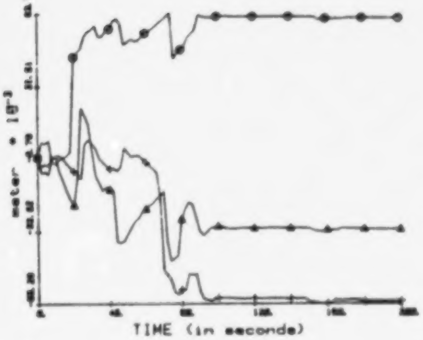


FIG (B): COUPLING DISPLACEMENT

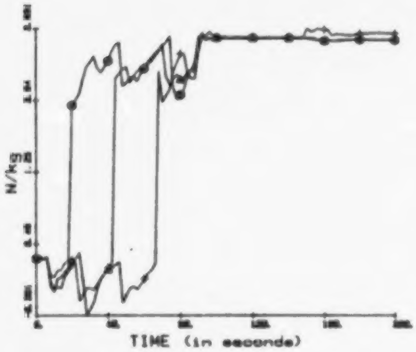


FIG (C): THROTTLING FORCE

B.7 Uphill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP4 Throttle Control (Pole=0.5)

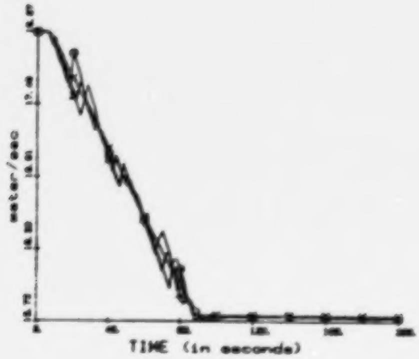


FIG (A): VELOCITY

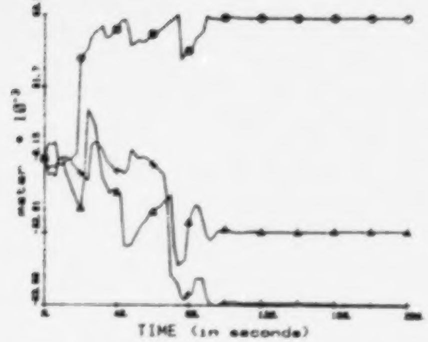


FIG (B): COUPLING DISPLACEMENT

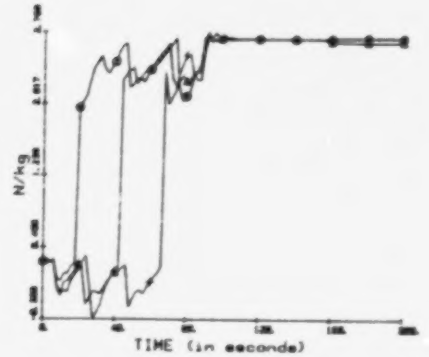


FIG (C): THROTTLING FORCE

B.8 Uphill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP4 Throttle Control (Pole=0.1)

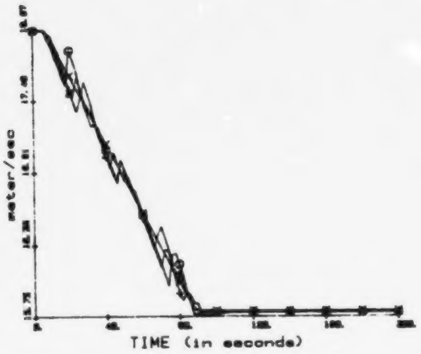


FIG (A): VELOCITY

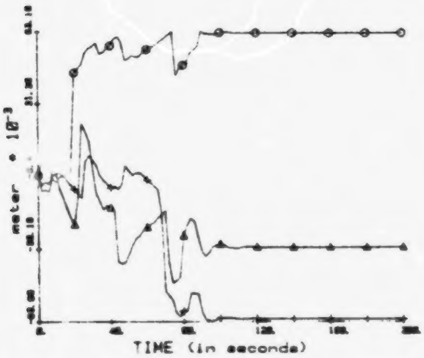


FIG (B): COUPLING DISPLACEMENT

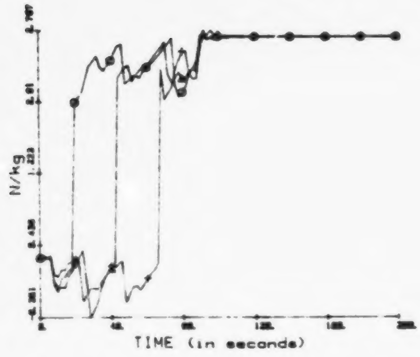


FIG (C): THROTTLING FORCE

B.9 Uphill Simulation of a Sixty-Three-Car Train
Using 3 SISO-STPP4 Throttle Control (Pole=0.2,0.3)

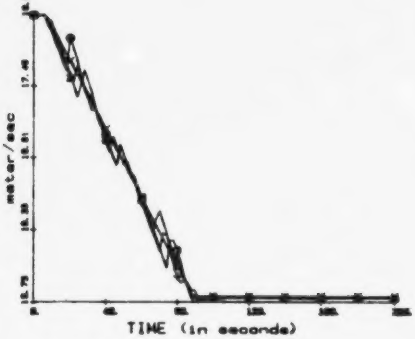


FIG (A): VELOCITY

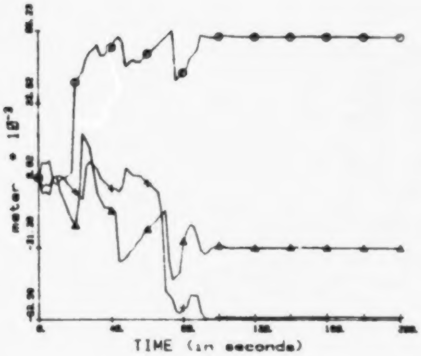


FIG (B): COUPLING DISPLACEMENT

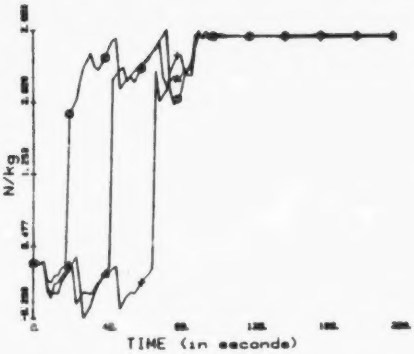


FIG (C): THROTTLING FORCE

B.10 Uphill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP4 Throttle Control (Pole=0.5,0.5)

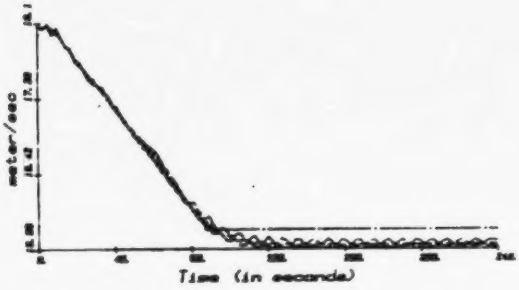


Fig. (a): Velocity

Fig. (a): Velocity

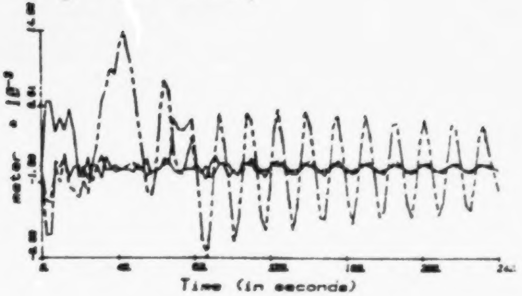


Fig. (b): Coupling displacement

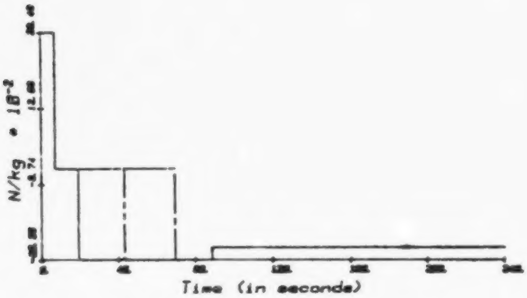


Fig. (c): Throttling force

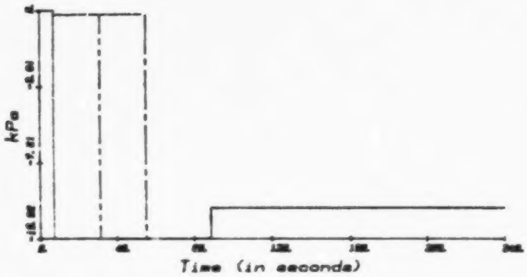


Fig. (d): Brake pipe pressure

B.11 Six-Three-Car Train, Downhill, Two Per Cent Grade, Off-Line Schedule Only

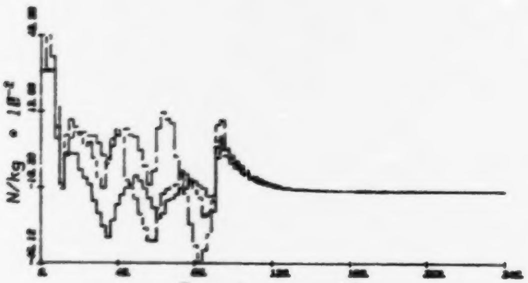


Fig. (c): Throttling force

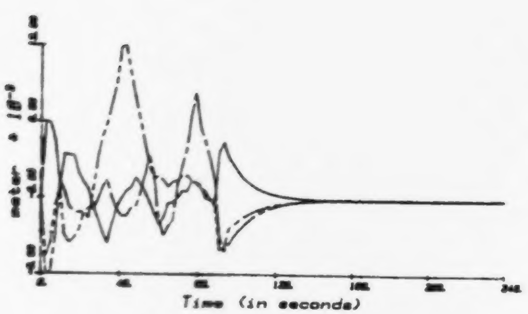


Fig. (b): Coupling displacement

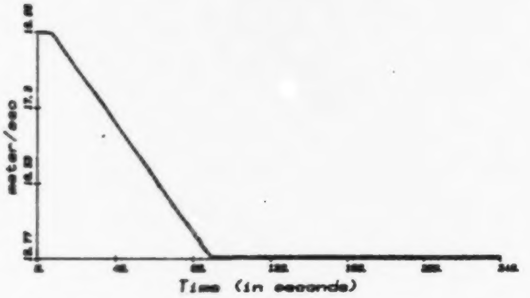


Fig. (a): Velocity

B.12 Six-Three-Car Train, Downhill, Two Per Cent Grade, Using 3 Clarke-Gawthrop STR (Q'=0.1)

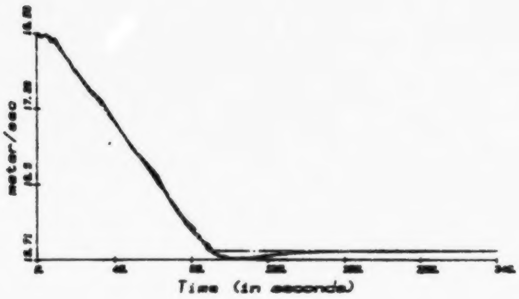


Fig. (a): Velocity

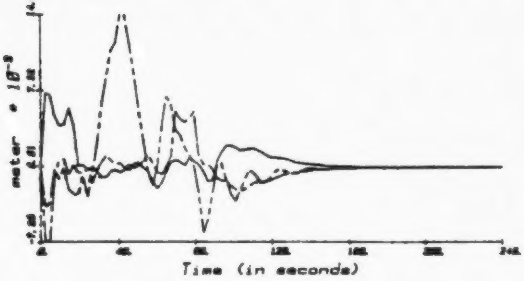


Fig. (b): Coupling displacement

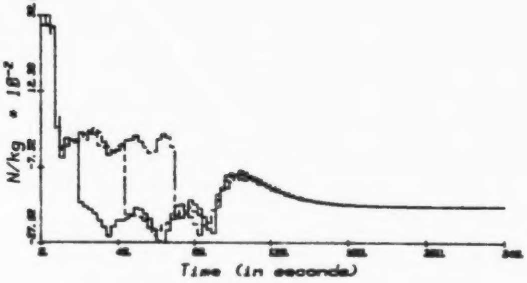


Fig. (c): Throttling force

B.13 Six-Three-Car Train, Downhill, Two Per Cent Grade, Using 3 Clarke-Gawthrop STR (Q'=0.1)

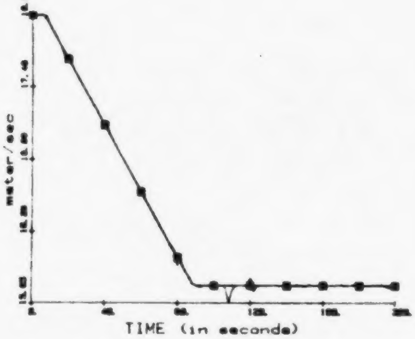


FIG (A): VELOCITY

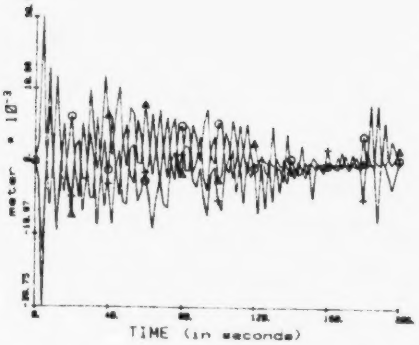


FIG (B): COUPLING DISPLACEMENT

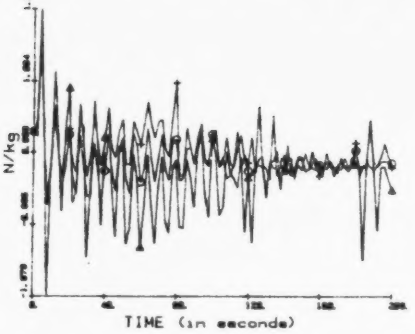


FIG (C): THROTTLING FORCE

B.14 Downhill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP2 Throttle Control (Pole=0.5)

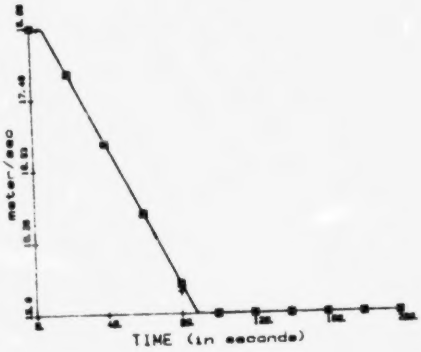


FIG (A): VELOCITY

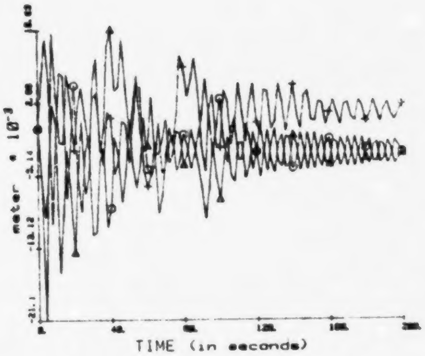


FIG (B): COUPLING DISPLACEMENT

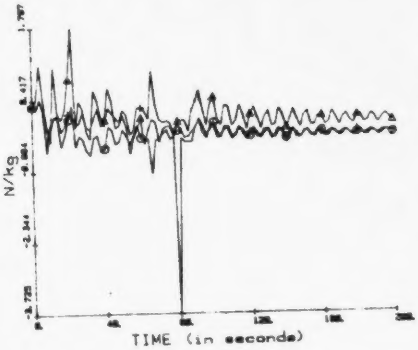


FIG (C): THROTTLING FORCE

B.15 Downhill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP2 Throttle Control (Pole=0.1)

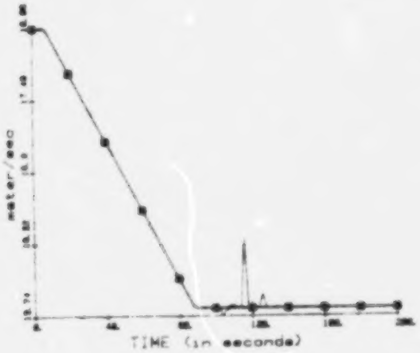


FIG (A): VELOCITY

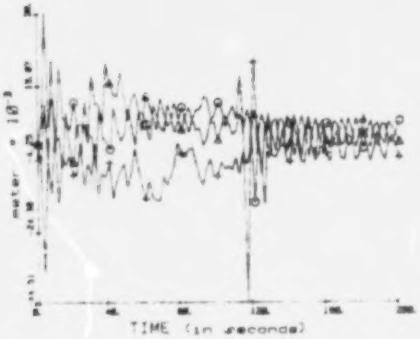


FIG (B): COUPLING DISPLACEMENT

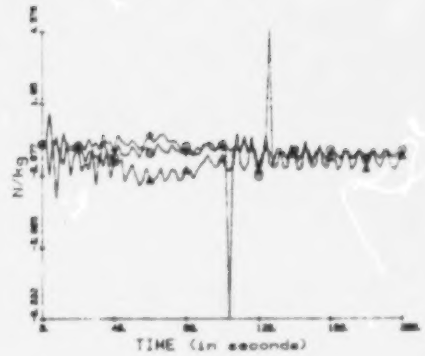


FIG (C): THROTTLING FORCE

B.16 Downhill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP2 Throttle Control (Pole=0.2,0.3)

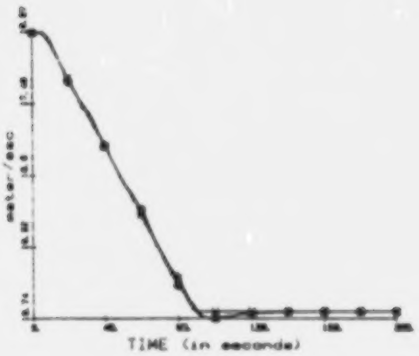


FIG (A): VELOCITY

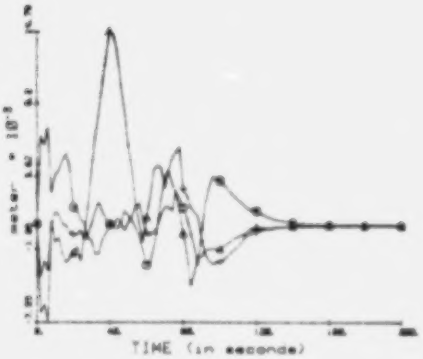


FIG (B): COUPLING DISPLACEMENT

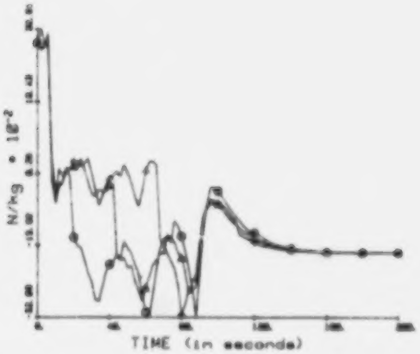


FIG (C): THROTTLING FORCE

B.17 Downhill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP4 Throttle Control (Pole=0.5)

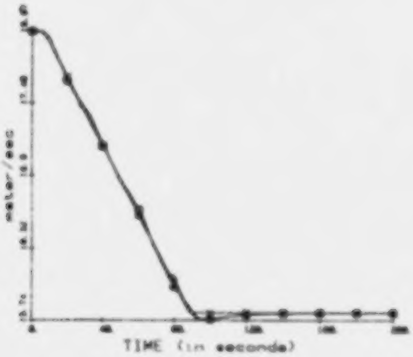


FIG (A): VELOCITY

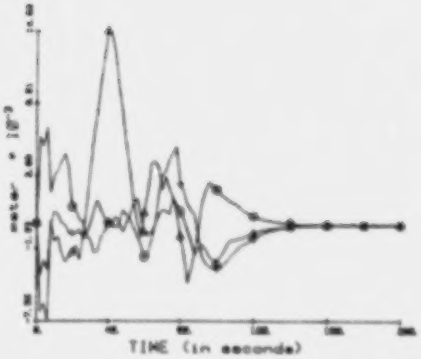


FIG (B): COUPLING DISPLACEMENT

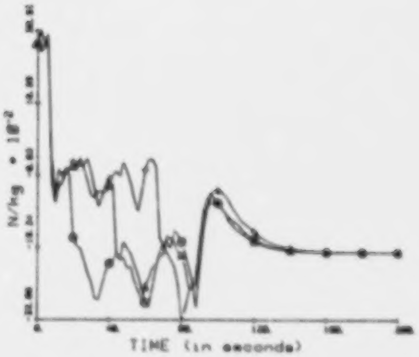


FIG (C): THROTTLING FORCE

B.18 Downhill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP4 Throttle Control (Pole=0.1)

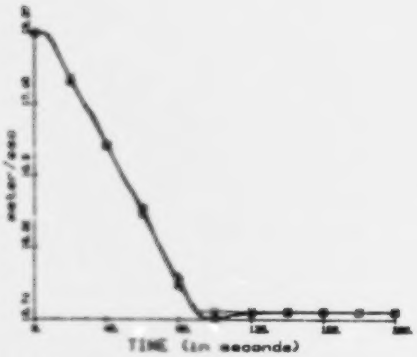


FIG (A). VELOCITY

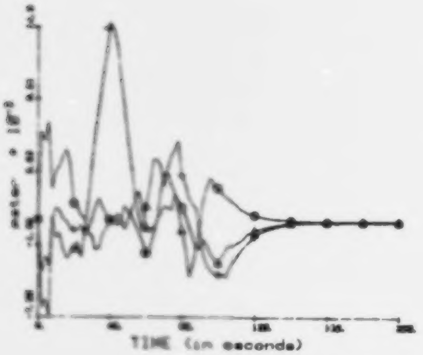


FIG (B). COUPLING DISPLACEMENT

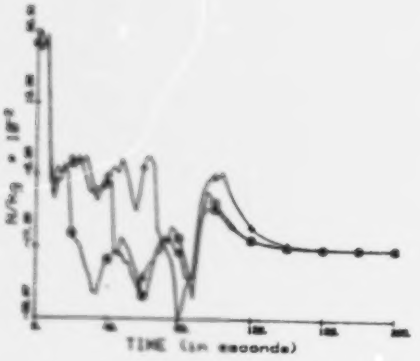


FIG (C). THROTTLING FORCE

B.19 Downhill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP4 Throttle Control (Pole=0.2,0.3)

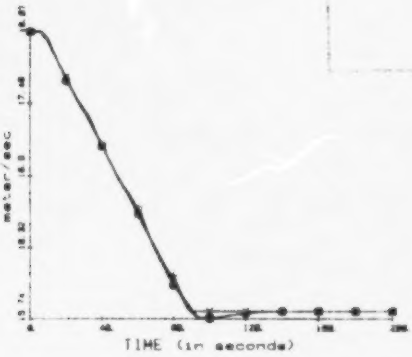


FIG (A): VELOCITY

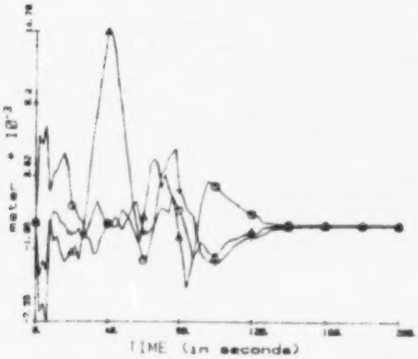


FIG (B): COUPLING DISPLACEMENT

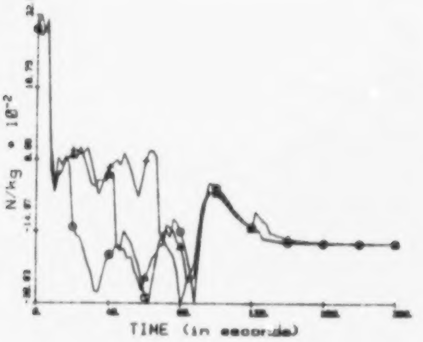


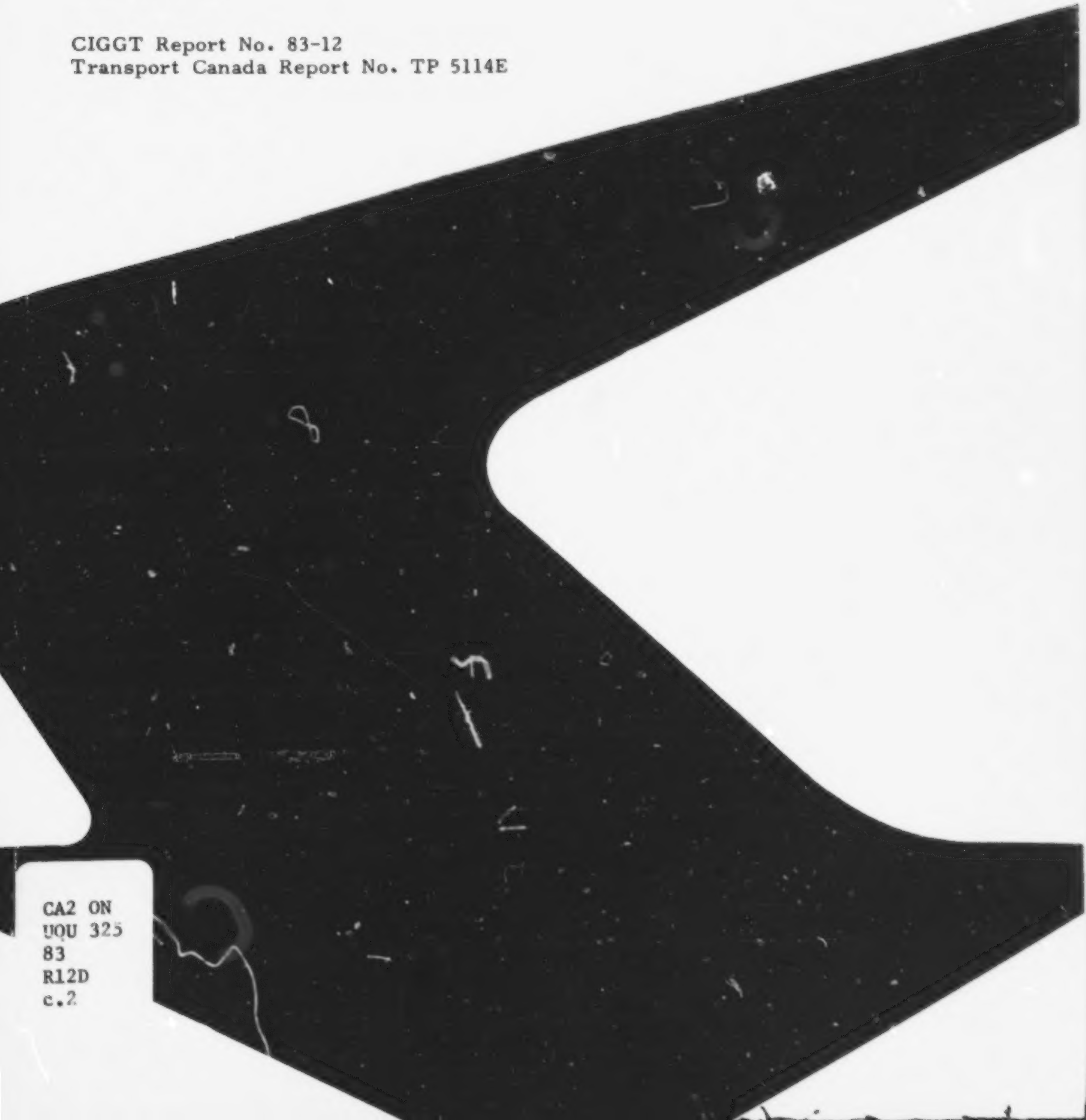
FIG (C): THROTTLING FORCE

B.20 Downhill Simulation of a Sixty-Three-Car Train Using 3 SISO-STPP4 Throttle Control (Pole=0.5,0.5)

Canadian Institute of
Guided Ground Transport
Queen's University
at Kingston, Ontario

K.Y. Wong
M.M. Bayoumi

CIGGT Report No. 83-12
Transport Canada Report No. TP 5114E



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EVALUATION OF DESIGN ALTERNATIVES FOR A FREIGHT TRAIN CONTROLLER

Appendix D

MINIMUM ENERGY CONSUMPTION CONTROL OF PASSENGER/FREIGHT TRAINS WITH FIXED TERMINAL TIME

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Department of Electrical Engineering
Queen's University

prepared for

Transportation Development Centre, Transport Canada

Canadian Institute of Guided Ground Transport
Queen's University at Kingston, Ontario
K7L 3N6

December 1983

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15. Supplementary Notes
Appendix A, "Automatic Control of Freight Trains," was originally a milestone report (TP 4476E, CIGGT 82-19) dated March 1983.

16. TDC Project Officer
W.G. Rowan

17. Abstract

This project on development and optimization of control schemes for freight trains was motivated by the need to find optimal or suboptimal closed-loop control schemes for long freight trains. Several on-line and off-line freight train control strategies have been studied and reported. The relation between these control strategies is discussed in this report. The structure of an overall train operation scheme which consists of an off-line energy saving strategy and an on-line control strategy to maintain the velocity profile specified by the off-line scheme is proposed. There are five separately-bound appendices: Appendix A: AUTOMATIC CONTROL OF FREIGHT TRAINS; Appendix B: A REDUCED-ORDER MODEL FOR LONG FREIGHT TRAINS; Appendix C: A COMPARATIVE STUDY OF SELF-TUNING CONTROLLERS FOR FREIGHT TRAINS; Appendix D: MINIMUM ENERGY CONSUMPTION CONTROL OF PASSENGER/FREIGHT TRAINS WITH FIXED TERMINAL TIME; Appendix E: OPTIMAL CONTROL STRATEGY FOR MINIMUM ENERGY AND MINIMUM FUEL CONSUMPTION OF FREIGHT TRAINS

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Long trains, automatic controller, braking, simulation program

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17. Résumé

L'étude dont ce rapport rend compte a trait aux programmes visant l'automatisation et l'optimalisation de la conduite des longs convois de marchandises, étude motivée par le besoin pressant de trouver des techniques en boucle fermée qui soient optimales, ou tout proche. Le rapport présente un certain nombre de programmes de marche prédétermines et de programmes d'optimalisation de la conduite; de plus, il traite de l'interaction entre eux. Il préconise un programme du premier type axé sur l'économie d'énergie, secondé par un programme du deuxième type assurant à la fois le respect des consignes imposées et une consommation minimale d'énergie. Le rapport contient cinq annexes distinctes, intitulées comme suit: A: Automatisation de la conduite des convois de marchandises; B: Logiciel en réduction approprié aux longs convois de marchandises; C: Etude comparative des logiciels tenant compte des variations dans les paramètres de conduite; D: Consommation minimale des trains de voyageurs et de marchandises à horaires fixes; E: Stratégie d'optimalisation de la consommation d'énergie et de carburant dans le cas des trains de marchandises.

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1. INTRODUCTION

Reducing energy consumption is a major objective of those responsible for train operations. Energy consumption can be considerably decreased by using the proper control strategy. Many operating controllers are, however, not designed from a minimum energy viewpoint.[1-5,7-9] The control objective has often been to minimize the velocity variation at a given velocity profile. The problem of choosing the velocity profile with fixed terminal time that would be most beneficial in terms of reducing energy consumption is investigated in this report. The optimal design approach which explicitly incorporates the minimum energy objective is presented. Some computation techniques for obtaining useful numerical results are also discussed. The optimal (ME) control strategy is compared with two suboptimal strategies which are comprised of the basic optimal control schemes, and is applied to simulated examples of a passenger train travelling from Kingston to Toronto and a long freight train travelling from Calgary to Field.

2. TRAIN MODEL

2.1 Non-Linear Model

The system under investigation is a multi-locomotive-powered train consisting of locomotives and gondola cars connected by couplers. The aim is to move the train from a source to a destination over a section of track in a pre-specified time. The important elements of this system are:

(1) Track information: distance and conditions (rolling resistance between wheel and rail, gradient and curvature).

(2) Train data: characteristics of locomotives and cars (mass and capacity), throttle and brake characteristics, and inter-car coupling and draft-gear characteristics.

(3) Train schedule: scheduled stop points, corresponding terminal times and possible stop points.

It is well known that the resistance of a moving vehicle may be estimated by a quadratic function of the speed of the vehicle, with constant coefficients R_0, R_1 and R_2:

$$R = R_0 + R_1 v + R_2 v^2$$

where v is the speed of the vehicle.

Typical resistance curves for the train members are shown in Figure 1.

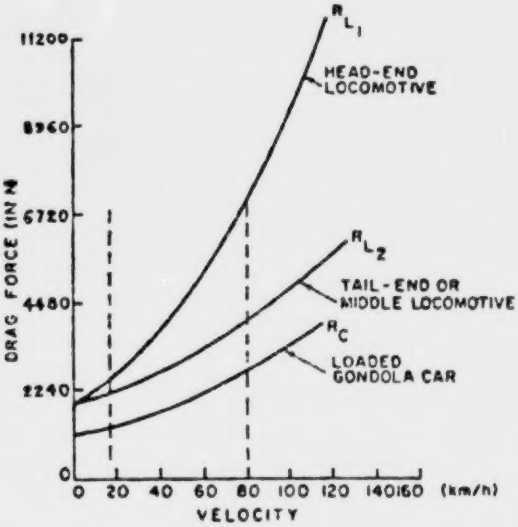

Figure 1 Aerodynamic Drag Forces and Rolling Resistance Curves

The throttle and dynamic brake characteristics are characterized by the following contraints:

Throttle limits:

$f \leq \mu W_L$ (adhesion limit) (1)

$fv \leq \eta P\max$ (motor power limit) (2)

Dynamic brake limits:

$$f \geq -\xi v \quad \text{(motor field current limit)} \tag{3}$$

$$f \geq -Z \quad \text{(extended range dynamic brake limit)} \tag{4}$$

$$fv \geq -\eta' P_{max} \quad \text{(motor heating limit)} \tag{5}$$

where: f = throttling/dynamic braking force of a locomotive

μ = tractive adhesion coefficient, $0.11 \leq \mu \leq 0.35$

W_L = weight of the locomotive

v = velocity of the train

η = transmission efficiency of the electro-mechanical drive system

P_{max} = maximum power rating of locomotive

ξ = low-speed dynamic brake build-up rate

Z = maximum extended range dynamic brake power

η' = locomotive braking efficiency

Typical tractive force constraints of a locomotive are shown in Figure 2. The air pressure brakes on each car are governed by the constraint:

$$0 \leq b \leq AB_{max} \quad \text{(air brake limit)} \tag{6}$$

where: b = braking force due to air pressure brakes of a car

AB_{max} = maximum air pressure braking power of a car.

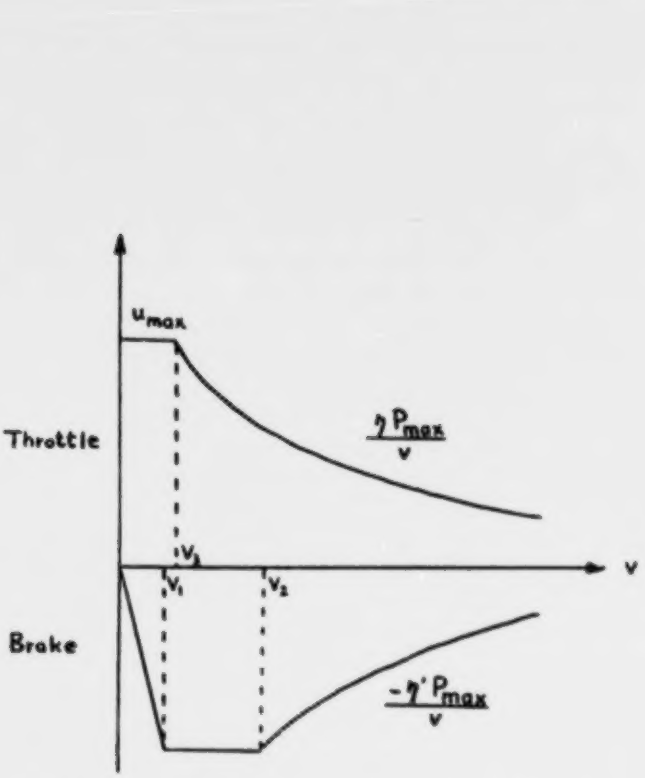

Figure 2 Typical Tractive Force Constraints

2.2 Simplified Model

If we consider a train as a lumped unit mass located at its centre of gravity and neglect the interaction between train members, the simplified train model can be described by:

$$\dot{\rho}(t) = v(t) \tag{7}$$

$$\dot{v}(t) = u(t) - r(t) - g(t) \tag{8}$$

where: ρ = distance travelled by the train

v = velocity of the train

u = total input force (throttles and brakes) applied per unit mass of the train

r = total non-linear rolling and wind resistance force per unit mass of the train

g = total inclinational resistance force per unit mass of train $g > 0$ per uphill, $g < 0$ for downhill train movement.

The corresponding throttle and brake constraints of the unit train can be expressed as:

$$u_{min}(v) \leq u(t) \leq u_{max}(v) \tag{9}$$

where:

$$u_{min}(v) = \begin{cases} -(n_L \xi v + n_B b_{max})/m & \text{if } 0 < v \leq V_1 \\ -(n_L Z + n_B b_{max})/m & \text{if } V_1 \leq v \leq V_2 \\ -(\eta' n_L p_{max}/v + n_B b_{max})/m & \text{if } v \geq V_2 \end{cases} \tag{10}$$

$$u_{max}(v) = \begin{cases} \eta n_L W_L/m & \text{if } 0 \leq v \leq V_3 \\ \eta n_L p_{max}/m v & \text{if } v \geq V_3 \end{cases} \tag{11}$$

n_L = number of locomotives in the train

n_B = number of air pressure brakes in the train

m = total mass of the train.

$$V_1 = X/\xi \tag{12}$$

$$V_2 = \eta' p_{max}/Z \tag{13}$$

$$V_3 = \eta p_{max}/W_L \tag{14}$$

3. MINIMUM ENERGY CONSUMPTION (ME) CONTROL

3.1 Minimum Energy Consumption Control Criterion

The objective of the minimum energy (ME) control problem is to minimize the total energy consumption criterion given by:

$$J_0 = \frac{1}{\eta} \int_0^T u^+(t)\, v(t)\, dt \tag{15}$$

subject to the train dynamic equations (equations (7) and (8)), the constraints equation (equation (9)) and the boundary conditions:

$$s(0) = 0 \qquad v(0) = 0 \tag{16}$$

$$s(T) = S \qquad v(T) = 0 \tag{17a}$$

where: S = distance between source and destination

T = pre-specified terminal time fixed by the schedule

u^+ = total positive input (throttling) force applied in per-unit mass of the train

Note that the total fuel consumption of the train is given by:

$$J = \int_0^T C_1 \left(\frac{u^+(t)\ v(t)}{\eta} + Q\right) dt = C_1 \ (J_0 + QT) \tag{17b}$$

where C_1 and Q are constants. Typical values of C_1 and Q are C_1 = 0.078 litres/MJ, Q = 0.081 MW.

Thus, the total fuel consumption of a train can be obtained from the linear relation (equation 17b) above for the fixed pre-specified time, T.

3.2 Optimal Solution

The optimal solution can be obtained according to the Pontrjagin's maximum principle by maximizing the following Hamiltonian function with respect to U:

$$H\ (u,\ s,\ v,\ \lambda_1,\ \lambda_2) = -\ u^+\ v + \lambda_1\ v + \lambda_2\ (u-r-g) \tag{18}$$

subject to:

$$\dot{s} = v \qquad \text{(state equations)} \tag{19}$$

$$\dot{v} = u - r - g \tag{20}$$

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial s} = 0 \qquad \text{(adjoint equation)} \tag{21}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial v} = u^+ - \lambda_1 + (r_1 + 2\ r_2\ v)\ \lambda_2 \tag{22}$$

$$s(0) = 0 \qquad s(T) = S \qquad \text{(boundary conditions)} \tag{23}$$

$$v(0) = 0 \qquad v(T) = 0 \tag{24}$$

$$u_{min}(v) \leq u \leq u_{max}(r) \qquad \text{(constraints)} \tag{25}$$

where $u_{min}(v)$ and $u_{max}(v)$ are given in equations (10) and (11).

The Hamiltonian takes the form:

$$H\ (u,\ s,\ v,\ \lambda_1,\ \lambda_2) = {}_2\ u - u^+ v + \text{tens in } s,\ v,\ \lambda_1 \text{ and } \lambda_2 \tag{26}$$

Thus, maximization of Hamiltonians of this form in the presence of amplitude constraints leads to the following expression for the optimal controller:

$$u^*(t) = \begin{cases} u_{min}(v) & \text{if } \lambda_2(t) < 0 \\ \text{undefined (any brake)} & \text{if } \lambda_2(t) = 0 \\ 0 & \text{if } 0 < \lambda_2(t) < v(t) \\ \text{undefined (any throttle)} & \text{if } \lambda_2(t) = v(t) \\ u_{max}(v) & \text{if } \lambda_2(t) > v(t) \end{cases} \tag{27}$$

The solution to the problem would be complete if we knew either the initial or the terminal values of λ_1 and λ_2, and hence λ_1, λ_2 can be obtained from equations (21) and (22) with the optimal controller in equation (27). Unfortunately, we do not know the initial nor the terminal values of λ_1 and λ_2. We are, therefore, reduced to the use of common sense and guesswork in choosing the appropriate values such that the controller (equation 27) generates a state trajectory originating and terminating at the required states at the required time.

The simplest approach to the problem is to use a physical argument to obtain a meaningful initial value of λ_2. Suppose a train starts from rest on level ground or on an uphill section. The only way to move the train forward is to apply a positive thrust to the train; hence, without loss of generality, we can assume that $\lambda_2(0) > v(0) = 0$, so that maximum thrust is applied at the beginning of the run. On the other hand, in order to stop the train on level ground or a downhill section, one must apply the brakes or let the train coast to a stop.

Therefore, we can assumed that $\lambda_2(T) < v(T) = 0$. For simplicity, consider a train travelling on level ground so that both end conditions hold. Due to the continuity of trajectories v and λ_2, there exists a time instant, t_1, between 0 and T such that $\lambda_2(t_1) = v(t_1) = v_1$. Note that λ_2 and v could remain equal to the same constant, v_1, for a period of time since the optimal control is undefined when $\lambda_2(t) = v(t)$. Suppose $\lambda_2(t) \equiv v(t)$ for $t_1 \leq t \leq t_2$. This could only happen if, and only if, $u^*(t) = r_0 + r_1 v_1 + r_2 v_1^2$ for $t_1 \leq t \leq t_2$ and $\lambda_1 = r_0 + 2 r_1 v_1 + 3 r_2 v_1^2$. It can be shown easily that v_1 is the only non-negative value and that λ_2 and v remain constant. This means that the time interval $[t_1, t_2]$ is the only period in which the train is travelling at a constant speed on level ground. Thus, the only possible optimal control strategy is composed of the following four references:

(i) driving with maximum force, $0 \leq t \leq t_1$

(ii) travelling with constant speed, $t_1 \leq t \leq t_2$

(iii) coasting, $t_2 \leq t \leq t_3$

(iv) braking with maximum force, $t_3 \leq t \leq T$

and the control law is given by:

$$u^*(t) = \begin{cases} u_{max}(v) & 0 \leq t < t_1 & \lambda_2(t) > v(t) \\ r_0 + r_1 v_1 + r_2 v_1^2 & t_1 \leq t \leq t_2 & \lambda_2(t) = v(t) = v_1 \\ 0 & t_2 \leq t < t_3 & 0 < \lambda_2(t) < v(t) \\ u_{min}(v) & t_3 < t \leq T & \lambda_2(t) < 0 \end{cases} \tag{28}$$

Hence, the corresponding minimum energy consumption is:

$$J_0^* = E_1 + E_2 \tag{29}$$

where: $$E_2 = \frac{1}{\eta} \int_0^{t_1} u_{max}(v)\, v(t)\, dt \tag{30}$$

$$E_2 = \frac{1}{\eta} (r_0 + r_1 v_1 + r_2 v_1^2)\, v_1\, (t_2 - t_1) \tag{31}$$

The optimal control problem is now reduced to the problem of choosing the nominal speed, v_1, and the switching times, t_1, t_2 amd t_3, such that the state trajectory satisfies the boundary conditions. Note that $\lambda_2(t_3) = 0$ gives the condition of obtaining the switching time, t_3, which is the moment of changing from the coasting scheme to the maximum braking scheme. The optimal trajectories v_1, 2, u^* and J_0^* are sketched in Figure 3. Note that the scheme shown in Figure 3 is the only possible candidate for the optimal control.

3.3 Switching Strategy

In order to find the optimal method of switching from one control scheme to another, all we need to do is calculate suitable values for the switching times t_1, t_2 and t_3.

Let $T_1 = t_1$, $T_2 = t_2 - t_1$, $T_3 = t_3 - t_2$ and $T4 = T - t_3$ be the time periods in which the train is: travelling with maximum thrust, at constant speed, coasting, and with maximum brake applied, respectively, and let S_1, S_2, S_3 and S_4 be the distance travelled using each of the schemes. These time periods and distances can be obtained by integrating equations (7) and (8) and by the following methods of changing vairables:

$$T_1 = \int_0^{v_1} \frac{dv}{u_{max}(v) - r_0 - r_1 v - r_2 v^2} \qquad (32)$$

$$S_1 = \int_0^{v_1} \frac{v\,dv}{u_{max}(v) - r_0 - r_1 v - v_2 v^2} \qquad (33)$$

$$T_3 = \int_{v_2}^{v_1} \frac{dv}{r_0 + r_1 v + r_2 v^2} \qquad (34)$$

$$S_3 = \int_{v_2}^{v_1} \frac{v\,dv}{r_0 + r_1 v + r_2 v^2} \qquad (35)$$

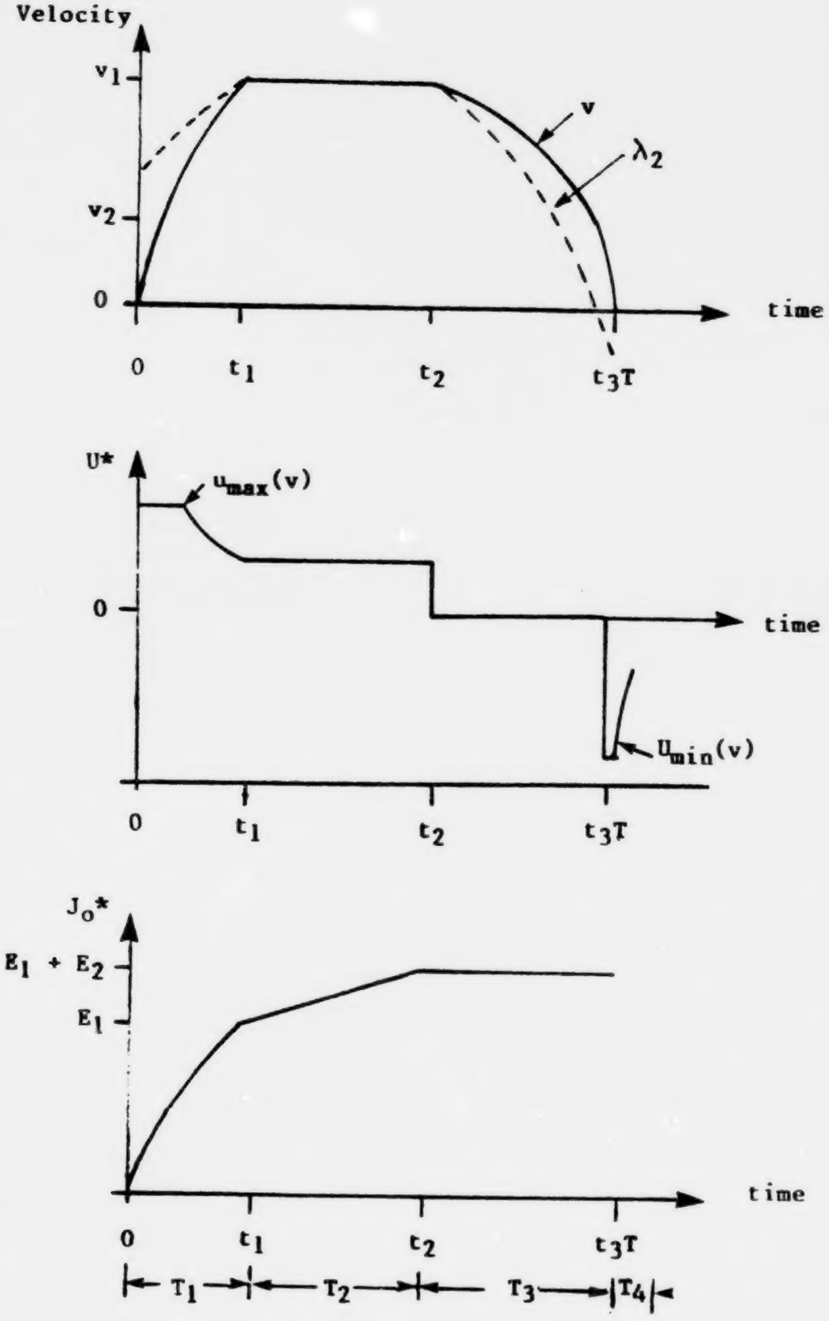

Figure 3 Optimal Trajectories

$$T_4 = \int_0^{v_2} \frac{dv}{- u_{min}(v) + r_0 + r_1 v + r_2 v^2} \tag{36}$$

$$S_4 = \int_0^{v_2} \frac{v\ d\ v}{- u_{min}(v) + r_0 + r_1 v + v_2 v^2} \tag{37}$$

$$T_2 = T - T_1 - T_3 - T_4 \tag{38}$$

$$S_2 = v_1\ T_2 = v_1\ (T - T_1 - T_3 - T_4) \tag{39}$$

The velocity, v_2, can be obtained as a function of v_1 by utilizing the conditon $\lambda_2(t_3) = 0$ and solving the non-linear equation:

$$\begin{aligned} T_3 &= \int_0^{v_1} \frac{d\ \lambda_2}{r_0 + 2\ r_1 v_1 + 3\ r_2 v_1^2 - (r_1 + 2r_2\ \bar{v})\ {}_2} \\ &= \int_{v_2}^{v_1} \frac{dv}{r_0\ r_1\ v_1\ v_2\ v^2} \end{aligned} \tag{40}$$

where $\bar{v} = v_1/2$.

The introducton of the approximation $\bar{v}$ simply reduces the complexity of solving a system of non-linear equations. If we neglect the terms with r_2, equation (40) becomes:

$$T_3 = \frac{1}{r_1} \log \left| \frac{r_0 + 2r_1\ v_1}{r_0 + r_1 v_1} \right| = \frac{1}{r_1} \log \left| \frac{r_1 v_1 + r_0}{r_1 v_2 + r_0} \right| \tag{41}$$

which can be easily solved to obtain:

$$v_2 = \frac{r_1\ v_1^2}{r_0 + 2r_1 v_1} \tag{42}$$

Equation (42) can be used as an approximation of v_2 if $r_2 \neq 0$. The optimal switching times and the optimal nominal velocity, v_1, can then be obtained by solving the non-linear equation of v_1 given by:

$$S_1 + S_2 + S_3 + S_4 = S \tag{43}$$

where S_1, S_2, S_3, S_4, T_1, T_2, T_3, T_4 and v_2 are expressed in equations (32) through (39) and (42).

Note that T_1, S_1, T_3, S_3, T_4, S_4 and v_2 are all positive for positive value of v_1. A necessary conditon for the existence of the solution is that $T_2 = T - T_1 - T_3 - T_4 \geq 0$. If $T_2 < 0$, the time schedule is too tight, and the trains cannot reach their destinations in the pre-specified time. The shortest time to reach the destination with the optimal (ME) control is $T_1 + T_3 + T_4$.

3.4 Overall Control Scheme

The optimal control schedule can be set-up using the following steps (track information, train data and train schedules are given):

Step 1: obtain the solution v_1 of the non-linear equation (equation 43)

Step 2: compute v_2, S_1, S_2, S_3, S_4 and T_1, T_2, T_3, T_4. If T_2 0, change the train schedule and go to Step 1.

Step 3: Compute the switching times, $t_1 = T_1$ $t_2 = T_1 + T_2$, $t_3 = T_1 + T_2 + T_3$ (note that $T = T_1 + T_2 + T_3 + T_4$).

Step 4: Apply the optimal control law (equation 28).

A train handled with this optimal (ME) control will have the most effective energy saving. The corresponding minimum energy consumption is $J_0{}^* = E_1 + E_2$ where:

$$E_1 = \frac{1}{\eta} \int_0^{v} \frac{u_{max}(v)\ v\ dv}{u_{max}(v) - r_0 - r_1 v - r_2 v^2} \tag{44}$$

$$E_2 = \frac{1}{\eta} (r_0 + r_1 v_1 + r_2 v_1{}^2)\ S^2 \tag{45}$$

4. NUMERICAL CONSIDERATIONS

4.1 Maximum Speed Computations

Before we try to solve the non-linar equation (equation 43) of v_1, we need to find the range of the variable v_1. The range of v_1 is given by:

$$0 \leq v_1 \leq v_{max} \tag{46}$$

where v_{max} is the maximum velocity of the train running on a given section.

The maximum velocity can be attained when a train is travelling with maximum thrust at the steady state. Hence, for a train running on level ground, we have the following condition at the steady state:

$$u_{max}(v_{max}) = \eta^{n}_{L} \; p_{max}/m \; v_{max} = r_0 + r_1 \; v_{max} + r_2 \; v_{max}^{2} \tag{47}$$

i.e. v_{max} is the positive real root of the cubic equation

$$- \eta^{n}_{L} \; p_{max}/m + r_0 v + v_1 \; v^2 + v_2 \; v^3 = 0 \tag{48}$$

The Newton-Raphson iteration method is used to obtain the roots of a real polynomial numerically. Given a polynomial:

$$f(z) = a_0 + a_1 \; z + \ldots + a_n \; z^n \tag{49}$$

let $z = x + j\, y$ be a starting value for a root of $f(z)$, and let

$$z^n = x_n + j\, y_n = (x + j\, y)^n \tag{50}$$

then:

$$x_0 = 1, \; y_o = 0$$

$$x_n = x \; x_{n-1} - y \; y_{n-1} \tag{51}$$

$$y_n = x \; y_{n-1} + y \; x_{n-1} \tag{52}$$

Let U be the real term of f(z) and V be the imaginary term of f(z), then:

$$U = a_0 + a_1\, x_1 + \ldots + a_n\, x_n \tag{53}$$

$$V = a_1,\ y_1 + \ldots + a_n\, y_n \tag{54}$$

$$\frac{\partial V}{\partial x} = \sum_{k=1}^{n} k\, a_k\, x_{k-1} \tag{55}$$

$$\frac{\partial V}{\partial y} = - \sum_{k-1}^{n} k\, a_k\, y_{k-1} \tag{56}$$

Applying the Canchy-Riemann equations, the next iteration using the Newton-Raphson method is:

$$x' = x + \Delta x, \quad y' = y + \Delta_j \tag{57}$$

where:

$$\Delta x = \left(v \frac{\partial U}{\partial y} - U \frac{\partial U}{\partial x}\right) \Big/ \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2 \right] \tag{58}$$

$$\Delta y = \left(U \frac{\partial U}{\partial y} + V \frac{\partial U}{\partial x}\right) \Big/ \left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2 \right] \tag{59}$$

Whenever a root is obtained to a desired accuracy, it is eliminated from the polynomial and the degree of the polynomial is reduced. The final iterations on each root are performed using the original polynomial than the reduced polynomial to avoid accumulated errors in the reduced polynomial.

4.2 Switching Time and Distance Computations

Numerical integration is required to compute the times and distances T_1, S_1, T_3, S_3, T_4 and S_4 in equations (32) to (37). Note that:

$$T_1 = T_{1a} + T_{1b} \tag{60}$$

where: $$T_{1a} = \int_0^{v_{1a}} \frac{d\ v}{u_{max} - r_0 - r_1 v - r_2 v^2} \tag{61}$$

$$v_{1a} = \min\ (v_1,\ V_2);\quad V_3 = \mu P_{max}/\mu W_L \tag{62}$$

$$U_{max} = \mu\, n_L\ W_L/m \tag{63}$$

$$T_{1b} = \int_{v_{1a}}^{v_1} \frac{dv}{u_{max}\ V_3/v - r_0 - r_1 V - r_2 V^2} \tag{64}$$

$$S_1 = S_{1a} + S_{1b} \tag{65}$$

where: $$S_{1a} = \int_0^{v_{1a}} \frac{v\ d\ v}{u_{max} - r_0 - r_1 v - r_2 v^2} \tag{66}$$

$$S_{1b} = \int_{v_{1a}}^{v_1} \frac{v\ d\ v}{u_{max}\ V_3/v - r_0 - r_1 v - r_2 v^2} \tag{67}$$

$$T_4 = T_{4a} + T_{4b} + T_{4c} \tag{68}$$

where: $$T_{4a} = \int_0^{v_{4a}} \frac{d\ v}{(n_B\ b_{max} + n_L\ \xi\ v)/m + r_0 + r_1 v + r_2 v^2} \tag{69}$$

$$v_{4a} = \min\ (v_2,\ V_1);\quad V_1 = z/\xi \tag{70}$$

$$T_{4b} = \int_{v_{4a}}^{v_{4b}} \frac{dv}{(n_B\ b_{max} + n_L\ Z)/m + r_0 + r_1 v + R_2 v^2} \tag{71}$$

$$v_{4b} = \min (v_{4a}, V_2) \tag{72}$$

$$T_{4c} = \int_{v_{4b}}^{v_2} \frac{d\,v}{(\,'\, n_L\, p_{max}/v + n_B\, b_{max})/m + r_0 + r_1 v + r_2 v^2} \tag{73}$$

$$S_4 = S_{4a} + S_{4b} + S_{4c} \tag{74}$$

where: $$S_{4a} = \int_0^{v_{4a}} \frac{v\,d\,v}{(n_B\, b_{max} + n_L\, \xi\, v)/m + r_0 + r_1 v + r_2 v^2} \tag{75}$$

$$S_{4b} = \int \frac{v\,d\,v}{(n_B\, b_{max} + n_L\, Z)/m + r_0 + r_1 v + r_2 v^2} \tag{76}$$

$$S_{4c} = \int \frac{v\,d\,v}{(\eta'\, n_L\, p_{max}/v + n_B\, b_{max})/m + r_0 + r_1 v + r_2 v^2} \tag{77}$$

The computations of T's and S's involve the integration of a rational function of up to order 3 in the denominator. Exact solutions for integrating rational functions of up to order 2 in the denominator can be obtained. They are given by the formula ($q = 4_{ac} - b^2$):

$$\int_{x_0}^{x_1} \frac{dx}{a + bx + cx^2} = \begin{cases} \frac{1}{\sqrt{-q}} \log \left| \frac{2cx + b = \sqrt{-q}}{2cx + b + \sqrt{-q}} \right|_{x_0}^{x_1} & \text{if } q<0,\ c\neq 0 \\ \frac{2}{\sqrt{-q}} \tan^{-1} \left(\frac{2cx + b}{\sqrt{q}}\right) \Big|_{x_0}^{x_1} & \text{if } q>0,\ c\neq 0 \\ \frac{1}{b} \log \left| \frac{a + bx_1}{a + bx_0} \right| & \text{if } b\neq 0,\ c=0 \end{cases} \tag{78}$$

$$\int_{x_0}^{x_1} \frac{x\,d\,x}{a + bx + cx^2} = \begin{cases} \frac{1}{2C} \log \left| \frac{a + bx_1 + cx_1^2}{a + bx_0 + cx_0^2} \right| \frac{b}{2c} \int_{x_0}^{x_1} \frac{dx}{a + bx + cx^2} & \text{if } c \neq 0 \\ \frac{1}{b^2} \left[b\,(x_1 - x_0) - a \log \left| \frac{a + bx_1}{a + bx_0} \right| \right] & \text{if } c = 0 \end{cases} \tag{79}$$

For integrating a rational function with a degree of denominator greater than two, the integration should be computed by numerical methods. One approach is the reduction of the rational function by partial fractions and with each term integrated using equations (78) and 79). However, this approach requires the computation of the roots of the denominator. Another approach is to use a numerical integration method, e.g. the trapezoidal rule, the Simpson rule, the Romberg integration, or the Gauss Quadrature formulas. The Gauss Quadratic method is chosen for the numerical integration since it requires a minimum of functional evaluations and calculations for attaining maximum accuracy. For a detailed discussion of the method, see Krylov.[6]

4.3 Nominal Speed Computation

In order to obtain the nominal speed, v_1, in the schedule, we need to solve a non-linear equation (equation 43) in the range $0 \leq v_1 \leq v_{max}$. If the nomial speed is found, then all other important times and distances, T_1, T_2, T_3, T_4, S_1, S_2, S_3, S_4, can be found easily by evaluation. The solution of a non-linear equation, $f(x) = 0$, can be done by Meuller's iteration method of successive bisections and inverse parabolic interpolation which starts at the initial bounds 0 and v_{max}. The method is as follows.

Starting with $x_1 = 0$, $x_2 = v_{max}$, assume that $f(x_1)\ f(x_2) < 0$, the midpoint of the interval $[x_1, x_2]$, is computed:

$$x_n = \frac{1}{2} (x_1 + x_2) \tag{80}$$

Let y_1, y_2, y_m be the functional values of x_1, x_2, x_m respectively.

If $y_m \, y_2 < 0$, x_1 and x_2 are interchanged to ensure that $y_m \, y_2 > 0$. Furthermore, if x_1, x_m and x_2 satisfy the condition:

$$\Delta = 2 \, y_m \, (y_m - y_1) - y_2 \, (y_2 - y_1) \geq 0 \tag{81}$$

x_2 is replaced by x_m and the bisection step is repeated. If the inequality (equation 81) is not satisfied, then the inverse parabolic interpolation is performed with:

$$x = x_1 + \frac{(y-y_1)\,(x_m-x_1)\,[\,(2\,y_m-y_2-y_1)\,y - \Delta\,]}{(y_m-y_1)\,(y_2-y_m)\,(y_2-y_1)} \tag{82}$$

and

$$x_0 = x_1 + \frac{y_1\,(x_m-x_1)\,\Delta}{(y_m-y_1)\,(y_2-y_m)\,(y_2-y_1)} \tag{83}$$

is chosen to be used for the next iteration. Note that x_0 is sure to be situated between x_1 and x_m. For the next iteration step, if $f(x_0) \, y_1 > 0$, then x_0 becomes x_1 and x_m becomes x_2, or if $f(x_0) \, y_1 < 0$, x_0 becomes x_2.

Convergence is either quadratic or linear if the multiplicity of the root to be determined is equal to one or greater than one, respectively, and if $f(x)$ canb e differentiated continuously at least twice in the range x_1 and x_2. Each iteration step requires two evaluations of $f(x)$.

The procedure is terminated when a given accuracy is satisfied.

5. SUBOPTIMAL CONTROL

Since the optimal control strategy is composed of four schemes: driving with maximum thrust, travelling with constant speed, coasting, and braking with maximum force, it is natural to consider suboptimal control strategies which are only composed of some of these schemes. The suboptimal control strategies considered in this section consist of three of the above schemes and is described below.

5.1 Maximum Thrust and Brake

This suboptimal strategy involves driving with maximum thrust at the beginning, then travelling at constant speed, and finally braking with maximum force. The difference between this and the optimal strategy is that this one leaves out the coasting scheme. Hence, $v_2 = v_1$ is used instead of equation (42), and, therefore, $T_3 = S_3 = 0$ in equations (34) and (35). The nominal speed, v_1, in this case is again solved by finding the positive real root of the non-linear equation (equation 43). Note that the nominal speed, v_1, for the suboptimal strategy should be smaller than the nominal speed for the optimal strategy if the time schedule is the same, because braking alone is used to stop the train rather than a combination of coasting and braking.

An interesting fact about this suboptimal strategy is that it involves an time optimal controller in which $T_2 = S_2 = 0$, i.e. the control strategy is a bang-bang control with maximum thrust and brake only. Since using the time optimal controller is the only feasible way to reach the destination in minimum time, it is also the optimal (ME) controller for the minimum time schedule.

5.2 Maximum Thrust and Coasting

This suboptimal strategy involves driving with maximum thrust at the beginning, then trvelling with constant speed, and finally coasting to rest. The velocity $v_2 = 0$ will be used instead of equation (42) so that $T_4 = S_4 = 0$ in equations (36) and (37). The nominal speed, v_1, would be obtained in the same way as before by solving the non-linear equation (equation 43) in the range 0 to v_{max}. The nominal speed, v_1, obtained for this suboptimal strategy should be larger than the one for the optimal strategy with a fixed time schedule.

6. OPTIMALITY OF THE ME CONTROL SCHEME

6.1 Comparison of Optimal (ME) and Suboptimal Controls

The optimal (ME) control strategy is introduced to reduce the energy comsumption of a train to a minimum when the train is travelling on a grade section with a given time schedule. A natural question is to ask how the optimal (ME) control strategy compares with the suboptimal control strategies mentioned in the previous section. This can be done with the help of a computer. A program package, MECFTT, has been created to obtain the optimal (ME) and suboptimal control schemes for given time schedules. The MECPTT program is listed in Annex A.

6.2 Illustrative Examples

A train composed of one locomotive and fifteen cars with total mass 2 M kg was simulated along a distance of 10 km over level ground. The track data are given by:

$$
\begin{aligned}
r_0 &= 0.012 \text{ N/kg} \\
r_1 &= 0.0005 \text{ Ns/m/kg} \\
r_2 &= 43.5\ \mu \text{ Ns}^2/\text{m}^2/\text{kg}
\end{aligned}
$$

and the train power and brake characteristics are given by:

$$
\begin{aligned}
\mu W_L &= 0.3470 \text{ MN} \\
P_{max} &= 2.2065 \text{ MW} \\
\eta &= 0.82 \\
B_{max} &= 0.3 \text{ NW for each air brake on a car} \\
\xi &= 0.0578 \text{ MNs/m} \\
Z &= 0.267 \text{ MN} \\
\eta' &= 1.2 \\
v_{max} &= 21.32 \text{ m/sec.}
\end{aligned}
$$

Various time schedules with ten minute increments were simulated for the range between ten minutes and one hour. It was found that the minimum times required for the optimal, maximum thrust/brake/maximum thrust, and throttle strategies were 14.51, 10.52 and 20.08 minutes, respectively. Hence, the control strategies were also compared for the schedules of 14.51 and 20.08 minutes. The simulation results are listed in Annex B. Corresponding energy consumption values are listed in Table 1 and plotted in Figure 4, where it is shown that when the time schedule is shorter, energy consumption is larger. Note that the suboptimal strategies tend to give energy consumption values close to the optimal value as the time schedule increases. This is because the constant speed section dominates the beginning and end sections in the simulation.

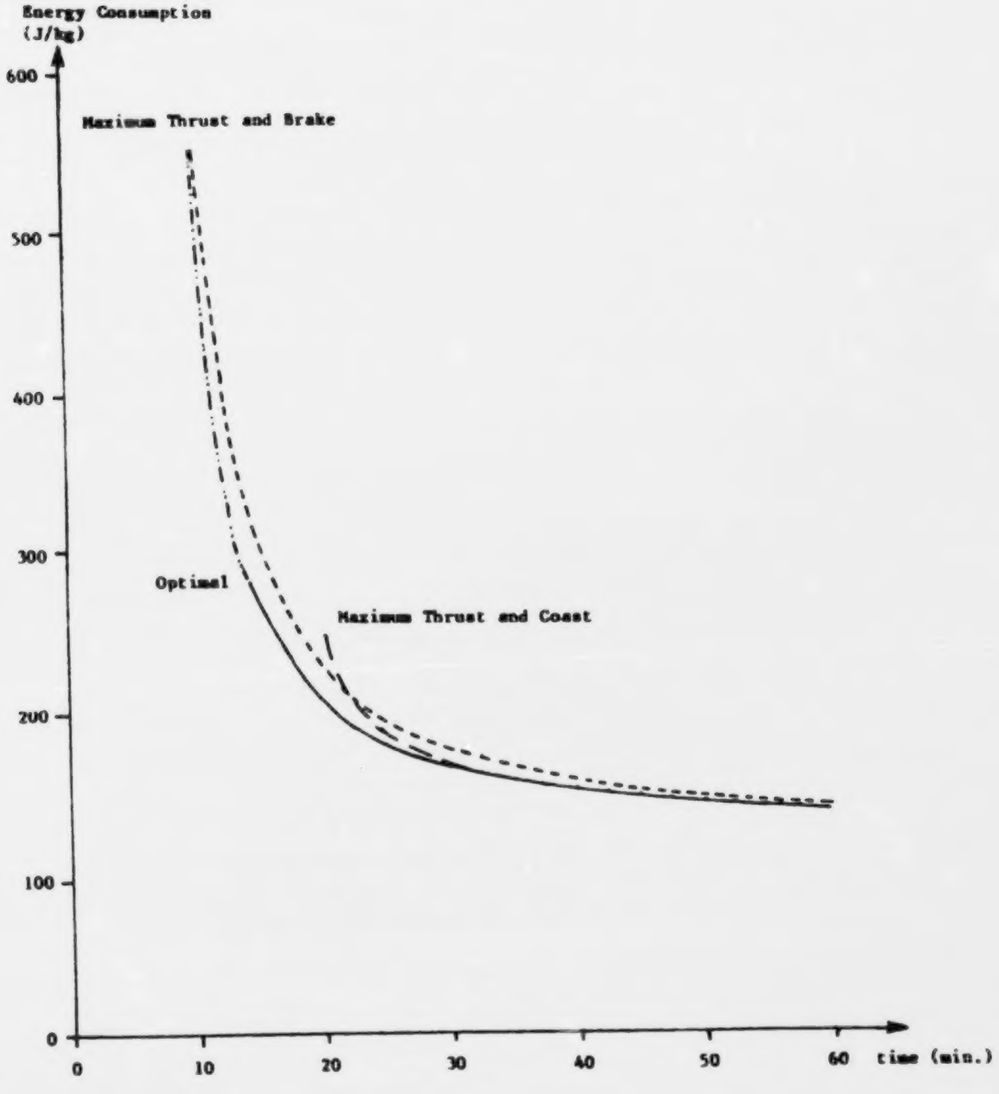

Figure 4 Energy Consumption Curves for Optimal and Suboptimal Control of a Train Travelling 10 km.

TABLE 1

ENERGY CONSUMPTION VALUES FOR OPTIMAL AND
SUBOPTIMAL CONTROL OF A TRAIN TRAVELLING 10 KM

		Energy Consumption (J/kg)		Per Cent Increase in Energy Consumption	
Time (min.)	Optimal	Maximum Thrust and Brake	Maximum Thrust and Coast	Maximum Thrust and Brake	Maximum Thrust and Coast
10.52	-	548.61	-	13.18	-
14.51	278.11	314.76	-	12.40	-
20.00	207.64	229.92	-	10.73	-
20.08	207.04	229.20	243.70	10.70	17.71
30.00	165.24	177.27	167.77	7.28	1.53
40.00	149.94	157.26	150.54	4.38	0.40
50.00	142.25	147.13	142.46	3.43	0.15
60.00	137.66	141.14	137.75	2.53	0.07

A comparison of a train with a typical time schedule of 30 minutes, running with optimal and suboptimal strategies, is shown in Figure 5.

7. APPLICATIONS

7.1 Light Short Train Example

An inter-city passenger train travelling between Kingston and Toronto is simulated. It is composed of one LRC locomotive, four coaches and one food car. The train is scheduled to depart from Kingston at 9:55, stop at Guildwood at 11:43 and arrive at Toronto at 12:00. The capacity of the train is 84 x 4 + 49 = 385 passengers, and the total mass of the train is approximately 0.34 x 10^6 kg. The distance and time schedule are as follows:

Kingston to Guildwood: 233.54 km in 108 minutes
Guildwood to Toronto: 20.34 km in 15 minutes
(allowing two minutes stop at Guildwood)

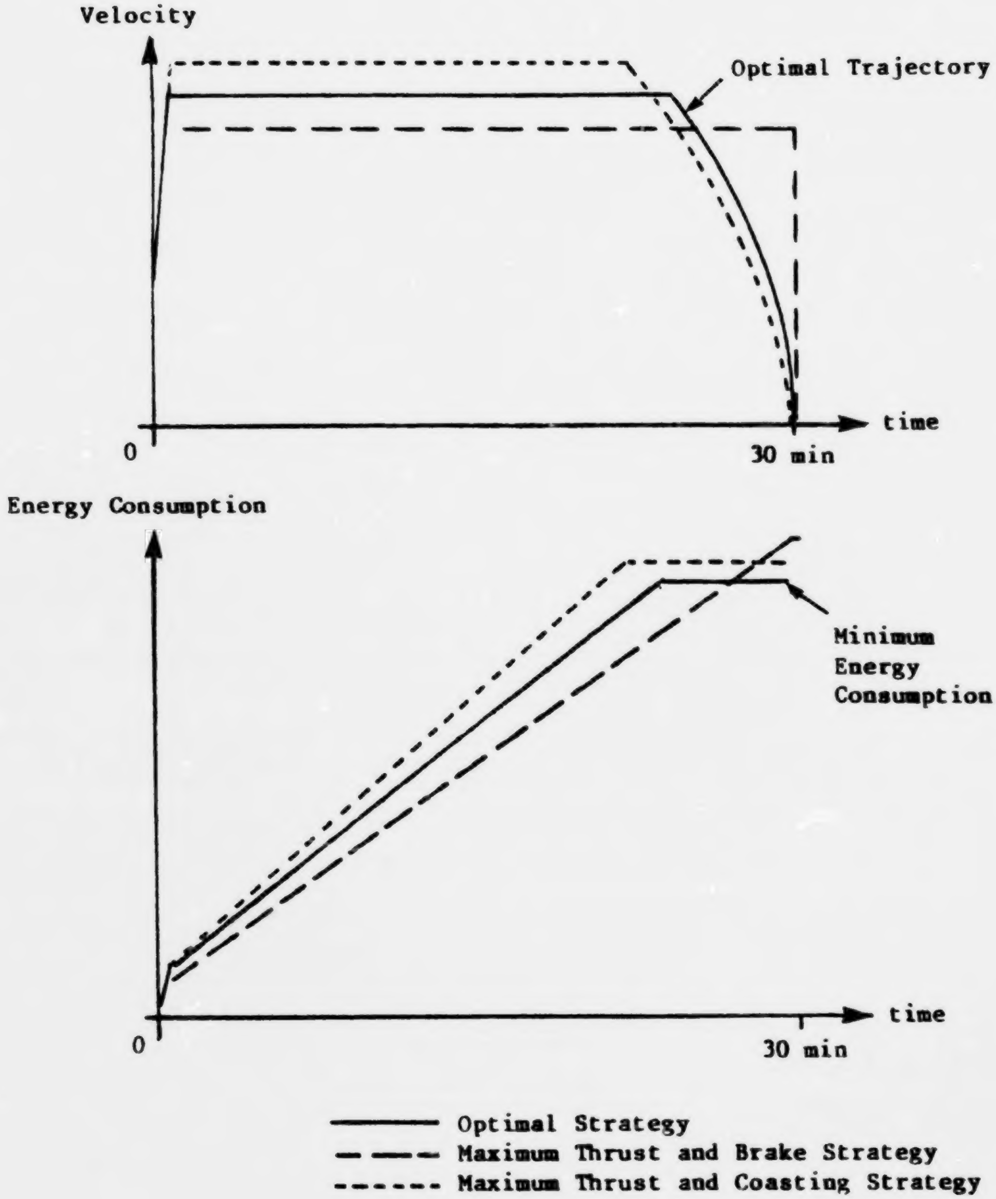

Figure 5 Comparison of a Train Moving with Optimal and Suboptimal Strategies

The track data is given by:

$$r_0 = 0.01624 \text{ N/kg}$$
$$r_1 = 0.00034 \text{ N sec/m/kg}$$
$$r_2 = 33.6 \times 10^{-6} \text{ Ns}^2\text{/m}^2\text{/kg}$$

and the train power and brake characteristics are given by:

$$\mu W_L = 0.2$$
$$P_{max} = 3{,}725 \text{ hp} = 2.7777 \text{ MW}$$
$$\eta = 0.7248$$
$$B_{max} = 0.3 \text{ MN/air brake}$$
$$v_{max} = 193 \text{ km/h}$$
$$\xi = 0.0578 \text{ MN /m}$$
$$Z = 0.267 \text{ MN}$$
$$\eta' = 1.16$$

Optimal and suboptimal strategies were simulated for the data and schedules given above. The simulation results are listed in Annex C and plotted in Figure 6. Note that this short train has the power-to-weight ratio of approximately 10 hp/ton (8.22 MW). The optimal value of total energy consumption from Kingston to Toronto for the given schedule is 17,614.6 + 1,054.6 = 18,679 J/kg.

Figure 6 that shows that the train is unable to meet the time schedule from Guildwood to Toronto using the maximum thrust and coasting strategy, and will be late by 7.7 minutes.

Suppose that the fuel consumption constants are:

$$C_1 = 0.078 \text{ litres/MJ}$$
$$Q = 0.081 \text{ MW}$$

The total energy and fuel consumption for a typical train from Kingston to Toronto is summarized in Table 2.

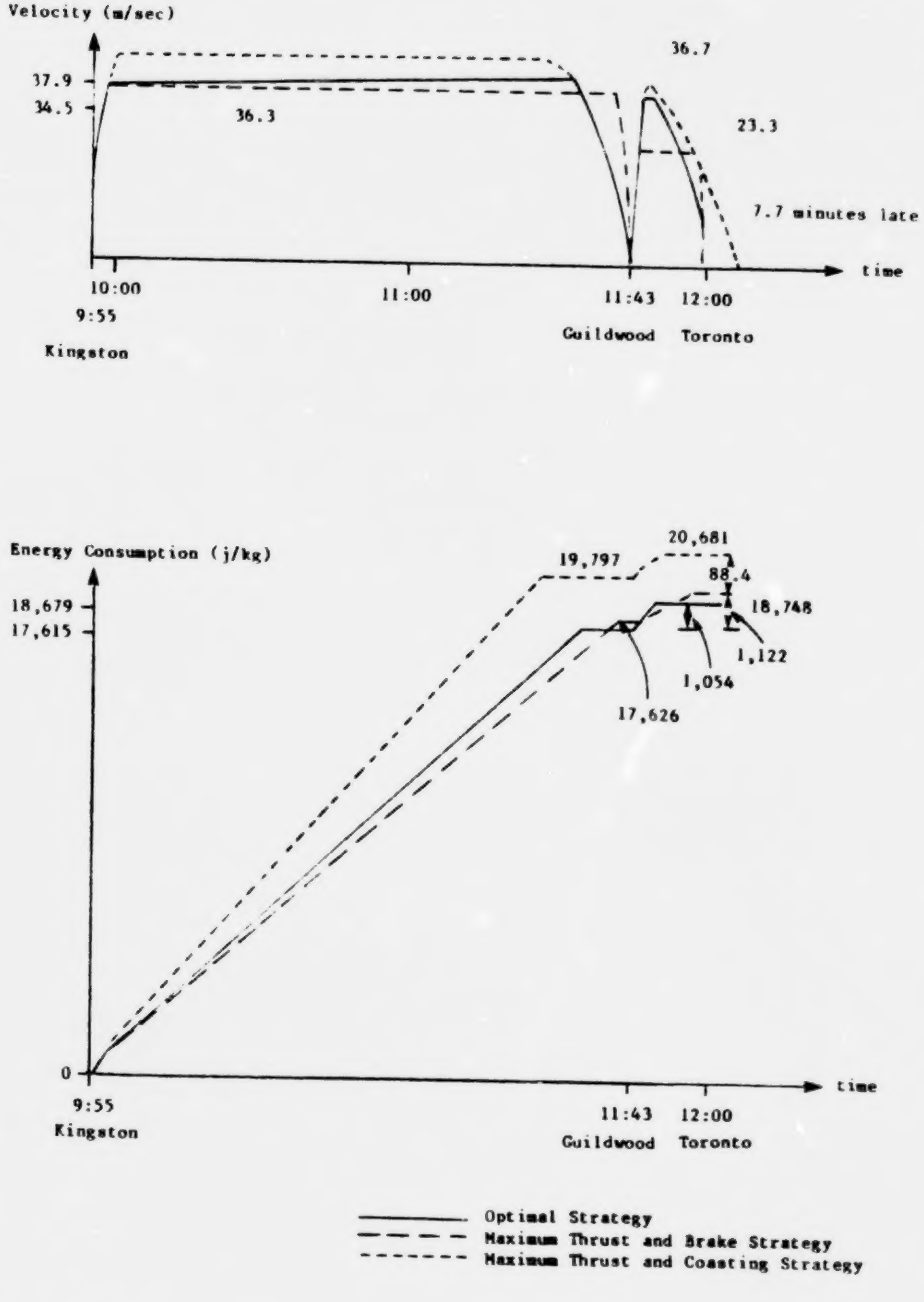

Figure 6 Simulation Results of a Short Passenger Train Moving with Optimal Strategy

TABLE 2

TOTAL FUEL/ENERGY CONSUMPTION, TYPICAL TRAIN, KINGTON-TORONTO

	Energy Consumption (J/kg) Kingston-Guildwood-Toronto	Total Fuel Consumption $C_1(J_0+QT)$ L
Optimal	17,615 + 1,054 = 18,679	540.4
Maximum thrust and brake	17,626 + 1,122 = 18,748	543.8 (0.63%)
Maximum thrust and coast	19,797 + 884* = 20,681	598.0 (10.7%)

* The train cannot meet the time schedule.

Note that a combination of the two suboptimal strategies can be applied and the total consumptions for this case are:

> maximum thrust and coast, 1st part E.C. = 19,797 + 1,122 = 20,,919, 1.078 x (7,112.46 + 597.78) F.C. = 601.4 (11.3%) and maximum thrust and brake, 2nd part.

8. CONCLUSION

The optimal (ME) control strategy has been designed to minimize the energy consumption of passenger/freight trains with a fixed time schedles. Power constraints and non-linear resistance force are considered in the design.

Numerical methods for the computation of the maximum speed by solving a cubic equation, computation of the switching time and distance by direct or numerical integration, and computation of the nominal speed by solving a non-linear equation have been presented.

Two suboptimal control strategies have also been introduced and compared with the optimal control strategy. The optimal control strategy was applied to the simulation of a light short passenger train and a long freight train using real data.

Although the solution is worked out for a train problem, the derivation is general enough to apply to any type of moving vehicle other than trains because the whole moving body of the train has been considered as a unit mass in the design.

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ANNEX A

MECFT PROGRAM LISTING

```
      PROGRAM MECFI
        COMMON/B1/S0,T0,R0,R1,R2,UMAX
        COMMON/B2/VV1,VV2,VV3,UMAXV3,UMIN,ABMAX,DBMAX,DBMV1,DBMV2
        COMMON/B3/T1,T2,T3,T4,S1,S2,S3,S4
        COMMON/B4/T1A,T1B,T4A,T4B,T4C,S1A,S1B,S4A,S4B,S4C
        COMMON/B5/LAW,ACC,V2     !create non-optimal control law
        COMMON/B6/VMAX           !
        CHARACTER*20 CTLAW(8)    !
        DATA CTLAW/'Optimal','Max. thrust & brake','Max. thrust & coast',!
     *  'Max. thrust & C.D.','C.A., coast & brake','C.A. & brake',       !
     *  'C.A. & coast','C.A. & C.D.'/                                    !
C1: Assign IO
        CALL ASSIGN(1,'MECFI.INP',0)
        CALL ASSIGN(2,'MECFI.OUT',0)
C2: Read data
        CALL LETIN
        WRITE(6,*) ' Control law:'      !
        DO 1 I=1,8                      !
    1   WRITE(6,*) I-1,' - ',CTLAW(I)   !
        READ(5,*) LAW                   !
        IF (LAW.LE.2) GOTO 2            !
        WRITE(6,*) 'Acceleration = ? (m/s**2)'  !
        READ(5,*) ACC                           !
        WRITE(2,21) 'Acc     =',ACC,'(m/s**2)'  !
   21   FORMAT(A,F10.4,1X,A)                    !
    2   CONTINUE                                !
C3: Initialize
        XLI=0.
        XRI=VMAX
        EPS=1.E-6
        IEND=500
C4: Solution of a non-linear equation
        CALL RIMI(X,F,XLI,XRI,EPS,IEND,IER)
C4c: Type out end conditions
        WRITE(6,*) IER
        V1=X
C       V2=R1*X*X/(R0+2.*R1*X)   !
        WRITE(6,*) 'v1 =',V1,'       v2 =',V2,'       F =',F
        U1A=UMAX
        U1B=UMAXV3/V1
        U2=R0+(R1+R2*V1)*V1
        U4A=UMIN
        U4B=UMIN
        IF (T4C.GT.0.) U4B=-(ABMAX+DBMV1/V2)
        IF (T4B.EQ.0.) U4B=-(ABMAX+DBMV1*V2)
        E1A=U1A*S1A
        E1B=UMAXV3*T1B
        E1=E1A+E1B
        E2=U2*S2
        WRITE(2,*) ' '
        WRITE(2,*) ' Switching strategy : ',CTLAW(LAW+1)
        WRITE(2,*) ' -------------------'
        WRITE(2,2001) ' v1     =',V1,'m/s','Vmax  =',VMAX,'m/s'
        WRITE(2,2001) ' v2     =',V2,'m/s'
        WRITE(2,*) ' '
      IF (V1.LT.VV3) GOTO 100
      WRITE(2,2001) ' T1a    =',T1A,'s','S1a    =',S1A*1.E-3,'km'
      WRITE(2,2001) ' T1b    =',T1B,'s','S1b    =',S1B*1.E-3,'km'
100   WRITE(2,2001) ' T1     =',T1,'s','S1     =',S1*1.E-3,'km'
      WRITE(2,2001) ' T2     =',T2,'s','S2     =',S2*1.E-3,'km'
      WRITE(2,2001) ' T3     =',T3,'s','S3     =',S3*1.E-3,'km'
      WRITE(2,2001) ' T4     =',T4,'s','S4     =',S4*1.E-3,'km'
      IF (V1.LT.VV1) GOTO 300
      WRITE(2,2001) ' T4a    =',T4A,'s','S4a    =',S4A*1.E-3,'km'
      WRITE(2,2001) ' T4b    =',T4B,'s','S4b    =',S4B*1.E-3,'km'
      IF (V1.LT.VV2) GOTO 300
      WRITE(2,2001) ' T4c    =',T4C,'s','S4c    =',S4C*1.E-3,'km'
300   WRITE(2,*) ' '
      WRITE(2,2001) ' U1a    =',U1A,'N/kg','E1a    =',E1A,'J/kg'
      IF (V1.LT.VV1) GOTO 400
      WRITE(2,2001) ' U1b    =',U1B,'N/kg','E1b    =',E1B,'J/kg'
400   WRITE(2,2001) ' U2     =',U2,'N/kg','E2     =',E2,'J/kg'
      WRITE(2,*) ' '
      WRITE(2,*) ' Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg)',
     *  ' E.C. (J/kg)'
      WRITE(2,*) ' --------  ---------  ----------  -----------',
     *  ' -----------'
      WRITE(2,2002) 0.,0.,0.,U1A,0.,'Max. force A'
      IF (V1.GT.VV3) WRITE(2,2002) T1A,S1A,VV3,U1B,E1A,'Max. force B'
      WRITE(2,2002) T1,S1,V1,U2,E1,'Const. speed'
      WRITE(2,2092) T1+T2,S1+S2,V1,0.,E1+E2,'Coasting'
      WRITE(2,2002) T1+T2+T3,S1+S2+S3,V2,U4B,E1+E2,'Max. brake'
      T=T1+T2+T3+T4C
      S=S1+S2+S3+S4C
      E=E1+E2
      IF (V2.GT.VV2) WRITE(2,2002) T,S,VV2,U4A,E,'Max. brake B'
      IF (V2.GT.VV1)WRITE(2,2002) T+T4B,S+S4B,VV1,U4A,E,'Max. brake A'
      T0=T1+T2+T3+T4
      WRITE(2,2002) T0,S1+S2+S3+S4,0.
      WRITE(2,*) ' '
      WRITE(2,2001) ' Dist.  =',S0*1.E-3,'km'
      WRITE(2,2001) ' Time   =',T0,'sec'
      WRITE(2,2001) ' E.C.   =',(E1+E2)*1.E+3/S0,'J/t/km'
      WRITE(6,2001) ' Time   =',T0/60.,'min'
      WRITE(6,2001) ' E.C.   =',E1+E2,'J/kg'
2001  FORMAT(2(A,F10.4,1X,A,T35))
2002  FORMAT(2F10.1,3F12.6,2X,A)
      STOP
      END
      FUNCTION FC1(V1)
      COMMON/B1/S0,T0,K0,K1,K2,UMAX
      COMMON/B2/VV1,VV2,VV3,UMAXV3,UMIN,ABMAX,DBMAX,DBMV1,DBMV2
      COMMON/B3/T1,T2,T3,T4,S1,S2,S3,S4
      COMMON/B4/T1A,T1B,T4A,T4B,T4C,S1A,S1B,S4A,S4B,S4C
      COMMON/B5/LAW,ACC,V2
      V2=K1*V1*V1/(K0+2.*K1*V1)
      IF (LAW.EQ.1) V2=V1
      IF (LAW.GE.2) V2=0.
      IF (V1.GT.VV3) GOTO 10
      T1A=RINTT(0.,V1,UMAX-K0,-K1,-K2)
      T1B=0.
      S1A=RINTV(0.,V1,UMAX-K0,-K1,-K2)
      S1B=0.
      GOTO 20
10    T1A=RINTT(0.,VV3,UMAX-K0,-K1,-K2)
      T1B=RINTT1(VV3,V1,UMAXV3,-K0,-K1,-K2)
      S1A=RINTV(0.,VV3,UMAX-K0,-K1,-K2)
      S1B=RINTV1(VV3,V1,UMAXV3,-K0,-K1,-K2)
20    T1=T1A+T1B
      S1=S1A+S1B
      T3=RINTT(V2,V1,K0,K1,K2)
      S3=RINTV(V2,V1,K0,K1,K2)
      IF (V2.GT.VV1) GOTO 30
      T4A=RINTT(0.,V2,ABMAX+K0,K1+DBMV1,K2)
      T4B=0.
      T4C=0.
      S4A=RINTV(0.,V2,ABMAX+K0,K1+DBMV1,K2)
      S4B=0.
      S4C=0.
      GOTO 50
30    IF (V2.GT.VV2) GOTO 40
      T4A=RINTT(0.,VV1,ABMAX+K0,K1+DBMV1,K2)
      T4B=RINTT(VV1,V2,-UMIN+K0,K1,K2)
      T4C=0.
      S4A=RINTV(0.,VV1,ABMAX+K0,K1+DBMV1,K2)
      S4B=RINTV(VV1,V2,-UMIN+K0,K1,K2)
      S4C=0.
      GOTO 50
40    T4A=RINTT(0.,VV1,ABMAX+K0,K1+DBMV1,K2)
      T4B=RINTT(VV1,VV2,-UMIN+K0,K1,K2)
      T4C=RINTT1(VV2,V2,DBMV2,ABMAX+K0,K1,K2)
      S4A=RINTV(0.,VV1,ABMAX+K0,K1+DBMV1,K2)
      S4B=RINTV(VV1,VV2,-UMIN+K0,K1,K2)
      S4C=RINTV1(VV2,V2,DBMV2,ABMAX+K0,K1,K2)
50    T4=T4A+T4B+T4C
      S4=S4A+S4B+S4C
      T2=T0-T1-T3-T4
      IF (T2.LT.0.) T2=0.
      S2=V1*T2
      FC1=S1+S2+S3+S4-S0
      RETURN
      END
      FUNCTION RINTT(X0,X1,A,B,C)
      DOUBLE PRECISION Q,D,E,Y1,Y0
      IF (C) 1,2,1
 1    Q=DBLE(4.*A*C-B*B)
      IF (Q) 10,10,30
10    D=DSQRT(-Q)
      E=DBLE(2.*C*X1+B)
      Y1=(E-D)/(E+D)
      E=DBLE(2.*C*X0+B)
      Y0=(E-D)/(E+D)
      RINTT=SNGL(DLOG(DABS(Y1/Y0))/D)
      RETURN
30    D=DSQRT(Q)
      E=DBLE(2.*C*X1+B)
      Y1=DATAN(E/D)
      E=DBLE(2.*C*X0+B)
      Y0=DATAN(E/D)
      RINTT=SNGL(2.*(Y1-Y0)/D)
      RETURN
 2    Y1=DBLE(A+B*X1)
      Y0=DBLE(A+B*X0)
      RINTT=SNGL((DLOG(DABS(Y1/Y0)))/B)
      RETURN
      END



      FUNCTION RINTV(X0,X1,A,B,C)
      IF (C) 10,20,10
10    X=(A+(B+C*X1)*X1)/(A+(B+C*X0)*X0)
      RINTV=(ALOG(ABS(X))-B*RINTT(X0,X1,A,B,C))/(2.*C)
      RETURN
20    Y1=A+B*X1
      Y0=A+B*X0
      RINTV=(Y1-Y0-A*ALOG(ABS(Y1/Y0)))/(B*B)
      RETURN
      END



      FUNCTION RINTT1(X0,X1,A,B,C,D)
      EXTERNAL RF1
      DIMENSION P(4)
      P(1)=A
      P(2)=B
      P(3)=C
      P(4)=D
      CALL QG10(X0,X1,RF1,P,Y)
      RINTT1=Y
      RETURN
      END
      FUNCTION RF1(X,P)
      DIMENSION P(1)
      S=0.
      DO 10 I=4,1,-1
      S=X*S
   10 S=S+P(I)
      RF1=X/S
      RETURN
      END

      FUNCTION RINTV1(X0,X1,A,B,C,D)
      EXTERNAL RF2
      DIMENSION P(4)
      P(1)=A
      P(2)=B
      P(3)=C
      P(4)=D
      CALL QG10(X0,X1,RF2,P,Y)
      RINTV1=Y
      RETURN
      END

      FUNCTION RF2(X,P)
      DIMENSION P(1)
      S=0.
      DO 10 I=4,1,-1
      S=X*S
   10 S=S+P(I)
      RF2=X*X/S
      RETURN
      END

C
C     To compute integral FCT(x,P), summed over x from xl to xu
C     by 10-point Gauss quadrature formula, which integrates
C     polynomials up to degree 19 exactly.
C
      SUBROUTINE QG10(XL,XU,FCT,P,Y)
      DIMENSION P(1)
      A=.5*(XU+XL)
      B=XU-XL
      C=.4869533*B
      Y=.03333567*(FCT(A+C,P)+FCT(A-C,P))
      C=.4325317*B
      Y=Y+.07472567*(FCT(A+C,P)+FCT(A-C,P))
      C=.3397048*B
      Y=Y+.1095432*(FCT(A+C,P)+FCT(A-C,P))
      C=.2166977*B
      Y=Y+.1346334*(FCT(A+C,P)+FCT(A-C,P))
      C=.07443717*B
      Y=B*(Y+.1477621*(FCT(A+C,P)+FCT(A-C,P)))
      RETURN
      END
      SUBROUTINE LETIN
        REAL N1,N2
        COMMON/B1/S0,T0,R0,R1,R2,UMAX
        COMMON/B2/VV1,VV2,VV3,UMAXV3,UMIN,ABMAX,DBMAX,DBMV1,DBMV2
        COMMON/B6/VMAX            !
        DIMENSION A(4),W(4),RR(4),RI(4) !
        READ(1,1000)
        READ(1,1000)
 1000   FORMAT()
        READ(1,1001) IMASS
        READ(1,1001) XLOC
        READ(1,1001) XBRK
        READ(1,1001) R0
        READ(1,1001) R1
        READ(1,1001) R2
        READ(1,1001) UW
        READ(1,1001) FMAX
        READ(1,1001) N1
        READ(1,1001) N2
        READ(1,1001) E1
        READ(1,1001) Z
        READ(1,1001) BMAX
C       READ(1,1001) UMAX
 1001   FORMAT(8X,G12.4)
        VV1=Z/E1
        VV2=N2*FMAX/Z
        VV3=N1*FMAX/UW
        UMAX=UW/IMASS
        UMAXV3=UMAX*VV3
        ABMAX=XBRK*BMAX/IMASS
        DBMAX=XLOC*Z/IMASS
        UMIN=-(DBMAX+ABMAX)
        DBMV1=DBMAX/VV1
        DBMV2=DBMAX*VV2
        IORD=3                     !
        IF (R2.EQ.0.) IORD=2       !
        A(1)=-N1*FMAX/IMASS        !
        A(2)=R0                    !
        A(3)=R1                    !
        A(4)=R2                    !
        CALL POLRT(A,W,IORD,RR,RI,IER)               !
        IF (IER.NE.0) WRITE(6,*) 'Error in Vmax'     !
        DO 10 I=1,IORD                               !
   10   IF (ABS(RI(I)).LT.1.E-6) VMAX=RR(I)          !
        WRITE(6,*) ' Dist. = ? km'
        READ(5,*) S0
        WRITE(6,2001) ' Dist. =',S0,'km'
        WRITE(6,*) ' Time  = ? min'
        READ(5,*) T0
        WRITE(6,2001) ' Time  =',T0,'min'
        WRITE(2,*) ' Schedule'
        WRITE(2,*) ' --------'
        WRITE(2,2001) ' Dist. =',S0,'km'
        WRITE(2,2001) ' Time  =',T0,'min'
        S0=S0*1.E+3      !meters
      TO=TO*60.        !seconds
      WRITE(2,*) ' '
      WRITE(2,*) ' Data of train'
      WRITE(2,*) ' --------------'
      WRITE(2,2001) ' Tmass =',TMASS*1.E-6,'Mkg'
      WRITE(2,2001) ' nLOC  =',XLOC,' '
      WRITE(2,2001) ' nBRK  =',XBRK,' '
      WRITE(2,2001) ' r0    =',R0,'N/kg',' a       =',[illegible],' '
      WRITE(2,2001) ' r1    =',R1,'Ns/m/kg',' a''      ',[illegible]
      WRITE(2,2001) ' r2    =',R2*1.E+6,'uNs2/m2/kg',
     *  ' e      =',E1*1.E-6,'MNs/m'
      WRITE(2,2001) ' uW    =',UW*1.E-6,'MN',
     *  ' Z      =',Z*1.E-6,'MN'
      WRITE(2,2001) ' Fmax  =',FMAX*1.E-6,'MW/locomotive',
     *  ' Bmax   =',BMAX*1.E-6,'MN/air brake'
      WRITE(2,*) ' '
      WRITE(2,2001) ' Umin  =',UMIN,'N/kg',' ABmax    =',ABMAX,'N/kg'
      WRITE(2,2001) ' Umax  =',UMAX,'N/kg',' DBmax    =',DBMAX,'N/kg'
      WRITE(2,2001) ' V1    =',VV1,'m/s',' DBmax/V1 =',DBMV1,'Ns/m/kg'
      WRITE(2,2001) ' V2    =',VV2,'m/s',' DBmax*V2 =',DBMV2,'W/kg'
      WRITE(2,2001) ' V3    =',VV3,'m/s',' Umax*V3  =',UMAXV3,'W/kg'
 2001 FORMAT(2(A,F10.4,1X,A,T35))
      RETURN
      END



C     Subroutine      RTMI
C     Purpose         To solve general non-linear equations of form
C                     FCT(x) = 0 by means of Mueller's iteration method.
C     Copied by       K.Y. Wong
C
      SUBROUTINE RTMI(X,F,XLI,XRI,EPS,IEND,IER)
C
      IER=0
      XL=XLI
      XR=XRI
      X=XL
      TOL=X
      F=FCT(TOL)
      IF(F)1,16,1
    1 FL=F
      X=XR
      TOL=X
      F=FCT(TOL)
      IF(F)2,16,2
    2 FR=F
      IF (SIGN(1.,FL)+SIGN(1.,FR)) 25,3,25
C
    3 I=0
      TOLF=100.*EPS
C
    4 I=I+1
C
```

```
      DO 13 K=1,IEND
      X=.5*(XL+XR)
      TOL=X
      F=FCT(TOL)
      IF (F) 5,16,5
    5 IF (SIGN(1.,F)+SIGN(1.,FR)) 7,6,7
C
    6 TOL=XL
      XL=XR
      XR=TOL
      TOL-FL
      FL=FR
      FR=TOL
    7 TOL=F-FL
      A=F*TOL
      A=A+A
      IF (A-FR*(FR-FL)) 8,9,9
    8 IF (I-IEND) 17,17,9
    9 XR=X
      FR=F
C
      TOL=EPS
      A=ABS(XR)
      IF (A-1.) 11,11,10
   10 TOL=TOL*A
   11 IF (ABS(XR-XL)-TOL) 12,12,13
   12 IF (ABS(FR-FL)-TOLF) 14,14,13
   13 CONTINUE
C
      IER=1
   14 IF (ABS(FR)-ABS(FL)) 16,16,15
   15 X=XL
      F=FL
   16 RETURN
C
   17 A=FR-F
      DX=(X-XL)*FL*(1.+F*(A-TOL)/(A*(FR-FL)))/TOL
      XM=X
      FM=F
      X=XL-DX
      TOL=X
      F=FCT(TOL)
      IF (F) 18,16,18
C
   18 TOL=EPS
      A=ABS(X)
      IF (A-1.) 20,20,19
   19 TOL=TOL*A
   20 IF (ABS(DX)-TOL) 21,21,22
   21 IF (ABS(F)-TOLF) 16,16,22
C
   22 IF (SIGN(1.,F)+SIGN(1.,FL)) 24,23,24
   23 XR=X
      FR=F
      GOTO 4
   24   XL=X
        FL=F
        XR=XM
        FR=FM
        GOTO 4
C
   25   IER=2
        RETURN
        END




C       SUBROUTINE       POLRT
C       PURPOSE          COMPUTE THE REAL AND COMPLEX ROOTS OF A REAL POLYNO
C
        SUBROUTINE POLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
        DIMENSION XCOF(1),COF(1),ROOTR(1),ROOTI(1)
        DOUBLE PRECISION X0,Y0,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,
      1 SUMSQ,DX,DY,TEMP,ALPHA
        IFIT=0
        N=M
        IER=0
        IF (XCOF(N+1)) 10,25,10
   10   IF (N) 15,15,32
   15   IER=1
   20   RETURN
   25   IER=4
        GOTO 20
   30   IER=2
        GOTO 20
   32   IF (N-36) 35,35,30
   35   NX=N
        NXX=N+1
        N2=1
        KJ1=N+1
        DO 40 L=1,KJ1
        MT=KJ1-L+1
   40   COF(MT)=XCOF(L)
   45   X0=0.00500101
        Y0=0.01000101
        IN=0
   50   X=X0
        X0=-10.0*Y0
        Y0=-10.0*X
        X=X0
        Y=Y0
        IN=IN+1
        GOTO 59
   55   IFIT=1
        XPR=X
        YPR=Y
   59   ICT=0
```

```
 60    UX=0.0
       UY=0.0
       V=0.0
       YT=0.0
       XT=1.0
       U=COF(N+1)
       IF (U) 65,130,65
 65    DO 70 I=1,N
       L=N-I+1
       TEMP=COF(L)
       XT2=X*XT-Y*YT
       YT2=X*YT+Y*XT
       U=U+TEMP*XT2
       V=V+TEMP*YT2
       FI=I
       UX=UX+FI*XT*TEMP
       UY=UY-FI*YT*TEMP
       XT=XT2
 70    YT=YT2
       SUMSQ=UX*UX+UY*UY
       IF (SUMSQ) 75,110,75
 75    DX=(V*UY-U*UX)/SUMSQ
       X=X+DX
       DY=-(U*UY+V*UX)/SUMSQ
       Y=Y+DY
 78    IF (DABS(DY)+DABS(DX)-1.0D-05) 100,80,80
 80    ICT=ICT+1
       IF (ICT-500) 60,85,85
 85    IF (IFIT) 100,90,100
 90    IF (IN-5) 50,95,95
 95    IER=3
       GOTO 20
100    DO 105 L=1,NXX
       MT=KJ1-L+1
       TEMP=XCOF(MT)
       XCOF(MT)=COF(L)
105    COF(L)=TEMP
       ITEMP=N
       N=NX
       NX=ITEMP
       IF (IFIT) 120,55,120
110    IF (IFIT) 115,50,115
115    X=XPR
       Y=YPR
120    IFIT=0
122    IF(DABS(Y)-1.0D-4*DABS(X)) 135,125,125
125    ALPHA=X+X
       SUMSQ=X*X+Y*Y
       N=N-2
       GOTO 140
130    X=0.0
       NX=NX-1
       NXX=NXX-1
```

```
135     Y=0.0
        SUMSQ=0.0
        ALPHA=X
        N=N-1
140     COF(2)=COF(2)+ALPHA*COF(1)
145     DO 150 L=2,N
150     COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
155     ROOTI(N2)=Y
        ROOTR(N2)=X
        N2=N2+1
        IF (SUMSQ) 160,165,160
160     Y=-Y
        SUMSQ=0.0
        GOTO 155
165     IF (N) 20,20,45
        END
```

ANNEX B

SIMULATION RESULTS OF THE MECFT PROGRAM

```
Data of train
-------------
Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   =   631.3000 sec
nLOC   =     1.0000
nBRK   =    15.0000
r0     =     0.0120 N/kg                n              0.8200
r1     =     0.0005 Ns/m/kg             n'             1.2000
r2     =    43.5000 uNs2/m2/kg          r              0.9578 [illegible]
nW     =     0.3470 MN                  Z      =       0.2670 [illegible]
Pmax   =     2.2065 MW/locomotive       Bmax           0.4000 MN [illegible]

Umin   =    -2.3835 N/kg                ABmax  =       2.2500 N/kg
Umax   =     0.1735 N/kg                DBmax          0.1335 N/kg
V1     =     4.6194 m/s                 DBmax/V1  =       0.0289 Ns/m/kg
V2     =     9.9169 m/s                 DBmax*V2          1.3239 W/kg
V3     =     5.2142 m/s                 Umax*V3           0.9047 W/kg

 Switching strategy : Max. thrust & brake
 ----------------------------------------
v1     =    20.4311 m/s
v2     =    20.4311 m/s

T1a    =    32.6307 s                   S1a    =    85.4086 m
T1b    =   590.0484 s                   S1b    =  9826.6660 m
T1     =   622.6791 s                   S1     =  9912.0742 m
T2     =     0.0000 s                   S2     =     0.0000 m
T3     =     0.0000 s                   S3     =     0.0000 m
T4     =     8.6235 s                   S4     =    87.9301 m
T4a    =     1.9830 s                   S4a    =     4.5441 m
T4b    =     2.2059 s                   S4b    =    16.0298 m
T4c    =     4.4346 s                   S4c    =    67.3661 m

U1a    =     0.1735 N/kg                E1a         14.8184 J/kg
U1b    =     0.0443 N/kg                E1b    -   533.7961 J/kg
U2     =     0.0404 N/kg                E2           0.0000 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     32.6       85.4    5.214208     0.044229   14.818391  Max. force B
    622.7     9912.1   20.431072     0.040374  548.614563  Const. speed
    622.7     9912.1   20.431072     0.000000  548.614563  Coasting
    622.7     9912.1   20.431072    -2.251415  548.614563  Max. brake
    627.1     9979.4    9.916854    -2.383500  548.614563  Max. brake B
    629.3     9995.5    4.619377    -2.383500  548.614563  Max. brake A
    631.3    10000.0    0.000000

Dist.  =   10.0000 km
Time   =  631.3026 sec
E.C.   =   54.8615 J/kg/km
```

```
 Data of train
 -------------
Tmass  =    2.0000 Mkg
Dist.  =   10.0000 km
Time   =  870.4000 sec
nLOC   =    1.0000
nBRK   =   15.0000
r0     =    0.0120 N/kg            a      =    0.8200
r1     =    0.0005 Ns/m/kg         a'     =    1.2000
r2     =   43.5000 uNs2/m2/kg      b      =    0.0578 kNs/m
nW     =    0.3470 MN              Z      =    0.2670 MN
Pmax   =    2.2065 MW/locomotive   Bmax   =    0.3000 MN/air brake

Umin   =   -2.3835 N/kg            ABmax  =    2.2500 N/kg
Umax   =    0.1735 N/kg            DBmax  =    0.1335 W/kg
V1     =    4.6194 m/s             DBmax/V1  =    0.0289 Ns/m/kg
V2     =    9.9169 m/s             DBmax*V2  =    1.3239 W/kg
V3     =    5.2142 m/s             Umax*V3   =    0.9047 W/kg

 Switching strategy : Optimal
 ----------------------------
v1     =   17.9484 m/s
v2     =    5.3783 m/s

T1a    =   32.6307 s              S1a    =   85.4086 m
T1b    =  291.0000 s              S1b    = 4005.6282 m
T1     =  323.6307 s              S1     = 4091.0369 m
T2     =    0.0447 s              S2     =    0.8021 m
T3     =  544.4254 s              S3     = 5902.0464 m
T4     =    2.2993 s              S4     =    6.1151 m
T4a    =    1.9830 s              S4a    =    4.5341 m
T4b    =    0.3164 s              S4b    =    1.5809 m
T4c    =    0.0000 s              S4c    =    0.0000 m

U1a    =    0.1735 N/kg           E1a    =   14.8184 J/kg
U1b    =    0.0504 N/kg           E1b    =  263.2576 J/kg
U2     =    0.0350 N/kg           E2     =    0.0281 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     32.6       85.4    5.214208     0.050404   14.818391  Max. force B
    323.6     4091.0   17.948391     0.034987  278.075958  Const. speed
    323.7     4091.8   17.948391     0.000000  278.104034  Coasting
    868.1     9993.9    5.378332    -2.383500  278.104034  Max. brake
    868.4     9995.5    4.619377    -2.383500  278.104034  Max. brake A
    870.4    10000.0    0.000000

Dist.  =   10.0000 km
Time   =  870.4001 sec
E.C.   =   27.8104 J/kg/km
```

```
 Data of train
 -------------
Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   =  1204.7000 sec
nLOC   =     1.0000
nBRK   =    15.0000
r0     =     0.0120 N/kg              a      =      0.8200
r1     =     0.0005 Ns/m/kg           a'     =      1.2000
r2     =    43.5000 uNs2/m2/kg        c      =      0.0578 MNs/m
nW     =     0.3470 MN                Z      =      0.2670 MN
Fmax   =     2.2065 MW/locomotive     Bmax   =      0.3000 MN/air brake

Umin   =    -2.3835 N/kg              ABmax  =      2.2500 N/kg
Umax   =     0.1735 N/kg              DBmax  =      0.1335 N/kg
V1     =     4.6194 m/s               DBmax/V1 =      0.0289 Ns/m/kg
V2     =     9.9169 m/s               DBmax*V2 =      1.3239 W/kg
V3     =     5.2142 m/s               Umax*V3  =      0.9047 W/kg

 Switching strategy : Optimal
 ----------------------
v1     =    10.2808 m/s
v2     =     2.3719 m/s

T1a    =    32.6307 s                 S1a    =    85.4086 m
T1b    =    52.4515 s                 S1b    =   424.8906 m
T1     =    85.0822 s                 S1     =   510.2992 m
T2     =   647.7644 s                 S2     =  6659.5449 m
T3     =   470.8206 s                 S3     =  2828.9375 m
T4     =     1.0327 s                 S4     =     1.2177 m
T4a    =     1.0327 s                 S4a    =     1.2177 m
T4b    =     0.0000 s                 S4b    =     0.0000 m
T4c    =     0.0000 s                 S4c    =     0.0000 m

U1a    =     0.1735 N/kg              E1a    =    14.8184 J/kg
U1b    =     0.0880 N/kg              E1b    =    47.4511 J/kg
U2     =     0.0217 N/kg              E2     =   144.7662 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     32.6       85.4    5.214208     0.087995   14.818391  Max. force B
     85.1      510.3   10.280813     0.021738   62.269474  Const. speed
    732.8     7169.8   10.280813     0.000000  207.035629  Coasting
   1203.7     9998.8    2.371886    -2.318547  207.035629  Max. brake
   1204.7    10000.0    0.000000

Dist.  =    10.0000 km
Time   =  1204.7000 sec
E.C.   =    20.7036 J/kg/km
```

```
 Data of train
 -------------
Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   =   870.4000 sec
nLOC   =     1.0000
nBRK   =    15.0000
r0     =     0.0120 N/kg             n       =    0.8200
r1     =     0.0005 Ns/m/kg          n'      =    1.2000
r2     =    43.5000 uNs2/m2/kg       c       =    0.0578 MN/m
nW     =     0.3470 MN               Z       =    0.2670 MN
Pmax   =     2.2065 MW/locomotive    Bmax    =    0.3000 MN/air brake

Umin   =    -2.3835 N/kg             ABmax   =    2.2500 N/kg
Umax   =     0.1735 N/kg             DBmax   =    0.1335 N/kg
V1     =     4.6194 m/s              DBmax/V1  =     0.0289 Ns/m/kg
V2     =     9.9169 m/s              DBmax*V2  =     1.3239 W/kg
V3     =     5.2142 m/s              Umax*V3         0.9047 W/kg

 Switching strategy : Max. thrust & brake
 ----------------------
v1     =    12.1613 m/s
v2     =    12.1613 m/s

T1a    =    32.6307 s              S1a   =    85.4086 m
T1b    =    85.2560 s              S1b   =   794.4482 m
T1     =   117.8867 s              S1    =   879.8568 m
T2     =   747.3867 s              S2    =  9089.2246 m
T3     =     0.0000 s              S3    =     0.0000 m
T4     =     5.1267 s              S4    =    30.9188 m
T4a    =     1.9830 s              S4a   =     4.5341 m
T4b    =     2.2059 s              S4b   =    16.0298 m
T4c    =     0.9379 s              S4c   =    10.3548 m

U1a    =     0.1735 N/kg           E1a   =    14.8184 J/kg
U1b    =     0.0744 N/kg           E1b   =    77.1281 J/kg
U2     =     0.0245 N/kg           E2    =   222.8155 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     32.6       85.4    5.214208     0.074389   14.818391  Max. force B
    117.9      879.9   12.161342     0.024514   91.946487  Const. speed
    865.3     9969.1   12.161342     0.000000  314.761963  Coasting
    865.3     9969.1   12.161342    -2.252376  314.761963  Max. brake
    866.2     9979.4    9.916854    -2.383500  314.761963  Max. brake B
    868.4     9995.5    4.619377    -2.383500  314.761963  Max. brake A
    870.4    10000.0    0.000000

Dist.  =    10.0000 km
Time   =   870.4000 sec
F.C.   =    31.4762 J/kg/km
```

```
 Data of train
 -------------
Tmass =     2.0000 Mkg
Dist. =    10.0000 km
Time  = 1204.7000 sec
nLOC  =     1.0000
nBRK  =    15.0000
r0    =     0.0120 N/kg              n       =     0.8200
r1    =     0.0005 Ns/m/kg           n'      =     1.2000
r2    =    43.5000 uNs2/m2/kg        e       =     0.0578 MNs/m
uW    =     0.3470 MN                Z       =     0.2670 MN
Pmax  =     2.2065 MW/locomotive     Bmax    =     0.3000 MN/air brake

Umin  =    -2.3835 N/kg              ABmax =       2.2500 N/kg
Umax  =     0.1735 N/kg              DBmax =       0.1335 N/kg
V1    =     4.6194 m/s               DBmax/V1 =       0.0289 Ns/m/kg
V2    =     9.9169 m/s               DBmax*V2 =       1.3239 W/kg
V3    =     5.2142 m/s               Umax*V3  =       0.9047 W/kg

 Switching strategy : Max. thrust & brake
 --------------------
v1    =     8.5091 m/s
v2    =     8.5091 m/s

T1a   =    32.6307 s                S1a    =    85.4086 m
T1b   =    29.0057 s                S1b    =   203.6815 m
T1    =    61.6364 s                S1     =   289.0901 m
T2    = 1139.4604 s                 S2     = 9695.7422 m
T3    =     0.0000 s                S3     =     0.0000 m
T4    =     3.6032 s                S4     =    15.1680 m
T4a   =     1.9830 s                S4a    =     4.5341 m
T4b   =     1.6202 s                S4b    =    10.6338 m

U1a   =     0.1735 N/kg             E1a    =    14.8184 J/kg
U1b   =     0,1063 N/kg             E1b    =    26.2404 J/kg
U2    =     0.0194 N/kg             E2     =   188.1373 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0   0.000000    0.173500    0.000000  Max. force A
     32.6       85.4   5.214208    0.106318   14.818391  Max. force B
     61.6      289.1   8.509065    0.019404   41.058815  Const. speed
   1201.1     9984.8   8.509065    0.000000  229.196121  Coasting
   1201.1     9984.8   8.509065   -2.383500  229.196121  Max. brake
   1202.7     9995.5   4.619377   -2.383500  229.196121  Max. brake A
   1204.7    10000.0   0.000000

Dist. =   10.0000 km
Time  = 1204.7000 sec
E.C.  =   22.9196 J/kg/km
```

```
 Data of train
 -------------

Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   =  1204.7000 sec
nLOC   =     1.0000
nBRK   =    15.0000
r0     =     0.0120 N/kg            n       =     0.8200
r1     =     0.0005 Ns/m/kg         n'      =     1.2000
r2     =    43.5000 uNs2/m2/kg      e       =     0.0578 uNs/m
nW     =     0.3470 MN              Z       =     0.2670 MN
Fmax   =     2.2065 MW/locomotive   Bmax    =     0.3000 MN/air brake

Umin   =    -2.3835 N/kg            ABmax   =     2.2500 N/kg
Umax   =     0.1735 N/kg            DBmax   =     0.1335 N/kg
V1     =     4.6194 m/s             DBmax/V1   =     0.0289 Ns/m/kg
V2     =     9.9169 m/s             DBmax*V2   =     1.3239 W/kg
V3     =     5.2142 m/s             Umax*V3    =     0.9047 W/kg

 Switching strategy : Max thrust & coast
 ---------------------

v1     =    17.3043 m/s
v2     =     0.0000 m/s

T1a    =    32.6307 s               S1a    =    85.4086 m
T1b    =   253.0022 s               S1b    =  3335.4822 m
T1     =   285.6329 s               S1     =  3420.8909 m
T2     =     0.0000 s               S2     =     0.0000 m
T3     =   919.0948 s               S3     =  6579.1128 m
T4     =     0.0000 s               S4     =     0.0000 m
T4a    =     0.0000 s               S4a    =     0.0000 m
T4b    =     0.0000 s               S4b    =     0.0000 m
T4c    =     0.0000 s               S4c    =     0.0000 m

U1a    =     0.1735 N/kg            E1a    =    14.8184 J/kg
U1b    =     0.0523 N/kg            E1b    =   228.8823 J/kg
U2     =     0.0337 N/kg            E2     =     0.0000 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     32.6       85.4    5.214208     0.052280   14.818391  Max. force B
    285.6     3420.9   17.304293     0.033678  243.700653  Const. speed
    285.6     3420.9   17.304293     0.000000  243.700653  Coasting
   1204.7    10000.0    0.000000    -2.250000  243.700653  Max. brake
   1204.7    10000.0    0.000000

Dist.  =    10.0000 km
Time   =  1204.7277 sec
E.C.   =    24.3701 J/kg/km
```

```
 Data of train
 -------------

Tmass  =    2.0000 Mkg
Dist.  =   10.0000 km
Time   = 1800.0000 sec
nLOC   =    1.0000
nBRK   =   15.0000
r0     =    0.0120 N/kg               n        =     0.8200
r1     =    0.0005 Ns/m/kg            n'       =     1.2000
r2     =   43.5000 uNs2/m2/kg         e        =     0.0578 MNs/m
nW     =    0.3470 MN                 Z        =     0.2670 MN
Pmax   =    2.2065 MW/locomotive      Bmax     =     0.3000 MN/air brake

Umin   =   -2.3835 N/kg               ABmax =       2.2500 N/kg
Umax   =    0.1735 N/kg               DBmax =       0.1335 N/kg
V1     =    4.6194 m/s                DBmax/V1 =       0.0289 Ns/m/kg
V2     =    9.9169 m/s                DBmax*V2 =       1.3239 W/kg
V3     =    5.2142 m/s                Umax*V3  =       0.9047 W/kg

 Switching strategy : Optimal
 ----------------------------

v1     =    6.1519 m/s
v2     =    1.0425 m/s

T1a    =   32.6307 s                 S1a    =     85.4086 m
T1b    =    6.5632 s                 S1b    =     37.3965 m
T1     =   39.1939 s                 S1     =    122.8051 m
T2     = 1404.4697 s                 S2     =   8640.1201 m
T3     =  355.8786 s                 S3     =   1236.8351 m
T4     =    0.4578 s                 S4     =      0.2402 m
T4a    =    0.4578 s                 S4a    =      0.2402 m
T4b    =    0.0000 s                 S4b    =      0.0000 m

U1a    =    0.1735 N/kg              E1a    =     14.8184 J/kg
U1b    =    0.1471 N/kg              E1b    =      5.9375 J/kg
U2     =    0.0167 N/kg              E2     =    144.4820 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     32.6       85.4    5.214208     0.147055   14.818391  Max. force B
     39.2      122.8    6.151874     0.016722   20.755896  Const. speed
   1443.7     8762.9    6.151874     0.000000  165.237869  Coasting
   1799.5     9999.8    1.042470    -2.280127  165.237869  Max. brake
   1800.0    10000.0    0.000000

Dist.  =   10.0000 km
Time   = 1800.0000 sec
E.C.   =   16.5238 J/kg/km
```

```
 Data of train
 -------------

Tmass =     2.0000 Mkg
Dist. =    10.0000 km
Time  = 1800.0000 sec
nLOC  =     1.0000
nBRK  =    15.0000
r0    =     0.0120 N/kg                n       =     0.8200
r1    =     0.0005 Ns/m/kg             n'      =     1.2000
r2    =    43.5000 uNs2/m2/kg          e       =     0.0578 MNs/m
uW    =     0.3470 MN                  Z       =     0.2670 MN
Pmax  =     2.2065 MW/locomotive       Bmax    =     0.3000 MN/air brake

Umin  =    -2.3835 N/kg                ABmax   =     2.2500 N/kg
Umax  =     0.1735 N/kg                DBmax   =     0.1335 N/kg
V1    =     4.6194 m/s                 DBmax/V1 =       0.0289 Ns/m/kg
V2    =     9.9169 m/s                 DBmax*V2 =       1.3239 W/kg
V3    =     5.2142 m/s                 Umax*V3  =       0.9047 W/kg

 Switching strategy : Max. thrust & brake
 --------------------

v1    =     5.6139 m/s
v2    =     5.6139 m/s

T1a   =    32.6307 s                   S1a    =    85.4086 m
T1b   =     2.6455 s                   S1b    =    14.3306 m
T1    =    35.2762 s                   S1     =    99.7392 m
T2    =  1762.3263 s                   S2     =  9893.6045 m
T3    =     0.0000 s                   S3     =     0.0000 m
T4    =     2.3975 s                   S4     =     6.6561 m
T4a   =     1.9830 s                   S4a    =     4.5341 m
T4b   =     0.4145 s                   S4b    =     2.1220 m

U1a   =     0.1735 N/kg                E1a    =    14.8184 J/kg
U1b   =     0.1611 N/kg                E1b    =     2.3933 J/kg
U2    =     0.0162 N/kg                E2     =   160.0581 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     32.6       85.4    5.214208     0.161146   14.818391  Max. force B
     35.3       99.7    5.613946     0.016178   17.211720  Const. speed
   1797.6     9993.3    5.613946     0.000000  177.269836  Coasting
   1797.6     9993.3    5.613946    -2.383500  177.269836  Max. brake
   1798.0     9995.5    4.619377    -2.383500  177.269836  Max. brake A
   1800.0    10000.0    0.000000

Dist. =    10.0000 km
Time  = 1800.0000 sec
E.C.  =    17.7270 J/kg/km
```

```
 Data of train
 -------------
Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   = 1800.0000 sec
nLOC   =     1.0000
nBRK   =    15.0000
r0     =     0.0120 N/kg               n      =    0.8200
r1     =     0.0005 Ns/m/kg            n'     =    1.2000
r2     =    43.5000 uNs2/m2/kg         e      =    0.0578 uNs/m
uW     =     0.3470 MN                 Z      =    0.2670 MN
Fmax   =     2.2065 MW/locomotive      Bmax   =    0.3000 MN/air brake

Umin   =    -2.3835 N/kg               ABmax  =    2.2500 N/kg
Umax   =     0.1735 N/kg               DBmax  =    0.1335 N/kg
V1     =     4.6194 m/s                DBmax/V1  =    0.0289 Ns/m/kg
V2     =     9.9169 m/s                DBmax*V2  =    1.3239 W/kg
V3     =     5.2142 m/s                Umax*V3   =    0.9047 W/kg

 Switching strategy : Max thrust & coast
 ---------------------
v1     =     6.5158 m/s
v2     =     0.0000 m/s

T1a    =    32.6307 s               S1a    =    85.4086 m
T1b    =     9.4540 s               S1b    =    55.7122 m
T1     =    42.0846 s               S1     =   141.1208 m
T2     = 1295.5831 s                S2     =  8441.8164 m
T3     =   462.3322 s               S3     =  1417.0626 m
T4     =     0.0000 s               S4     =     0.0000 m
T4a    =     0.0000 s               S4a    =     0.0000 m
T4b    =     0.0000 s               S4b    =     0.0000 m

U1a    =     0.1735 N/kg            E1a    =    14.8184 J/kg
U1b    =     0.1388 N/kg            E1b    =     8.5527 J/kg
U2     =     0.0171 N/kg            E2     =   144.3953 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     32.6       85.4    5.214208     0.138841   14.818391  Max. force B
     42.1      141.1    6.515843     0.017105   23.371063  Const. speed
   1337.7     8582.9    6.515843     0.000000  167.766373  Coasting
   1800.0    10000.0    0.000000    -2.250000  167.766373  Max. brake
   1800.0    10000.0    0.000000

Dist.  =    10.0000 km
Time   = 1800.0000 sec
E.C.   =    16.7766 J/kg/km
```

```
 Data of train
 -------------
Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   =  2400.0000 sec
nLOC   =     1.0000
nBKK   -    15.0000
r0     =     0.0120 N/kg                n       -      0.8200
r1     =     0.0005 Ns/m/kg             n'      -      1.2000
r2     =    43.5000 uNs2/m2/kg          e       -      0.0578 MNs/m
uW     =     0.3470 MN                  Z       -      0.2670 MN
Fmax   =     2.2065 MW/locomotive       Bmax    -      0.3000 MN/air brake

Umin   -    -2.3835 N/kg                ABmax  -       2.2500 N/kg
Umax   =     0.1735 N/kg                DBmax  -       0.1335 N/kg
V1     =     4.6194 m/s                 DBmax/V1  =      0.0289 Ns/m/kg
V2     =     9.9169 m/s                 DBmax*V2  -      1.3239 W/kg
V3     =     5.2142 m/s                 Umax*V3   -      0.9047 W/kg

 Switching strategy : Optimal
 ---------------------
v1     =     4.4258 m/s
v2     =     0.5963 m/s

T1     =    27.6433 s                  S1      -    61.3674 m
T2     =  2089.1902 s                  S2      =  9246.3955 m
T3     =   282.9038 s                  S3      =   692.1602 m
T4     =     0.2626 s                  S4      =     0.0776 m

U1a    =     0.1735 N/kg               E1a     -    10.6472 J/kg
U2     =     0.0151 N/kg               E2      -   139.2969 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0   0.000000    0.173500     0.000000  Max. force A
     27.6       61.4   4.425828    0.015065    10.647232  Const. speed
   2116.8     9307.8   4.425828    0.000000   149.944092  Coasting
   2399.7     9999.9   0.596255   -2.267232   149.944092  Max. brake
   2400.0    10000.0   0.000000

Dist.  =    10.0000 km
Time   =  2400.0000 sec
E.C.   =    14.9944 J/kg/km
```

```
 Data of train
 -------------

Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   = 2400.0000 sec
nLOC   =     1.0000
nBRK   =    15.0000
r0     =     0.0120 N/kg                n        =     0.8200
r1     =     0.0005 Ns/m/kg             n'       -     1.2000
r2     =    43.5000 uNs2/m2/kg          e        -     0.0578 MNs/m
nW     =     0.3470 MN                  Z        -     0.2670 MN
Pmax   =     2.2065 MW/locomotive       Bmax     -     0.3000 MN/air brake

Umin   =    -2.3835 N/kg                ABmax =        2.2500 N/kg
Umax   =     0.1735 N/kg                DBmax -        0.1335 N/kg
V1     =     4.6194 m/s                 DBmax/V1 =        0.0289 Ns/m/kg
V2     =     9.9169 m/s                 DBmax*V2 =        1.3239 W/kg
V3     =     5.2142 m/s                 Umax*V3  -        0.9042 W/kg

 Switching strategy : Max. thrust & brake
 ----------------------

v1     =     4.1910 m/s
v2     =     4.1910 m/s

T1     =    26.1623 s                 S1     =    54.9861 m
T2     = 2372.0337 s                 S2     -  9941.2686 m
T3     =     0.0000 s                 S3     =     0.0000 m
T4     =     1.8039 s                 S4     =     3.7467 m

U1a    =     0.1735 N/kg              E1a    =     9.5401 J/kg
U2     =     0.0149 N/kg              E2     =   147.7231 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)   Ctr. (N/kg) E.C. (J/kg)
 --------- ---------- -----------  ----------- -----------
      0.0        0.0    0.000000     0.173500     0.000000  Max. force A
     26.2       55.0    4.191031     0.014860     9.540095  Const. speed
   2398.2     9996.3    4.191031     0.000000   157.263199  Coasting
   2398.2     9996.3    4.191032    -2.371121   157.263199  Max. brake
   2400.0    10000.0    0.000000

Dist. =    10.0000 km
Time   = 2400.0000 sec
E.C.   =    15.7263 J/kg/km
```

```
 Data of train
 -------------
Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   = 2400.0000 sec
nLOC   =     1.0000
nBRK   =    15.0000
r0     =     0.0120 N/kg                n       =      0.8200
r1     =     0.0005 Ns/m/kg             n'      =      1.2000
r2     =    43.5000 uNs2/m2/kg          e       =      0.0578 MWs/m
nW     =     0.3470 MN                  Z       =      0.2620 MW
Fmax   =     2.2065 MW/locomotive       Bmax           0.3000 MN/air brake

Umin   =    -2.3835 N/kg                ABmax  =       2.2500 N/kg
Umax   =     0.1735 N/kg                DBmax  =       0.1335 N/kg
V1     =     4.6194 m/s                 DBmax/V1 =      0.0289 Ns/m/kg
V2     =     9.9169 m/s                 DBmax*V2 =      1.3239 W/kg
V3     =     5.2142 m/s                 Umax*V3  =      0.9047 W/kg

 Switching strategy : Max thrust & coast
 -------------------
v1     =     4.5247 m/s
v2     =     0.0000 m/s

T1     =    28.2679 s                   S1     =    64.1629 m
T2     = 2033.2211 s                    S2     =  9199.8135 m
T3     =   338.5111 s                   S3     =   736.0239 m
T4     =     0.0000 s                   S4     =     0.0000 m

U1a    =     0.1735 N/kg                E1a    =    11.1323 J/kg
U2     =     0.0152 N/kg                E2     =   139.4045 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
     0.0        0.0    0.000000    0.173500     0.000000  Max. force A
    28.3       64.2    4.524748    0.015153    11.132264  Const. speed
  2061.5     9264.0    4.524748    0.000000   150.536728  Coasting
  2400.0    10000.0    0.000000   -2.250000   150.536728  Max. brake
  2400.0    10000.0    0.000000

Dist.  =    10.0000 km
Time   = 2400.0002 sec
E.C.   =    15.0537 J/kg/km
```

```
 Data of train
 -------------

Tmass =     2.0000 Mkg
Dist. =    10.0000 km
Time  = 3000.0000 sec
nLOC  =     1.0000
nBRK  =    15.0000
r0    =     0.0120 N/kg                n      =      0.8200
r1    =     0.0005 Ns/m/kg             n'     =      1.2000
r2    =    43.5000 uNs2/m2/kg          e      =      0.0578 MNs/m
uW    =     0.3470 MN                  Z      =      0.2670 MN
Fmax  =     2.2065 MW/locomotive       Bmax   =      0.3000 MN/air brake

Umin  =    -2.3835 N/kg                ABmax =       2.2500 N/kg
Umax  =     0.1735 N/kg                DBmax =       0.1335 N/kg
V1    =     4.6194 m/s                 DBmax/V1 =       0.0289 Ns/m/kg
V2    =     9.9169 m/s                 DBmax*V2 =       1.3239 W/kg
V3    =     5.2142 m/s                 Umax*V3  =       0.9047 W/kg

 Switching strategy : Optimal
 --------------------

v1    =     3.4695 m/s
v2    =     0.3891 m/s

T1    =    21.6231 s                   S1     =     37.5985 m
T2    = 2743.6685 s                    S2     =  9519.2607 m
T3    =  234.5369 s                    S3     =   443.1068 m
T4    =     0.1716 s                   S4     =     0.0340 m

U1a   =     0.1735 N/kg                E1a    =      6.5233 J/kg
U2    =     0.0143 N/kg                E2     =    135.7295 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force a
     21.6       37.6    3.469537     0.014258    6.523338  Const. speed
   2765.3     9556.9    3.469537     0.000000  142.252838  Coasting
   2999.8    10000.0    0.389077    -2.261244  142.252838  Max. brake
   3000.0    10000.0    0.000000

Dist. =    10.0000 km
Time  = 3000.0000 sec
E.C.  =    14.2253 J/kg/km
```

```
 Data of train
 -------------
Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   = 3000.0000 sec
nLOC   =     1.0000
nBRK   =    15.0000
r0     =     0.0120 N/kg              n       =     0.8200
r1     =     0.0005 Ns/m/kg           n'      =     1.2000
r2     =    43.5000 uNs2/m2/kg        p       =     0.0578 MNs/m
gW     =     0.3470 MN                Z       =     0.2670 MN
Fmax   =     2.2065 MW/locomotive     Bmax    =     0.3000 MN/air brake

Umin   =    -2.3835 N/kg              ABmax   =     2.2500 N/kg
Umax   =     0.1735 N/kg              DBmax   =     0.1335 N/kg
V1     =     4.6194 m/s               DBmax/V1  =     0.0289 Ns/m/kg
V2     =     9.9169 m/s               DBmax*V2  =     1.3239 W/kg
V3     =     5.2142 m/s               Umax*V3   =     0.9047 W/kg

 Switching strategy : Max. thrust & brake
 -------------------
v1     =     3.3457 m/s
v2     =     3.3457 m/s

T1     =    20.8459 s                S1      =      34.9500 m
T2     = 2977.7063 s                S2      =    9962.6445 m
T3     =     0.0000 s                S3      =       0.0000 m
T4     =     1.4477 s                S4      =       2.4053 m

U1a    =     0.1735 N/kg             E1a     =       6.0638 J/kg
U2     =     0.0142 N/kg             E2      =     141.0692 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     20.8       34.7    3.345744     0.014160    6.063821  Const. speed
   2998.6     9997.6    3.345744     0.000000  147.132996  Coasting
   2998.6     9997.6    3.345744    -2.346692  147.132996  Max. brake
   3000.0    10000.0    0.000000

Dist.  =    10.0000 km
Time   = 3000.0000 sec
E.C.   =    14.7133 J/kg/km
```

```
 Data of train
 -------------

Tmass =     2.0000 Mkg
Dist. =    10.0000 km
Time  - 3000.0000 sec
nLOC  =     1.0000
nBRK  =    15.0000
r0    =     0.0120 N/kg              n      -      0.8200
r1    =     0.0005 Ns/m/kg           n'     =      1.2000
r2    =    43.5000 uNs2/m2/kg        e      -      0.0578 MNs/m
uW    =     0.3470 MN                Z      -      0.2670 MN
Pmax  =     2.2065 MW/locomotive     Bmax   =      0.3000 MN/air brake

Umin  =    -2.3835 N/kg              ABmax =       2.2500 N/kg
Umax  =     0.1735 N/kg              DBmax -       0.1335 N/kg
V1    =     4.6194 m/s               DBmax/V1 -       0.0289 Ns/m/kg
V2    =     9.9169 m/s               DBmax*V2 -       1.3249 W/kg
V3    =     5.2142 m/s               Umax*V3  =       0.9047 W/kg

 Switching strategy : Max thrust & coast
 -------------------

v1    =     3.5082 m/s
v2    =     0.0000 m/s

T1    =    21.8659 s                 S1     -     38.4463 m
T2    = 2708.7314 s                  S2     = 9502.7607 m
T3    =   269.4026 s                 S3     -   458.7944 m
T4    =     0.0000 s                 S4     =     0.0000 m

U1a   =     0.1735 N/kg              E1a    =      6.6704 J/kg
U2    =     0.0143 N/kg              E2     =  135.7894 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000 Max. force A
     21.9       38.4    3.508196     0.014289    6.670440 Const. speed
   2730.6     9541.2    3.508196     0.000000  142.459854 Coasting
   3000.0    10000.0    0.000000    -2.250000  142.459854 Max. brake
   3000.0    10000.0    0.000000

Dist. =    10.0000 km
Time  = 3000.0000 sec
E.C.  =    14.2460 J/kg/km
```

```
 Data of train
 -------------
Tmass =    2.0000 Mkg
Dist. =   10.0000 km
Time  = 3600.0000 sec
nLOC  =    1.0000
nBRK  =   15.0000
r0    =    0.0120 N/kg              n      =     0.8200
r1    =    0.0005 Ns/m/kg           n'     =     1.2000
r2    =   43.5000 uNs2/m2/kg        e      =     0.0578 MNs/m
oW    =    0.3470 MN                Z      =     0.2670 MN
Fmax  =    2.2065 MW/locomotive     Bmax   =     0.3000 MN/air brake

Umin  =   -2.3835 N/kg              ABmax =     2.2500 W/kg
Umax  =    0.1735 N/kg              DBmax -     0.1335 W/kg
V1    =    4.6194 m/s               DBmax/V1 =     0.0289 Ns/m/kg
V2    =    9.9169 m/s               DBmax*V2 =     1.3239 W/kg
V3    =    5.2142 m/s               Umax*V3  -     0.9047 W/kg

 Switching strategy : Optimal
 ----------------------------
v1    =    2.8583 m/s
v2    =    0.2749 m/s

T1    =   17.7905 s                S1     =    25.4720 m
T2    = 3381.7769 s                S2     -  9666.1768 m
T3    =  200.3112 s                S3     -   308.3336 m
T4    =    0.1213 s                S4     =     0.0182 m

U1a   =    0.1735 N/kg             E1a    -     4.4194 J/kg
U2    =    0.0138 N/kg             E2     -   133.2439 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     17.8       25.5    2.858313     0.013785    4.419400  Const. speed
   3399.6     9691.6    2.858313     0.000000  137.663300  Coasting
   3599.9    10000.0    0.274929    -2.257946  137.663300  Max. brake
   3600.0    10000.0    0.000000

Dist. =   10.0000 km
Time  = 3600.0000 sec
F.C.  =   13.7663 J/kg/km
```

```
 Data of train
 -------------
Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   =  3600.0000 sec
nLOC   =     1.0000
nBRK   =    15.0000
r0     =     0.0120 N/kg                n        =    0.8200
r1     =     0.0005 Ns/m/kg             n'       =    1.2000
r2     =    43.5000 uNs2/m2/kg          e        =    0.0578 MNs/m
nW     =     0.3470 MN                  Z        =    0.2670 MN
Fmax   =     2.2065 MW/locomotive       Bmax     =    0.3000 MN/air brake

Umin   =    -2.3835 N/kg                ABmax    =    2.2500 N/kg
Umax   =     0.1735 N/kg                DBmax    =    0.1335 N/kg
V1     =     4.6194 m/s                 DBmax/V1   =     0.0289 Ns/m/kg
V2     =     9.9169 m/s                 DBmax*V2   =     1.3239 W/kg
V3     =     5.2142 m/s                 Umax*V3    =     0.9047 W/kg

 Switching strategy : Max. thrust & brake
 ------------------------
v1     =     2.7849 m/s
v2     =     2.7849 m/s

T1     =    17.3312 s                  S1      =    24.1762 m
T2     =  3581.4595 s                  S2      =  9974.1494 m
T3     =     0.0000 s                  S3      =     0.0000 m
T4     =     1.2094 s                  S4      =     1.6748 m

U1a    =     0.1735 N/kg               E1a     =     4.1946 J/kg
U2     =     0.0137 N/kg               E2      =   136.9436 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) F.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.173500    0.000000  Max. force A
     17.3       24.2    2.784940     0.013730    4.194521  Const. speed
   3598.8     9998.3    2.784940     0.000000  141.138153  Coasting
   3598.8     9998.3    2.784940    -2.330485  141.138153  Max. brake
   3600.0    10000.0    0.000000

Dist.  =   10.0000 km
Time   = 3600.0000 sec
F.C.   =   14.1138 J/kg/km
```

```
 Data of train
 --------------
Tmass  =     2.0000 Mkg
Dist.  =    10.0000 km
Time   =  3600.0000 sec
nLOC   =     1.0000
nBRK   =    15.0000
r0     =     0.0120 N/kg              n        =     0.8200
r1     =     0.0005 Ns/m/kg           n'       =     1.2000
r2     =    43.5000 uNs2/m2/kg        e        =     0.0578 mN/m
nW     =     0.3470 MN                Z        =     0.2670 MN
Fmax   =     2.2065 MW/locomotive     Bmax           0.3000 MN/car brake

Umin   =    -2.3835 N/kg              ABmax  =       2.2500 N/kg
Umax   =     0.1735 N/kg              DBmax  =       0.1335 N/kg
V1     =     4.6194 m/s               DBmax/V1  =       0.0289 Ns/m/kg
V2     =     9.9169 m/s               DBmax*V2          1.3239 W/kg
V3     =     5.2142 m/s               Umax*V3   =       0.9047 W/kg

 Switching strategy : Max thrust & coast
 ---------------------

v1     =     2.8767 m/s
v2     =     0.0000 m/s

T1     =    17.9054 s                 S1     =    25.8015 m
T2     =  3357.6736 s                 S2     =  9658.9219 m
T3     =   224.4208 s                 S3     =   315.2759 m
T4     =     0.0000 s                 S4     =     0.0000 m

U1a    =     0.1735 N/kg              E1a    =     4.4766 J/kg
U2     =     0.0138 N/kg              E2     =   133.2768 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- ----------
      0.0        0.0    0.000000     0.173500    0.000000 Max. force a
     17.9       25.8    2.876671     0.013798    4.476553 Const. speed
   3375.6     9684.7    2.876671     0.000000  137.753326 Coasting
   3600.0    10000.0    0.000000    -2.250000  137.753326 Max. brake
   3600.0    10000.0    0.000000

Dist.  =    10.0000 km
Time   =  3600.0000 sec
E.C.   =    13.7753 J/kg/km
```

ANNEX C

SIMULATION RESULTS FOR A SHORT PASSENGER TRAIN TRAVELLING FROM KINGSTON TO TORONTO

```
 Schedule
 --------
Dist. =  233.5400 km        Kingston → Gildwood
Time  =  108.0000 min

 Data of train
 -------------
Tmass  =     0.3400 Mkg
nLOC   -     1.0000
nBRK   -     5.0000
r0     =     0.0162 N/kg              n      -       0.7248
r1     =     0.0003 Ns/m/kg           n'     -       1.1600
r2     =   33.6000 uNs2/m2/kg         e      -       0.0578 MNs/m
nW     =     0.2000 MN                Z      -       0.2670 MN
Fmax   =     2.7777 MW/locomotive     Bmax   -       0.3000 MN/air brake

Umin   =    -5.1971 N/kg              ABmax  -       4.4118 N/kg
Umax   =     0.5882 N/kg              DBmax  -       0.7853 N/kg
V1     =     4.6194 m/s               DBmax/V1  -       0.1700 Ns/m/kg
V2     =    12.0679 m/s               DBmax*V2  -       9.4769 W/kg
V3     -    10.0664 m/s               Umax*V3   -       5.9214 W/kg

 Switching strategy : Optimal
 --------------------
v1     -    37.8744 m/s              Vmax    -    50.1959 m/s
v2     =    11.6139 m/s

T1a    -    17.6869 s                S1a     -      0.0892 km
T1b    =   155.0849 s                S1b     =      4.3411 km
T1     =   172.7718 s                S1      =      4.4303 km
T2     =  5683.8208 s                S2      -    215.2711 km
T3     =   621.1068 s                S3      -     13.8255 km
T4     =     2.3005 s                S4      =      0.0130 km
T4a    =     0.9602 s                S4a     =      0.0022 km
T4b    =     1.3403 s                S4b     -      0.0109 km
T4c    =     0.0000 s                S4c     =      0.0000 km

U1a    =     0.5882 N/kg             E1a     =     52.4707 J/kg
U1b    =     0.1563 N/kg             E1b     -    918.3199 J/kg
U2     =     0.0773 N/kg             E2     -16643.7695 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000   0.588235     0.000000  Max. force A
     17.7       89.2   10.066384   0.156343    52.470718  Max. force B
    172.8     4430.3   37.874363   0.077315   970.790588  Const. speed
   5856.6   219701.4   37.874363   0.00000017614.560547  Coasting
   6477.7   233527.0   11.613857  -5.19705917614.560547  Max. brake
   6479.0   233537.8    4.619377  -5.19705917614.560547  Max. brake A
   6480.0   233540.0    0.000000

Dist. =  233.5400 km
Time  = 6480.0000 sec
E.C.  =    75.4242 J/kg/km
```

```
 Schedule
 --------
Dist. =  233.5400 km
Time  =  108.0000 min

 Data of train
 -------------
Tmass  =     0.3400 Mkg
nLOC   =     1.0000
nBRK   =     5.0000
r0     =     0.0162 N/kg              n       =      0.7248
r1     =     0.0003 Ns/m/kg           n'      =      1.1600
r2     =    33.6000 uNs2/m2/kg        e       =      0.0578 MNs/m
uW     =     0.2000 MN                Z       =      0.2670 MN
Fmax   =     2.7777 MW/locomotive     Bmax    =      0.3000 MN/air brake

Umin   =    -5.1971 N/kg              ABmax =        4.4118 N/kg
Umax   =     0.5882 N/kg              DBmax =        0.7853 N/kg
V1     =     4.6194 m/s               DBmax/V1  =      0.1700 Ns/m/kg
V2     =    12.0679 m/s               DBmax*V2  =      9.4769 W/kg
V3     =    10.0664 m/s               Umax*V3   =      5.9214 W/kg

 Switching strategy : Max. thrust & brake
 --------------------
v1     =    36.3481 m/s               Vmax   =    50.1959 m/s
v2     =    36.3481 m/s

T1a    =    17.6869 s                 S1a    =      0.0892 km
T1b    =   136.9852 s                 S1b    =      3.6691 km
T1     =   154.6721 s                 S1     =      3.7583 km
T2     =  6317.9697 s                 S2     =    229.6464 km
T3     =     0.0000 s                 S3     =      0.0000 km
T4     =     7.3581 s                 S4     =      0.1353 km
T4a    =     0.9602 s                 S4a    =      0.0022 km
T4b    =     1.4273 s                 S4b    =      0.0119 km
T4c    =     4.9707 s                 S4c    =      0.1212 km

U1a    =     0.5882 N/kg              E1a    =     52.4707 J/kg
U1b    =     0.1629 N/kg              E1b    =    811.1447 J/kg
U2     =     0.0730 N/kg              E2     =16761.9473 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000   0.588235     0.000000  Max. force A
     17.7       89.2   10.066384   0.162908    52.470218  Max. force B
    154.7     3758.3   36.348133   0.072990   863.615356  Const. speed
   6472.6   233404.8   36.348133   0.00000017625.562500  Coasting
   6472.6   233404.8   36.348133  -4.41644117625.562500  Max. brake
   6477.6   233525.9   12.067910  -5.19705917625.562500  Max. brake B
   6479.0   233537.8    4.619377  -5.19705917625.562500  Max. brake A
   6480.0   233540.0    0.000000

Dist. =  233.5400 km
Time  = 6480.0000 sec
E.C.  =    75.4713 J/kg/km
```

```
 Schedule
 --------
Dist. =  233.5400 km
Time  =  108.0000 min

 Data of train
 -------------
Tmass =     0.3400 Mkg
mLOC  =     1.0000
mBRK  =     5.0000
r0    =     0.0162 N/kg              n       =    0.7248
r1    =     0.0003 Ns/m/kg           n'      =    1.1600
r2    =    33.6000 uNs2/m2/kg        e       =    0.0578 MNs/m
mW    =     0.2000 MN                Z       =    0.2670 MN
Fmax  =     2.7777 MW/locomotive     Bmax    =    0.3000 MN/air brake

Umin  =    -5.1971 N/kg              ABmax =      4.4118 N/kg
Umax  =     0.5882 N/kg              DBmax =      0.7853 N/kg
V1    =     4.6194 m/s               DBmax/V1 =     0.1700 Ns/m/kg
V2    =    12.0679 m/s               DBmax*V2 =     9.4769 W/kg
V3    =    10.0664 m/s               Umax*V3  =     5.9214 W/kg

 Switching strategy : Max thrust & coast
 ---------------------
v1    =    41.7144 m/s               Vmax    =   50.1959 m/s
v2    =     0.0000 m/s

T1a   =    17.6869 s                 S1a     =     0.0892 km
T1b   =   214.1292 s                 S1b     =     6.6983 km
T1    =   231.8161 s                 S1      =     6.7875 km
T2    = 4982.8784 s                  S2      =   207.8579 km
T3    = 1265.3057 s                  S3      =    18.8946 km
T4    =     0.0000 s                 S4      =     0.0000 km
T4a   =     0.0000 s                 S4a     =     0.0000 km
T4b   =     0.0000 s                 S4b     =     0.0000 km
T4c   =     0.0000 s                 S4c     =     0.0000 km

U1a   =     0.5882 N/kg              E1a     =    52.4707 J/kg
U1b   =     0.1420 N/kg              E1b     = 1267.9449 J/kg
U2    =     0.0889 N/kg              E2      =18476.4941 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0   0.000000    0.588235    0.000000  Max. force A
     17.7       89.2  10.066384    0.141951   52.470718  Max. force B
    231.8     6787.5  41.714420    0.088890 1320.415649  Const. speed
   5214.7   214645.4  41.714420    0.00000019796.910156  Coasting
   6480.0   233540.0   0.000000   -4.41176519796.910156  Max. brake
   6480.0   233540.0   0.000000

Dist. =  233.5400 km
Time  = 6480.0000 sec
E.C.  =    84.7688 J/kg/km
```

Gildwood -> Toronto

```
 Schedule
 --------
Dist. =   20.3400 km
Time  =   15.0000 min

 Data of train
 -------------
Tmass =    0.3400 Mkg
nLOC  =    1.0000
nBRK  =    5.0000
r0    =    0.0162 N/kg              n      =     0.7248
r1    =    0.0003 Ns/m/kg           n'     =     1.1600
r2    =   33.6000 uNs2/m2/kg        e      =     0.0578 MNs/m
uW    =    0.2000 MN                Z      =     0.2670 MW
Fmax  =    2.7777 MW/locomotive     Bmax   =     0.3000 MN/air brake

Umin  =   -5.1971 N/kg              ABmax  =     4.4118 N/kg
Umax  =    0.5882 N/kg              DBmax  =     0.7853 N/kg
V1    =    4.6194 m/s               DBmax/V1 =     0.1700 Ns/m/kg
V2    =   12.0679 m/s               DBmax*V2 =     9.4769 W/kg
V3    =   10.0664 m/s               Umax*V3  =     5.9214 W/kg

 Switching strategy : Optimal
 ----------------------------
v1    =   34.5457 m/s               Vmax   -   50.1959 m/s
v2    =   10.2126 m/s

T1a   =   17.6869 s                 S1a    =    0.0892 km
T1b   =  118.2905 s                 S1b    =    3.0061 km
T1    =  135.9774 s                 S1     =    3.0953 km
T2    =  128.2635 s                 S2     =    4.4310 km
T3    =  633.7269 s                 S3     =   12.8036 km
T4    =    2.0321 s                 S4     =    0.0101 km
T4a   =    0.9602 s                 S4a    =    0.0022 km
T4b   =    1.0720 s                 S4b    =    0.0079 km
T4c   =    0.0000 s                 S4c    =    0.0000 km

U1a   =    0.5882 N/kg              E1a    =   52.4207 J/kg
U1b   =    0.1714 N/kg              E1b    =  700.4457 J/kg
U2    =    0.0681 N/kg              E2     =  301.6777 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000    0.588235     0.000000  Max. force A
     17.7       89.2   10.066384    0.171408    52.420718  Max. force B
    136.0     3095.3   34.545746    0.068064   752.916443  Const. speed
    264.2     7526.2   34.545746    0.000000  1054.594238  Coasting
    898.0    20329.9   10.212626   -5.197059  1054.594238  Max. brake
    899.0    20332.8    4.619327   -5.197059  1054.594238  Max. brake A
    900.0    20340.0    0.000000

Dist. =   20.3400 km
Time  =  900.0001 sec
E.C.  =   51.8483 J/kg/km
```

```
Schedule
--------
Dist. =    20.3400 km
Time  =    15.0000 min

 Data of train
 -------------
Tmass =     0.3400 Mkg
nLOC  =     1.0000
nBRK  =     5.0000                          n               0.7248
r0    =     0.0162 N/kg                     n'              1.1600
r1    =     0.0003 Ns/m/kg                  e       -       0.0578 MNs/m
r2    =    33.6000 uNs2/m2/kg               Z       =       0.2670 MN
uW    =     0.2000 MN                       Bmax            0.3000 MN/air brake
Pmax  =     2.7777 MW/locomotive
                                            ABmax  =        4.4118 N/kg
Umin  =    -5.1971 N/kg                     DBmax  -        0.7853 N/kg
Umax  =     0.5882 N/kg                     DBmax/V1 =         0.1700 Ns/m/kg
V1    =     4.6194 m/s                      DBmax*V2 =         9.4769 W/kg
V2    =    12.0679 m/s                      Umax*V3  =         5.9214 W/kg
V3    =    10.0664 m/s

 Switching strategy : Max. thrust & brake
 ---------------------
                                            Vmax   -    50.1959 m/s
v1    =    23.2753 m/s
v2    =    23.2753 m/s
                                            S1a     =      0.0892 km
T1a   =    17.6869 s                        S1b     -      0.7333 km
T1b   =    41.4459 s                        S1      -      0.8225 km
T1    =    59.1329 s                        S2      =     19.4637 km
T2    =   836.2374 s                        S3      -      0.0000 km
T3    =     0.0000 s                        S4      =      0.0538 km
T4    =     4.6297 s                        S4a     -      0.0022 km
T4a   =     0.9602 s                        S4b     -      0.0119 km
T4b   =     1.4273 s                        S4c     :      0.0398 km
T4c   =     2.2423 s
                                            E1a     =     52.4707 J/kg
U1a   =     0.5882 N/kg                     F1b     -    245.4181 J/kg
U1b   =     0.2544 N/kg                     E2      =    824.4026 J/kg
U2    =     0.0424 N/kg

 Time (s)   Dist. (m)   Vel. (m/s)   Ctr. (N/kg) E.C. (J/kg)
 --------   ---------   ----------   ----------- -----------
      0.0         0.0     0.000000     0.588235     0.000000  Max. force A
     17.7        89.2    10.066384     0.254407    52.470718  Max. force B
     59.1       822.5    23.275274     0.042356   297.888794  Const. speed
    895.4     20286.2    23.275274     0.000000  1122.291504  Coasting
    895.4     20286.2    23.275274    -4.419068  1122.291504  Max. brake
    897.6     20325.9    12.067910    -5.197059  1122.291504  Max. brake B
    899.0     20337.8     4.619377    -5.197059  1122.291504  Max. brake A
    900.0     20340.0     0.000000

Dist. =   20.3400 km
Time  =  900.0000 sec
F.C.  -   55.1766 J/kg/km
```

```
 Schedule
 --------
Dist. =    20.3400 km
Time  =    15.0000 min

 Data of train
 -------------
Tmass =     0.3400 Mkg
nLOC  =     1.0000
nBRK  =     5.0000
r0    =     0.0162 N/kg                n       =    0.7249
r1    =     0.0003 Ns/m/kg             n'      =    1.1600
r2    =    33.6000 uNs2/m2/kg          e       =    0.0578 MNs/m
uW    =     0.2000 MN                  Z       =    0.2670 MN
Fmax  =     2.7777 MW/locomotive       Bmax    =    0.3000 MN/air brake

Umin  =    -5.1971 N/kg                ABmax =      4.4118 N/kg
Umax  =     0.5882 N/kg                DBmax =      0.7853 N/kg
V1    =     4.6194 m/s                 DBmax/V1  =     0.1700 Ns/m/kg
V2    =    12.0679 m/s                 DBmax*V2  =     9.4769 W/kg
V3    =    10.0664 m/s                 Umax*V3   =     5.9214 W/kg

 Switching strategy : Max thrust & coast
 --------------------
v1    =    36.6591 m/s                 Vmax    =   50.1959 m/s
v2    =     0.0000 m/s

T1a   =    17.6869 s                   S1a     =    0.0892 km
T1b   =   140.4880 s                   S1b     =    3.7970 km
T1    =   158.1749 s                   S1      =    3.8862 km
T2    =     0.0000 s                   S2      =    0.0000 km
T3    =  1202.8943 s                   S3      =   16.4538 km
T4    =     0.0000 s                   S4      =    0.0000 km
T4a   =     0.0000 s                   S4a     =    0.0000 km
T4b   =     0.0000 s                   S4b     =    0.0000 km
T4c   =     0.0000 s                   S4c     =    0.0000 km

U1a   =     0.5882 N/kg                E1a     =   52.4707 J/kg
U1b   =     0.1615 N/kg                E1b     =  831.8860 J/kg
U2    =     0.0739 N/kg                E2      =    0.0000 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000     0.588235    0.000000  Max. force A
     17.7       89.2   10.066384     0.161526   52.470718  Max. force B
    158.2     3886.2   36.659130     0.073859  884.356689  Const. speed
    158.2     3886.2   36.659130     0.000000  884.356689  Coasting
   1361.1    20340.0    0.000000    -4.411765  884.356689  Max. brake
   1361.1    20340.0    0.000000

Dist. =    20.3400 km
Time  = 1361.0692 sec
F.C.  =    43.4787 J/kg/km
```

22.7 min.

ANNEX D

SIMULATION RESULTS FOR A LONG FREIGHT TRAIN FROM CALGARY TO FIELD

Calgary → Field

```
 Schedule
 --------
Dist. =  219.9800 km
Time  =  320.0000 min

 Data of train
 -------------
Tmass  =     5.9800 Mkg
nLOC   =     3.0000
nBRK   =   100.0000
r0     =     0.0154 N/kg                 n        =     0.8000
r1     =     0.0003 Ns/m/kg              n'       =     1.1500
r2     =    18.2000 uNs2/m2/kg           c        =     0.0578 MN/m
uW     =     0.3520 MN                   Z        =     0.2670 MN
Pmax   =     2.2371 MW/locomotive        Bmax     =     0.3000 MN/[illegible]

Umin   =    -5.1507 N/kg                 ABmax    =     5.0187 N/kg
Umax   =     0.0589 N/kg                 DBmax    =     0.1339 N/kg
V1     =     4.6194 m/s                  DBmax/V1 =       0.0290 Ns/m/kg
V2     =     9.7192 m/s                  DBmax*V2 =       1.3019 W/kg
V3     =     5.0843 m/s                  Umax*V3  =       0.2993 W/kg

 Switching strategy : Optimal
 ----------------------------
v1     =    11.7192 m/s                  Vmax   =   13.2608 m/s
v2     =     1.8376 m/s

T1a    =   119.5029 s                    S1a    =     0.3062 km
T1b    =   525.5540 s                    S1b    =     5.0303 km
T1     =   645.0569 s                    S1     =     5.3364 km
T2     =-18013.9102 s                    S2     =   211.1081 km
T3     =   540.6702 s                    S3     =     3.5351 km
T4     =     0.3632 s                    S4     =     0.0003 km
T4a    =     0.3632 s                    S4a    =     0.0003 km
T4b    =     0.0000 s                    S4b    =     0.0000 km
T4c    =     0.0000 s                    S4c    =     0.0000 km

U1a    =     0.0589 N/kg                 E1a    =    18.0225 J/kg
U1b    =     0.0255 N/kg                 E1b    =   157.2865 J/kg
U2     =     0.0214 N/kg                 E2     =  4518.8374 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)   Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------   ----------- -----------
     0.0        0.0    0.000000     0.058863      0.000000  Max. force A
   119.5      306.2    5.084318     0.025537     18.022339  Max. force B
   645.1     5336.4   11.719172     0.021405    175.308884  Const. speed
 18659.0   216444.5   11.719172     0.000000   4694.146484  Coasting
 19199.6   219979.7    1.837597    -5.070006   4694.146484  Max. brake
 19200.0   219980.0    0.000000

Dist. =  219.9800 km
Time  =19200.0000 sec
F.C.  =   21.3390 J/kg/km
```

```
 Schedule
 --------
Dist. =  219.9800 km
Time  =  320.0000 min

 Data of train
 -------------
Tmass =     5.9800 Mkg
nLOC  =     3.0000
nBRK  =   100.0000
r0    =     0.0154 N/kg           η      =    0.8000
r1    =     0.0003 Ns/m/kg        η'     =    1.1600
r2    =    18.2000 uNs2/m2/kg     e      =    0.0578 MNs/m
nW    =     0.3520 MN             Z      =    0.2670 MN
Pmax  =     2.2371 MW/locomotive  Bmax   =    0.3000 MN/car brake

Umin  =    -5.1507 N/kg           ABmax  =    5.0167 N/kg
Umax  =     0.0589 N/kg           DBmax  =    0.1339 N/kg
V1    =     4.6194 m/s            DBmax/V1  =    0.0290 Ns/m/kg
V2    =     9.7192 m/s            DBmax*V2  =    1.3019 W/kg
V3    =     5.0843 m/s            Umax*V3   =    0.2993 W/kg

 Switching strategy : Max. thrust & brake
 --------------------
v1    =    11.5688 m/s           Vmax   =   13.2608 m/s
v2    =    11.5688 m/s

T1a   =   119.5029 s             S1a    =     0.3062 km
T1b   =   490.9811 s             S1b    =     4.6276 km
T1    =   610.4840 s             S1     =     4.9338 km
T2    =18587.2656 s              S2     =   215.0432 km
T3    =     0.0000 s             S3     =     0.0000 km
T4    =     2.2509 s             S4     =     0.0130 km
T4a   =     0.9058 s             S4a    =     0.0021 km
T4b   =     0.9866 s             S4b    =     0.0071 km
T4c   =     0.3585 s             S4c    =     0.0038 km

U1a   =     0.0589 N/kg          E1a    =    18.0223 J/kg
U1b   =     0.0259 N/kg          E1b    =   146.9396 J/kg
U2    =     0.0213 N/kg          E2     =  4579.4570 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000   0.058863    0.000000  Max. force A
    119.5      306.2    5.084318   0.025869   18.022339  Max. force B
    610.5     4933.8   11.568846   0.021297  164.961975  Const. speed
  19197.8   219967.0   11.568846   0.000000 4744.418945  Coasting
  19197.8   219967.0   11.568846  -5.019228 4744.418945  Max. brake
  19198.1   219970.0    9.719236  -5.150669 4744.418945  Max. brake B
  19199.1   219977.9    4.619377  -5.150669 4744.418945  Max. brake A
  19200.0   219980.0    0.000000

Dist. =  219.9800 km
Time  =19200.0000 sec
E.C.  =    21.5675 J/kg/km
```

```
 Schedule
 --------
Dist. =  219.9800 km
Time  =  320.0000 min

 Data of train
 -------------
Tmass =     5.9800 Mkg
nLOC  =     3.0000
nBKK  =   100.0000
r0    =     0.0154 N/kg                 n       =     0.8000
r1    =     0.0003 Ns/m/kg              n'      =     1.1600
r2    =    18.2000 uNs2/m2/kg           e       =     0.0578 MNs/m
nW    =     0.3520 MN                   Z       =     0.2670 kN
Fmax  =     2.2371 MW/locomotive        Bmax    =     0.3000 MN/air brake

Umin  =    -5.1507 N/kg                 ABmax   =     5.0167 N/kg
Umax  =     0.0589 N/kg                 DBmax   =     0.1339 N/kg
V1    =     4.6194 m/s                  DBmax/V1  =     0.0290 Ns/m/kg
V2    =     9.7192 m/s                  DBmax*V2  =     1.3019 W/kg
V3    =     5.0843 m/s                  Umax*V3   =     0.2993 W/kg

 Switching strategy : Max thrust & coast
 -----------------------
v1    =    11.7897 m/s                Vmax    =    13.2608 m/s
v2    =     0.0000 m/s

T1a   =   119.5029 s                  S1a     =     0.3062 km
T1b   =   543.0685 s                  S1b     =     5.2361 km
T1    =   662.5714 s                  S1      =     5.5423 km
T2    =17876.3027 s                   S2      =   210.7569 km
T3    =   661.1257 s                  S3      =     3.6808 km
T4    =     0.0000 s                  S4      =     0.0000 km
T4a   =     0.0000 s                  S4a     =     0.0000 km
T4b   =     0.0000 s                  S4b     =     0.0000 km
T4c   =     0.0000 s                  S4c     =     0.0000 km

U1a   =     0.0589 N/kg               E1a     =    18.0224 J/kg
U1b   =     0.0254 N/kg               E1b     =   162.5282 J/kg
U2    =     0.0215 N/kg               E2      =  4522.1440 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000    0.058863    0.000000  Max. force A
    119.5      306.2    5.084318    0.025385   18.022339  Max. force B
    662.6     5542.3   11.789737    0.021457  180.550568  Const. speed
  18538.9   216299.2   11.789737    0.000000 4702.694824  Coasting
  19200.0   219980.0    0.000000   -5.016722 4702.694824  Max. brake
  19200.0   219980.0    0.000000

Dist. =  219.9800 km
Time  =19200.0000 sec
E.C.  =    21.3778 J/kg/km
```

```
 Schedule
 --------
Dist. =  110.5600 km
Time  =  164.4000 min
```

Calgary → Canmore

```
 Data of train
 -------------
Tmass =     5.9800 Mkg
nLOC  =     3.0000
nBRK  =   100.0000
r0    =     0.0154 N/kg            n       =     0.8000
r1    =     0.0003 Ns/m/kg         n'      =     1.1600
r2    =    18.2000 uNs2/m2/kg      e       =     0.0578 MNs/m
nW    =     0.3520 MN              Z       =     0.2670 MN
Pmax  =     2.2371 MW/locomotive   Bmax    =     0.3000 MN/air brake

Umin  =    -5.1507 N/kg            ABmax   =     5.0167 N/kg
Umax  =     0.0589 N/kg            DBmax   =     0.1339 W/kg
V1    =     4.6194 m/s             DBmax/V1  =     0.0290 Ns/m/kg
V2    =     9.7192 m/s             DBmax*V2  =     1.3019 W/kg
V3    =     5.0843 m/s             Umax*V3   =     0.2993 W/kg

 Switching strategy : Optimal
 --------------------
v1    =    11.7181 m/s            Vmax   =    13.2608 m/s
v2    =     1.8373 m/s

T1a   =   119.5029 s              S1a    =     0.3062 km
T1b   =   525.2859 s              S1b    =     5.0271 km
T1    =   644.7888 s              S1     =     5.3333 km
T2    =  8678.2109 s              S2     =   101.6918 km
T3    =   540.6367 s              S3     =     3.5346 km
T4    =     0.3632 s              S4     =     0.0003 km
T4a   =     0.3632 s              S4a    =     0.0003 km
T4b   =     0.0000 s              S4b    =     0.0000 km
T4c   =     0.0000 s              S4c    =     0.0000 km

U1a   =     0.0589 N/kg           E1a    =    18.0223 J/kg
U1b   =     0.0255 N/kg           E1b    =   157.2063 J/kg
U2    =     0.0214 N/kg           E2     =  2176.6643 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- ----------
      0.0        0.0    0.000000    0.058863    0.000000  Max. force A
    119.5      306.2    5.084318    0.025540   18.022339  Max. force B
    644.8     5333.3   11.718062    0.021405  175.228638  Const. speed
   9323.0   107025.1   11.718062    0.000000 2351.893066  Coasting
   9863.6   110559.7    1.837304   -5.069998 2351.893066  Max. brake
   9864.0   110560.0    0.000000

Dist. =  110.5600 km
Time  = 9864.0000 sec
E.C.  =    21.2725 J/kg/km
```

```
 Schedule
 --------
Dist. =  110.5600 km
Time  =  164.4000 min

 Data of train
 -------------
Tmass =     5.9800 Mkg
nLOC  =     3.0000
nBRK  =   100.0000
r0    =     0.0154 N/kg              n       =     0.8000
r1    =     0.0003 Ns/m/kg           n'      =     1.1600
r2    =    18.2000 uNs2/m2/kg        e       =     0.0578 MNs/m
uW    =     0.3520 MN                Z       =     0.2670 MN
Pmax  =     2.2371 MW/locomotive     Bmax    =     0.3000 MN/axle brake

Umin  =    -5.1507 N/kg              AKmax   =     5.0167 N/kg
Umax  =     0.0589 N/kg              DBmax   =     0.1339 N/kg
V1    =     4.6194 m/s               DBmax/V1 =     0.0290 Ns/m/kg
V2    =     9.7192 m/s               DBmax*V2 =     1.3019 W/kg
V3    =     5.0843 m/s               Umax*V3  =     0.2993 W/kg

 Switching strategy : Max. thrust & brake
 ------------------
v1    =    11.4163 m/s             Vmax   =    13.2608 m/s
v2    =    11.4163 m/s

T1a   =   119.5029 s               S1a    =     0.3062 km
T1b   =   459.1847 s               S1b    =     4.2622 km
T1    =   578.6876 s               S1     =     4.5684 km
T2    =  9283.0908 s               S2     =   105.9790 km
T3    =     0.0000 s               S3     =     0.0000 km
T4    =     2.2213 s               S4     =     0.0126 km
T4a   =     0.7058 s               S4a    =     0.0021 km
T4b   =     0.9866 s               S4b    =     0.0071 km
T4c   =     0.3288 s               S4c    =     0.0035 km

U1a   =     0.0589 N/kg            E1a    =    18.0223 J/kg
U1b   =     0.0262 N/kg            E1b    =   137.4237 J/kg
U2    =     0.0212 N/kg            E2     =  2245.3738 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000    0.058863     0.000000  Max. force A
    119.5      306.2    5.084318    0.026215    18.022339  Max. force B
    578.7     4568.4   11.416349    0.021187   155.446030  Const. speed
   9861.8   110547.4   11.416349    0.000000  2400.819824  Coasting
   9861.8   110547.4   11.416349   -5.019262  2400.819824  Max. brake
   9862.1   110550.8    9.719236   -5.150669  2400.819824  Max. brake B
   9863.1   110557.9    4.619377   -5.150669  2400.819824  Max. brake A
   9864.0   110560.0    0.000000

Dist. =  110.5600 km
Time  = 9864.0000 sec
E.C.  =   21.7151 J/kg/km
```

```
 Schedule
 --------
Dist. =  110.5600 km
Time  =  164.4000 min

 Data of train
 -------------
Tmass =     5.9800 Mkg
nLOC  =     3.0000
nBRK  =   100.0000
r0    =     0.0154 N/kg                n              0.8000
r1    =     0.0003 Ns/m/kg             n'      =      1.1600
r2    =    18.2000 uNs2/m2/kg          e       =      0.0578 MNs/m
nW    =     0.3520 MN                  Z       =      0.2670 MN
Fmax  =     2.2371 MW/locomotive       Bmax    =      0.3000 MN/air brake

Umin  =    -5.1507 N/kg                ABmax =      5.0167 N/kg
Umax  =     0.0589 N/kg                DBmax =      0.1339 N/kg
V1    =     4.6194 m/s                 DBmax/V1 =     0.0290 Ns/m/kg
V2    =     9.7192 m/s                 DBmax*V2 -     1.3019 W/kg
V3    =     5.0843 m/s                 Umax*V3  =     0.2993 W/kg

 Switching strategy : Max thrust & coast
 -------------------
v1    =    11.8658 m/s                 Vmax   =    13.2608 m/s
v2    =     0.0000 m/s

T1a   =  119.5029 s                    S1a    =     0.3062 km
T1b   =  563.0012 s                    S1b    =     5.4719 km
T1    =  682.5042 s                    S1     =     5.7781 km
T2    = 8516.8291 s                    S2     =   101.0593 km
T3    =  664.6675 s                    S3     -     3.7227 km
T4    =     0.0000 s                   S4     =     0.0000 km
T4a   =     0.0000 s                   S4a    =     0.0000 km
T4b   =     0.0000 s                   S4b    -     0.0000 km
T4c   =     0.0000 s                   S4c    -     0.0000 km

U1a   =     0.0589 N/kg                E1a    -    18.0223 J/kg
U1b   =     0.0252 N/kg                E1b    =   168.4937 J/kg
U2    =     0.0215 N/kg                E2     =  2174.0142 J/kg

 Time (s)   Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------   ---------  ----------  ----------- -----------
      0.0         0.0    0.000000    0.058863     0.000000  Max. force A
    119.5       306.2    5.084318    0.025222    18.022339  Max. force B
    682.5      5778.1   11.865830    0.021512   186.515991  Const. speed
   9199.3    106837.3   11.865830    0.000000  2360.530273  Coasting
   9864.0    110560.0    0.000000   -5.016722  2360.530273  Max. brake
   9864.0    110560.0    0.000000

Dist. =  110.5600 km
Time  = 9864.0010 sec
E.C.  =    21.3507 J/kg/km
```

Canmore → Field

```
 Schedule
 --------
Dist. =  109.4200 km
Time  =  150.0000 min

 Data of train
 -------------
Tmass =     5.9800 Mkg
nLOC  =     3.0000
nBRK  =   100.0000
r0    =     0.0154 N/kg           n      -      0.8000
r1    =     0.0003 Ns/m/kg        n'     =      1.1600
r2    =    18.2000 uNs2/m2/kg     e      =      0.0578 mNs/m
nW    =     0.3520 MN             Z      =      0.2670 MN
Fmax  =     2.2371 MW/locomotive  Bmax   =      0.3000 MN/air brake

Umin  =    -5.1507 N/kg           ABmax  =      5.0167 N/kg
Umax  =     0.0589 N/kg           DBmax  =      0.1339 N/kg
V1    =     4.6194 m/s            DBmax/V1 =  .  0.0290 Ns/m/kg
V2    =     9.7192 m/s            DBmax*V2 =     1.3019 W/kg
V3    =     5.0843 m/s            Umax*V3  =     0.2993 W/kg

 Switching strategy : Optimal
 --------------------
v1    =    12.8751 m/s           Vmax   =    13.2608 m/s
v2    =     2.1514 m/s

T1a   =   119.5029 s             S1a    =     0.3062 km
T1b   =  1061.4752 s             S1b    =    11.6926 km
T1    =  1180.9781 s             S1     =    11.9988 km
T2    =  7244.5181 s             S2     -    93.2737 km
T3    =   574.0793 s             S3     =     4.1471 km
T4    =     0.4249 s             S4     =     0.0005 km
T4a   =     0.4249 s             S4a    =     0.0005 km
T4b   =     0.0000 s             S4b    =     0.0000 km
T4c   =     0.0000 s             S4c    =     0.0000 km

U1a   =     0.0589 N/kg        , E1a    =    18.0223 J/kg
U1b   =     0.0232 N/kg          E1b    =   317.6758 J/kg
U2    =     0.0223 N/kg          E2     =  2077.1582 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000   0.058863    0.000000  Max. force A
    119.5      306.2    5.084318   0.023245   18.022339  Max. force B
   1181.0    11998.8   12.875074   0.022269  335.698090  Const. speed
   8425.5   105272.5   12.875074   0.000000 2412.856201  Coasting
   8999.6   109419.5    2.151424  -5.079106 2412.856201  Max. brake
   9000.0   109420.0    0.000000

Dist. =  109.4200 km
Time  = 9000.0000 sec
E.C.  =    22.0513 J/kg/km
```

```
 Schedule
 --------
Dist. =  109.4200 km
Time  =  150.0000 min

 Data of train
 -------------
Tmass =     5.9800 Mkg
nLOC  =     3.0000
nBRK  =   100.0000
r0    =     0.0154 N/kg               n      -      0.8000
r1    =     0.0003 Ns/m/kg            n'     =      1.1600
r2    =    18.2000 uNs2/m2/kg         e      =      0.0578 MNs/m
nW    =     0.3520 MN                 Z      =      0.2670 MN
Fmax  =     2.2371 MW/locomotive      Bmax   -      0.3000 MN/air brake

Umin  =    -5.1507 N/kg               ABmax -       5.0162 N/kg
Umax  =     0.0589 N/kg               DBmax -       0.1339 N/kg
V1    =     4.6194 m/s                DBmax/V1 -      0.0290 Ns/m/kg
V2    =     9.7192 m/s                DBmax*V2 -      1.3019 W/kg
V3    =     5.0843 m/s                Umax*V3  -      0.2993 W/kg

 Switching strategy : Max. thrust & brake
 --------------------
v1     =    12.4696 m/s              Vmax          13.2608 m/s
v2     =    12.4696 m/s

T1a    =  119.5029 s                 S1a    =      0.3062 km
T1b    =  779.9625 s                 S1b    -      8.1183 km
T1     =  899.4654 s                 S1     =      8.4245 km
T2     = 8098.1089 s                 S2     -    100.9804 km
T3     -    0.0000 s                 S3     -      0.0000 km
T4     =    2.4259 s                 S4     -      0.0151 km
T4a    =    0.9058 s                 S4a    -      0.0021 km
T4b    =    0.9866 s                 S4b    -      0.0071 km
T4c    =    0.5335 s                 S4c    -      0.0059 km

U1a    =    0.0589 N/kg              E1a    -     18.0223 J/kg
U1b    =    0.0240 N/kg              E1b    =    233.4253 J/kg
U2     =    0.0220 N/kg              E2     =   2217.6150 J/kg

 Time (s)  Dist. (m)  Vel. (m/s)  Ctr. (N/kg) E.C. (J/kg)
 --------  ---------  ----------  ----------- -----------
      0.0        0.0    0.000000    0.058863    0.000000  Max. force A
    119.5      306.2    5.084318    0.024001   18.022339  Max. force B
    899.5     8424.5   12.469631    0.021961  251.447632  Const. speed
   8997.6   109404.9   12.469631    0.000000 2469.062744  Coasting
   8997.6   109404.9   12.469631   -5.019048 2469.062744  Max. brake
   8998.1   109410.8    9.719236   -5.150669 2469.062744  Max. brake B
   8999.1   109417.9    4.619377   -5.150669 2469.062744  Max. brake A
   9000.0   109420.0    0.000000

Dist. =  109.4200 km
Time  = 9000.0000 sec
E.C.  =    22.5650 J/kg/km
```

```
Schedule
--------
Dist. =  109.4200 km
Time  =  150.0000 min

Data of train
-------------
Tmass =     5.9800 Mkg
nLOC  =     3.0000
nBRK  =   100.0000
r0    =     0.0154 N/kg               n        =      0.8000
r1    =     0.0003 Ns/m/kg            n'       =      1.1600
r2    =    18.2000 uNs2/m2/kg         e        =      0.0578 MNs/m
uW    =     0.3520 MN                 Z        =      0.2670 MN
Pmax  =     2.2371 MW/locomotive      Bmax     =      0.3000 MN/air brake

Umin  =    -5.1507 N/kg               ABmax =        5.0167 N/kg
Umax  =     0.0589 N/kg               DBmax =        0.1339 N/kg
V1    =     4.6194 m/s                DBmax/V1 =  .    0.0290 Ns/m/kg
V2    =     9.7192 m/s                DBmax*V2 =       1.3019 W/kg
V3    =     5.0843 m/s                Umax*V3  =       0.2993 W/kg

Switching strategy : Max thrust & coast
--------------------
v1    =    13.1068 m/s                Vmax     =   13.2608 m/s
v2    =     0.0000 m/s

T1a   =   119.5029 s                  S1a      =     0.3062 km
T1b   =  1420.1935 s                  S1b      =    16.3573 km
T1    =  1539.6964 s                  S1       =    16.6635 km
T2    =  6739.1572 s                  S2       =    88.3289 km
T3    =   721.1465 s                  S3       =     4.4276 km
T4    =     0.0000 s                  S4       =     0.0000 km
T4a   =     0.0000 s                  S4a      =     0.0000 km
T4b   =     0.0000 s                  S4b      =     0.0000 km
T4c   =     0.0000 s                  S4c      =     0.0000 km

U1a   =     0.0589 N/kg               E1a      =    18.0223 J/kg
U1b   =     0.0228 N/kg               E1b      =   425.0321 J/kg
U2    =     0.0224 N/kg               E2       =  1982.8590 J/kg

 Time (s)   Dist. (m)   Vel. (m/s)   Ctr. (N/kg) E.C. (J/kg)
 --------   ---------   ----------   ----------- -----------
      0.0         0.0     0.000000     0.058863     0.000000  Max. force A
    119.5       306.2     5.084318     0.022834    18.022339  Max. force B
   1539.7     16663.5    13.106812     0.022449   443.054443  Const. speed
   8278.9    104992.4    13.106812     0.000000  2425.913574  Coasting
   9000.0    109420.0     0.000000    -5.016722  2425.913574  Max. brake
   9000.0    109420.0     0.000000

Dist.  =  109.4200 km
Time   = 9000.0000 sec
E.C.   =   22.1707 J/kg/km
```

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at Kingston, Ontario

K.Y. Wong
M.M. Bayoumi

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EVALUATION OF DESIGN ALTERNATIVES FOR A FREIGHT TRAIN CONTROLLER

Appendix E

Optimal Control Strategy for Minimum Energy and Minimum Fuel Consumption of Freight Trains

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Appendix E

OPTIMAL CONTROL STRATEGY FOR MINIMUM ENERGY AND MINIMUM FUEL CONSUMPTION OF FREIGHT TRAINS

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17. Abstract

This project on development and optimization of control schemes for freight trains was motivated by the need to find optimal or suboptimal closed-loop control schemes for long freight trains. Several on-line and off-line freight train control strategies have been studied and reported. The relation between these control strategies is discussed in this report. The structure of an overall train operation scheme which consists of an off-line energy saving strategy and an on-line control strategy to maintain the velocity profile specified by the off-line scheme is proposed. There are five separately-bound appendices: Appendix A: AUTOMATIC CONTROL OF FREIGHT TRAINS; Appendix B: A REDUCED-ORDER MODEL FOR LONG FREIGHT TRAINS; Appendix C: A COMPARATIVE STUDY OF SELF-TUNING CONTROLLERS FOR FREIGHT TRAINS; Appendix D: MINIMUM ENERGY CONSUMPTION CONTROL OF PASSENGER/FREIGHT TRAINS WITH FIXED TERMINAL TIME; Appendix E: OPTIMAL CONTROL STRATEGY FOR MINIMUM ENERGY AND MINIMUM FUEL CONSUMPTION OF FREIGHT TRAINS

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16. Agent de projet — CDT
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17. Résumé

L'étude dont ce rapport rend compte a trait aux programmes visant l'automatisation et l'optimalisation de la conduite des longs convois de marchandises, étude motivée par le besoin pressant de trouver des techniques en boucle fermée qui soient optimales, ou tout proche. Le rapport présente un certain nombre de programmes de marche prédétermines et de programmes d'optimalisation de la conduite; de plus, il traite de l'interaction entre eux. Il préconise un programme du premier type axé sur l'économie d'énergie, secondé par un programme du deuxième type assurant à la fois le respect des consignes imposées et une consommation minimale d'énergie. Le rapport contient cinq annexes distinctes, intitulées comme suit: A: Automatisation de la conduite des convois de marchandises; B: Logiciel en réduction approprié aux longs convois de marchandises; C: Etude comparative des logiciels tenant compte des variations dans les paramètres de conduite; D: Consommation minimale des trains de voyageurs et de marchandises à horaires fixes; E: Stratégie d'optimalisation de la consommation d'énergie et de carburant dans le cas des trains de marchandises.

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1. INTRODUCTION

Great advances in the computer field in the past decade have facilitated the economical implementation of many control strategies for improving railway operations. The controller design for long multi-locomotive-powered trains with braking operation was studied by Gruber and Bayoumi,[1,2,3,4,5] Wong, Bayoumi and Gruber,[6] and Wong and Bayoumi.[8] The control criteria of all these approaches is to maintain a train at a given velocity pattern, to keep the absolute value of all coupler forces below a given limit, and to minimize the effect of noise due to grade change.

Other approaches dealt with the criterion of minimizing the energy consumption of the train operation (Strobel, Kosemand and Horn;[7] Wong and Bayoumi.[8]

The purpose of this report is to obtain an off-line minimum energy/fuel consumption control schedule and to test it for long-train simulation with throttle and brakes.

The controller is chosen to minimize an energy/fuel consumption criterion subject to train dynamics, constraints of travelling speed, tractive forces and brake power. Optimality of the ME/MF controller will be evaluated by comparing it to non-optimal controllers.

2. TRAIN MODEL

2.1 Non-Linear Model

The train system under investigation is a multi-locomotive-powered train in which the locomotives and gondola cars are connected together by couplers with spring and damping characteristics. Such a train can be modelled as a series of non-linear, spring-dashpot coupler systems of point masses described by the following set of non-linear differential equations:

$$\dot{\rho}_i = v_i$$

$$m_i \dot{v}_i = F_i(t) - F_{i+1}(t) - R_i(t) - G_i(t) + f_i(t) + U_i(t)$$

$$\dot{\Delta}_i = v_{i-1}(t) - v_i(t)$$

for $i = 1, 2, \ldots, n$

where: n = number of train members

ρ_i = distance travelled by train member i

v_i = velocity of train member i, $v_0(t) = v_{n+1}(t) = 0$

m_i = mass of train member i

F_i = draft gear force of the coupler between train member i-1 and train i, $F_1(t) = F_{n+1}(t) = 0$

R_i = non-linear rolling and aerodynamic resistance force of train member i

G_i = tangential component of the gravitational force of train member i in the direction of motion

f_i = dynamic braking/throttling force of train member i if it is a locomotive

U_i = braking force of train member i if it is a gondola car. The sign of U_i is non-positive to indicate that the braking force is in the direction opposite to the train movement

$_i$ = coupler displacement between train member i-1 and train member i, $\Delta_1 = \Delta_{n+1} = 0$.

The draft gear force is given by:

$$F_i(t) = k_i \Delta_i(t) + c_i \dot{\Delta}_i(t) = k_i \Delta_i(t) + c_i [v_{i-1}(t) - v_i(t)] \quad (4)$$

where: k_i = coupler spring constant between train member i-1 and train member i, typical value is k_i = 7 MN/m (20 ton/inch)

c_i = coupler damping coefficient between train member i-1 and train member i, typical value is c_i = 0.203 MN sec/m (0.58 ton/inch).

The rolling and aerodynamic resistance force is given by:

$$R_i = R_{0i} + R_{1i}\ v_i + R_{2i}\ v_i^2 \qquad (5)$$

where R_{0i}, R_{1i} and R_{2i} are constants which depend on the mass and the number of axles of train member i. They also depend on the location in the train if train member i is a locomotive. Typical values of these constants for locomotives with mass 176,900 kg (195 tons) and cars with mass 99,800 kg (110 tons) are:

Member i	R_{0i}(N)	R_{1i}(N sec/m)	R_{2i}(Nsec^2/m^2)
Front locomotive		86.57	6.41
Middle and end locomotives	1,903.84	86.57	0.9037
Gondola car	1,152.09	49.25	1.0016

The drag and resistance curves of these member categories are shown in Figure 1.

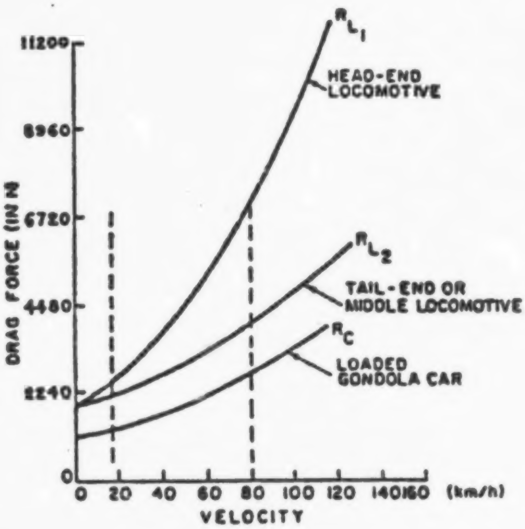


Figure 1 Aerodynamic Drag Force and Rolling Resistance Curves

The tangential component of the gravitational force in the direction of motion is given by:

$$G_i = m_i\ g\ \sin\Theta_i = m_i\ g\ \frac{\tan 0}{\sqrt{1 + \tan^2 0}} \qquad (6)$$

where: g = gravity constant, g = 9.80665 N/kg (m/sec^2)

0_i = angle of the grade on which train member i lies.

The existence constraints of the train system are travelling speed, coupler, throttle and braking power:

$0 \leq v_i(t) \leq v_{max}$ (travelling speed limit) (7)

$|\Delta_i(t)| \quad \Delta_{limit}$ (coupler displacement limit) (8)

$f_i \leq \mu m_L g$ (adhesion limit) (9)

$f_i v_i \leq \eta \text{ max}$ (motor power limit) (10)

$f_i \geq - \xi v_i$ (motor field current limit) (11)

$f_i \geq - Z$ (extended range dynamic brake limit) (12)

$f_i v_i \geq - \eta' P_{max}$ (motor heating limit) (13)

where: v_{max} = maximum trvelling speed

Δ_{limit} = coupler displacement limit, Δ_{limit} = 0.159 m (0.52 ft)

μ = tractive adhesion coefficient, $0.11 \leq \mu \leq 0.35$ dependent on track conditions

m_L = mass of locomotive

η = transmission efficiency of the electro-mechanical drive system, $\eta = 0.82$

P_{max} = maximum power rating of locomotive

ξ = low-speed dynamic brake build-up rate, typical value for an extended range dynamic brake is ξ = 0.578 MNρ/m (3,960 lb sec/ft)

Z = maximum extended range dynamic brake power over the speed range of approximately 4 to 10 m/sec, typical value for an SD-40 unit (six axles, 3,000 hp) is $Z \cong$ 0.257 MN (60,000 lb)

η' = locomotive braking efficiency, $\eta' = 1.16$

The braking constraints of each gondola car having an air pressure brake is governed by the dynamic equation and the constraints:

$$U_i(t) = - \eta_2 \, (p_i(t) \tag{14}$$

$$\dot{p}_i = [\, \eta_1 \, b_i \, (t-\tau_i) - p_i(t) \,] \, / \, \sigma_i \tag{15}$$

$$b_i(t) = 0 \text{ or } b_{min} \leq b_i(t) \leq b_i \, (t+\Delta t) \leq b_{max}, \; \Delta t > 0 \quad \text{(air brake limit)} \tag{16}$$

where: p_i = braking cylinder pressure of freight car i

b_i = braking pipe pressure input of freight car i

η_1 = brake cylinder constant

η_2 = brake pressure to force conversion constant

σ_i = time constant of the braking system

τ_i = time delay of the braking system

b_{min} = minimum braking input value, b_{min} = 48.2 kPa (7 psi)

b_{max} = maximum braking input value, b_{max} = 158.5 kPa (23 psi)

Details of the train model can be found in Gruber and Bayoumi.[3]

Major difficulties of the controller design of an unsimplified train system are:

(a) The number of input and state variables are large. A typical three-locomotive, sixty-car train with individual air pressure brakes on each car can be modelled by a dynamic system of 63 inputs and 185 states.

(b) There are constraints on the inputs and states, namely on the travelling speed, coupler displacements, throttling power and braking power.

(c) Not all states are measurable. The coupler displacements are difficult to measure on-line due to technical problems, and minimum car velocities can be measured for to econmical reasons.

2.2 Simplifying Assumptions and the Reduced Order Model

If we consider a train in the large, the length of the train (around 1 km for a sixty-car train) is negligible when it is being compared with the distance between stations (in terms of hundreds or thousands of kilometres). Hence, a train can be considered as a unit mass in the design of an open-loop control strategy.

Considering the whole train as a unit mass body, the following simplifying assumptions can be made:

(a) The travelling distances of all train members are the same.

(b) The velocities of all train members are the same.

(c) Coupler displacements between train members are neglible. Moreover, the draft gear force of the couplers are also negligible.

The simplified train model can be described by:

$$\dot{s} = v \tag{17}$$

$$m_T \dot{v} = F(t) + U(t) - R(t) - G(t) \tag{18}$$

where: s = distance travelled by the train

v = velocity of the train

m_T = mass of the train

$F = \sum_{i=1}^{n} f_i$, the total throttling/dyamic braking force applied to the train

$U = \sum_{i=1}^{n} U_i$, the total air pressure braking force applied to the train

$R = \sum_{i=1}^{n} R_i$, the total non-linear rolling and aerodynamic resistance force of the train

$G = \sum_{i=1}^{n} G_i$, the total inclinational resistance force, $G_T > 0$ for uphill, $G_T < 0$ for downhill movement

Note that under the assumption that the whole train can be considered as a unit mass located at its centre of mass, the large-scale train model can be reduced to a second-order train model. This reduced-order model only provides information about the overall system which does not provide interaction of members in the train configuration.

The constraints of the reduced-order model corresponding to equations (7) through (13) are:

$$0 \leq v(t) \leq v_{max} \tag{19}$$

$$F \leq \mu W \tag{20}$$

$$F\, v \leq \eta P_{Tmax} \tag{21}$$

$$F \geq - \ell \xi v \tag{19}$$

$$F \geq - \ell Z \tag{20}$$

$$F\, v \geq - \eta' P_{Tmax} \tag{21}$$

where: ℓ = number of locomotives in the train

$W = \ell m_L g$, the total weight of locomotives

P_{Tmax} = total maximum power of locomotives

The constraint of the air pressure braking power limit obtained from equations (14) to (16) is given by:

$$0 \geq U(t) \geq - n_B \eta_1 \eta_2 b_{max} \tag{25}$$

where n_B = number of air pressure brakes on the train.

3. OFF-LINE MINIMUM ENERGY CONSUMPTION (ME) CONTROL

3.1 Minimum Energy Consumption Control Criterion

The power consumption of a train at time t is:

$$p(t) = F^{+}(t)\ v(t)/\eta \tag{26}$$

where: $F^{+} = \frac{1}{2}(F + |F|)$ is the total throttling force applied to the train

v = velocity of the train

η = transmission efficiency of the electro-mechanical drive system, $\eta = 0.82$

The total energy consumption of the train is:

$$\begin{aligned} J_0 &= \int_0^T p(t)\,dt = \frac{1}{\eta}\int_0^T F^{+}(t)\ v(t)\,dt \\ &= \frac{1}{2\eta}\int_0^T [\,F(t) + |F(t)|\,]\ v(t)\,dt \end{aligned} \tag{27}$$

The objective of the minimum energy control problem is to minimize J_0 subject to train dynamics (equations (17) and (18)), constraints (equations (19) through (25)), and boundary conditions:

$$s(0) = 0 \tag{28}$$

$$v(0) = 0 \tag{29}$$

$$s(T) = S \tag{30}$$

$$v(T) = 0 \tag{31}$$

3.2 Optimal Solution

The optimal solution can be obtained, according to Pontrjagin's maximum principle, by selecting from among all admissible control F and U, which maximizes the Hamiltonian function:

$$H_0\ (F,\ U,\ s,\ v,\ \lambda_1,\ \lambda_2) = -\frac{1}{2}\,(F+|F|)v + \lambda_1\, v + \lambda_2\ (F+U-R-G)$$

$$= \begin{cases} \lambda_2\ (F+U) + \lambda_1\ v - \lambda_2\ (R+G) & \text{for } F < 0 \\ (\lambda_2 - v)\ F + \lambda_2\ U + \lambda_1\ v - \lambda_2\ (R+G) & \text{for } F > 0 \end{cases} \qquad (32)$$

subject to:

$$\dot{s} = v \qquad \text{(state equation)} \qquad (33)$$

$$m_T\dot{v} = F + U - R - G \qquad \text{(state equation)} \qquad (34)$$

$$\dot{\lambda}_1 = \frac{\partial H_0}{\partial s} = 0 \qquad \text{(adjoint equation)} \qquad (35)$$

$$\dot{\lambda}_2\ \frac{\partial H_0}{\partial v} = \frac{1}{2}\,(F+|F|) - \lambda_1 + \frac{\partial R}{\partial v} \qquad \text{(adjoint equation)} \qquad (36)$$

$$s(0) = 0,\ v(0) = 0,\ s(T) = S,\ v(T) = 0 \quad \text{(braking conditions)} \qquad (37)$$

$$\lambda_1(T) = \vartheta_1,\ \lambda_2(T) = \vartheta_2 \qquad \text{(transversality conditions)} \qquad (38)$$

and all the constraints, equations (19) through (25):

$$0 \leq v \leq v_{max}$$

$$F \leq \mu W$$

$$F\ v \leq \eta P_{Tmax}$$

$$F \geq -\ell \xi v$$

$$F \geq -\ell z$$

$$F\,v \geq -\,\eta'\,P_{Tmax}$$

$$0 \geq U \geq -\,n_B\,\eta_1\,\eta_2\,b_{max}$$

The maximum of H w.r.t F and V is attended depends on λ_2 by the following cases:

Case	Condition	Optimal Input
1	$\lambda_2 \quad 0$	F + U = braking with maximum force
2	$\lambda_2 = 0$	F + U = any braking value (1^{st} singular solution)
3	$0 < \lambda < 2 \quad v$	U = 0, F = 0 (coasting)
4	$\lambda_2 = v$	U = 0, F = any throttling force (2^{nd} singular solution)
5	$\lambda_2 > v$	U = 0, F = driving with maximum throttling

The optimal control strategy therefore includes the following schemes:

(1) braking with maximum force

(2) coasting

(3) travelling at constant speed

(4) driving with maximum force

Note that control scheme (3) may be applied to case (2) (when the train goes downhill) and to case (4) above.

The optimal solution can be obtained by solving the boundary value problems of each case, since λ_2 is a continuous function (see previous report ?? Appendix ? of this report ??).

It is obvious that switching from one control scheme to another must occur in singular cases and most of the time the train must be running at a constant speed, depending on the grade angle of the section. This speed is referred to as the nominal velocity of a given grade section.

3.3 Switching Strategy

Given the train data, grade sections and nominal speed of each section, a very effective switching strategy to meet the speed requirement should be achieved. Note that if a train is to be stopped, we can set the nominal speed of the last section to zero.

Now let us consider a train travelling from section 1 to secton 2. Let θ_1, θ_2, v_1 and v_2 be the grade angles and nominal velocities of the two sections, respectively. Note that $\theta_1 < \theta_2$ implies that $v_1 < v_2$ and vice versa. Since more energy will be saved when a train accelerates on a smaller grade angle and decelerates on a larger one, the following is a very energy efficient switching stategy:

(a) If $\theta_1 \geq \theta_2$, begin acceleration just past the grade or velocity change point. Upon reaching the new nominal velocity, v_2, continue travelling at constant speed, v_2. The priority for acceleration is to apply the coasting scheme if possible, otherwise apply the maximum throttle scheme.

(b) If $\theta_1 \leq \theta_2$, begin deceleration at the retardation distance before the velocity change point. When the grade or velocity change point is reached, the velocity becomes the new nominal velocity, v_2, so that the train continues travelling at this speed, v_2, on the next section. The priority for deceleration is to apply the maximum braking scheme. The retardation distance is estimated by solving the BVP of the appropriate control scheme.

3.4 Overall Control Scheme

The overall control strategy is:

(1) fill in the grade information and the nominal velocity of each section

(2) compute the retardation and acceleration distances for each section

(3) fill inthe optimal control scheme based on the switching strategy.

3.5 Illustrative Examples

A train with 2 M kg mass was simulated over a distance of 10 km with various grades and nominal velocities using the optimal (ME) control strategy and the simulation package LFTSIM (see Annex B for the program listing).

Example 1: Travelling on Level Ground

The grade and nominal velocity data for the simulation is:

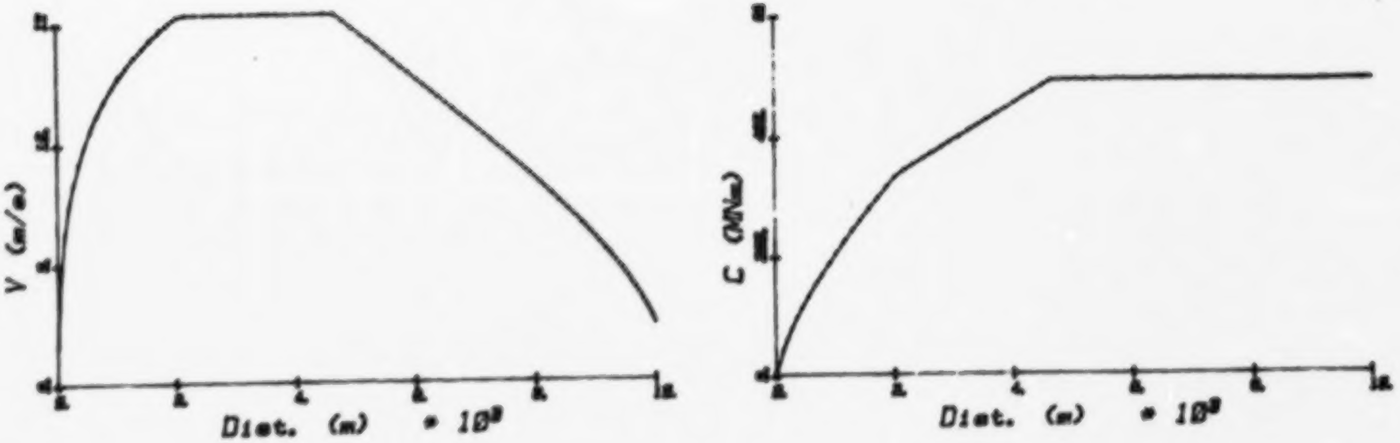
Distance	10 km
Grade	0%
Nominal Speed	15 m/sec

The acceleration distance for accelerating a train from start to a nominal speed of 15 m/sec is found to be 2.058 km by solving the BVP for control scheme 4 (driving with maximum throttling).

The stopping distance for a train coasting from 15 m/sec to a complete stop is found to be 5.357 km. Hence, the optimal switching strategy is:

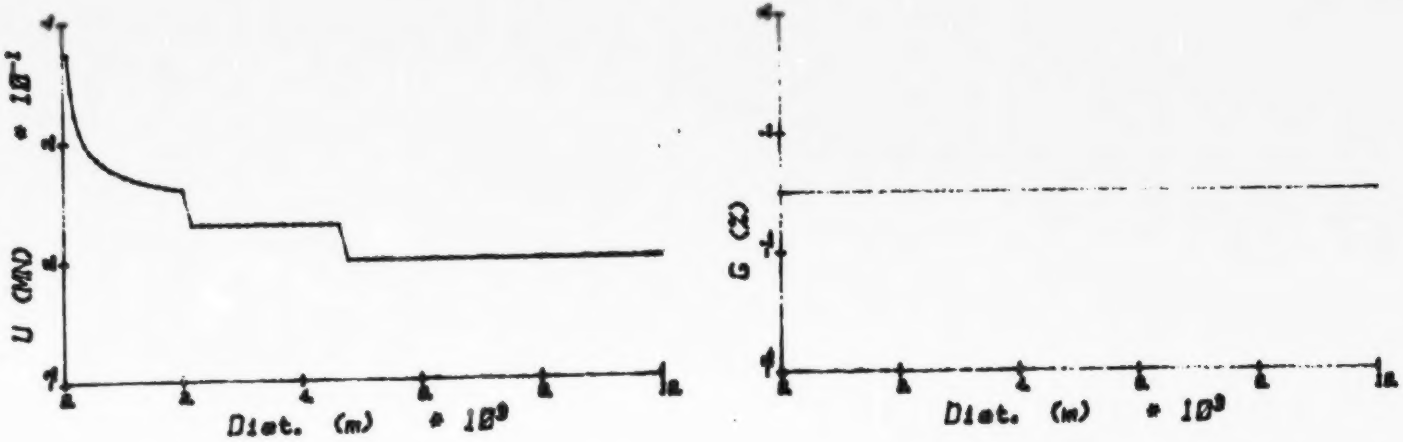
Distance (km)	Control Scheme
0 to 2.058 km	maximum throttling
2.058 to 4.644 km	nominal speed 15 m/sec
4.644 to 10 km	coasting (dec.)

The simulation results are presented in Figure 2. Figure 2(a) shows the velocity curve, Figure 2(b) shows the input force, Figure 2(c) shows the energy consumption, and Figure 2(d) shows the grade of sections.



(a): Velocity

(c): Energy consumption



(b): Input force

(d): Grade of sections

Figure 2 Train Simulation Results -- Travelling on Level Ground

Example 2: Travelling from Level Ground to Uphill

The grade and nominal velocity data for the simulation is:

	Section 1	Section 2
Distance	7 km	3 km
Grade	0%	1%
Nominal Speed	15 m/sec	5 m/sec

The optimal switching strategy is:

Distance (km)	Control Scheme
0 to 2.06	maximum throttle
2.06 to 2.53	15 m/sec
2.53 to 7.00	coasting (dec.)
7.00 to 9.89	5 m/sec
9.89 to 10.0	coasting (dec.)

The simultion results are presented in Figure 3.

Example 3: Travelling from Level Ground to Downhill

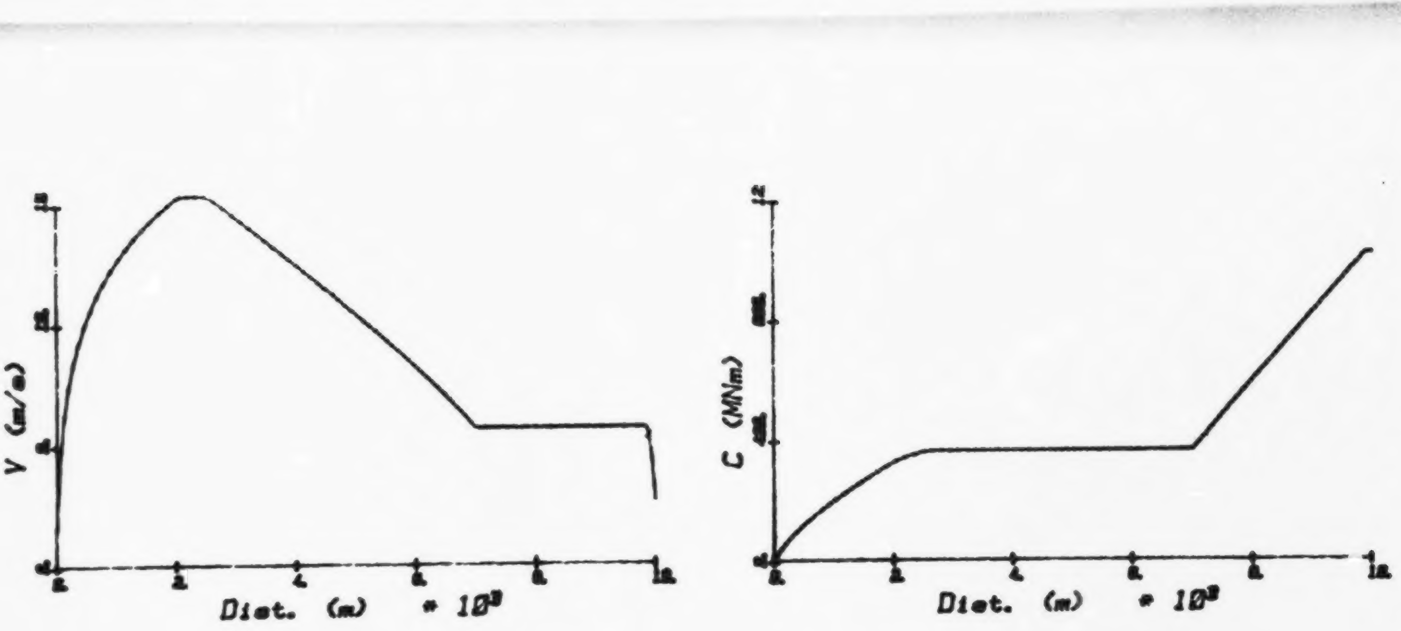
The grade and nominal velocity data for the simulation is:

	Section 1	Section 2
Distance	7 km	3 km
Grade	0%	-1%
Nominal speed	15 m/sec	18 m/sec

The optimal switching strategy is:

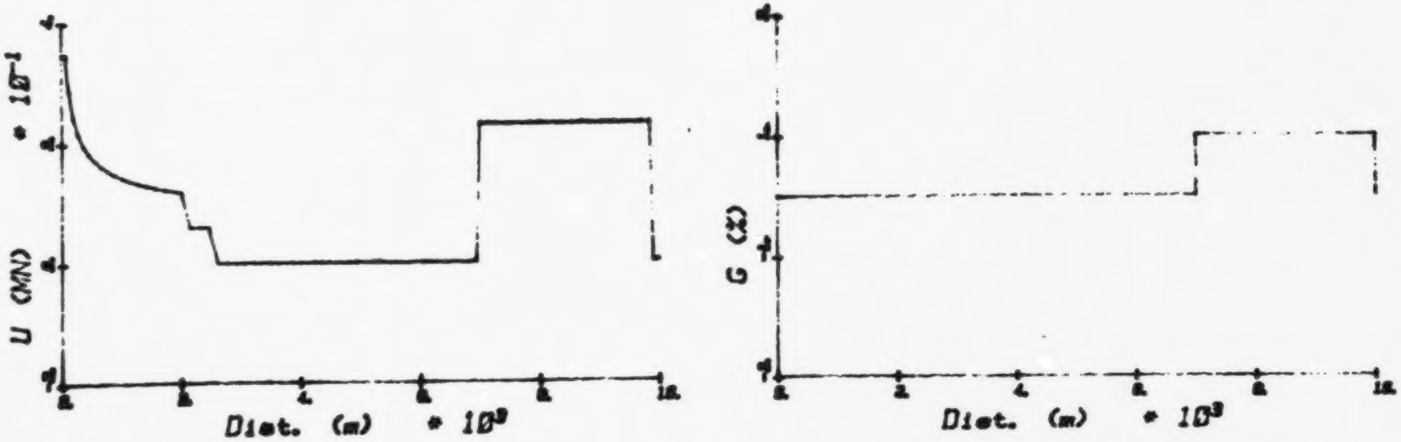
Distance (km)	Control Scheme
0 to 2.06	maximum throttle
2.06 to 7.0	15 m/sec
7.0 to 7.74	coasting (acc.)
7.74 to 9.91	18 m/sec
9.91 to 10.0	maximum brake

The simulation results are presented in Figure 4.



(a): Velocity

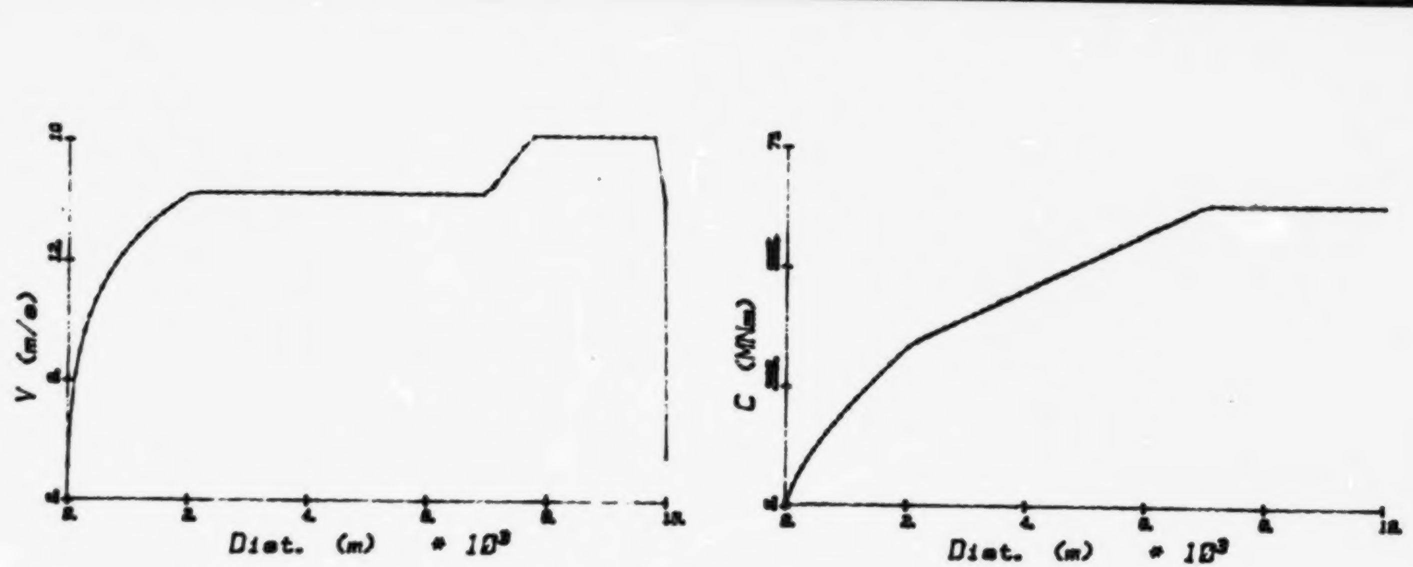
(c): Energy consumption



(b): Input force

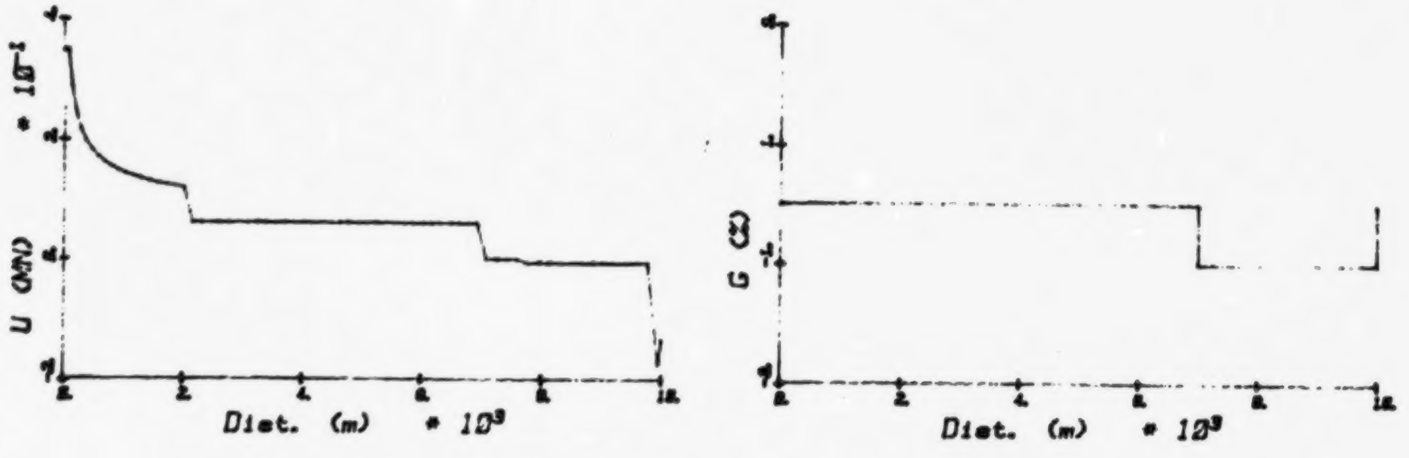
(d): Grade of sections

Figure 3 Train Simulation Results -- Travelling from Level Ground to Uphill



(a): Velocity

(c): Energy consumption



(b): Input force

(d): Grade of sections

Figure 4 Train Simulation Results -- Travelling from Level Ground to Downhill

Example 4: Travelling Over a Mountain

The grade and nominal velocity data for the simulation is:

	Section 1	Section 2	Section 3	Section 4	Section 5
Distance	0.2 km	0.3 km	0.2 km	0.3 km	1 km
Grade	0%	1%	0%	-1%	0%
Nominal Speed	5 m/sec	5 m/sec	5 m/sec	5 m/sec	5 m/sec

The optimal switching strategy is:

Distance (km)	Control Scheme
0 to 0.0786	maximum throttle
0.0786 to 1.12	5 m/sec
1.12 to 2	coasting (dec.)

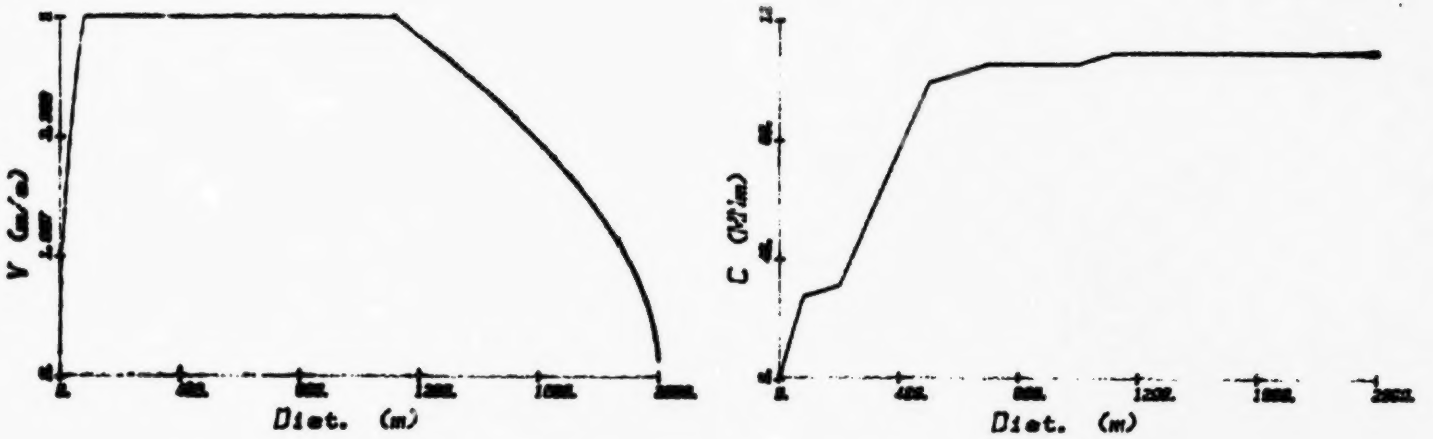
The simulation results are presented in Figure 5.

Example 5: Travelling Across a Valley

The grade and nominal velocity data for the simulation are:

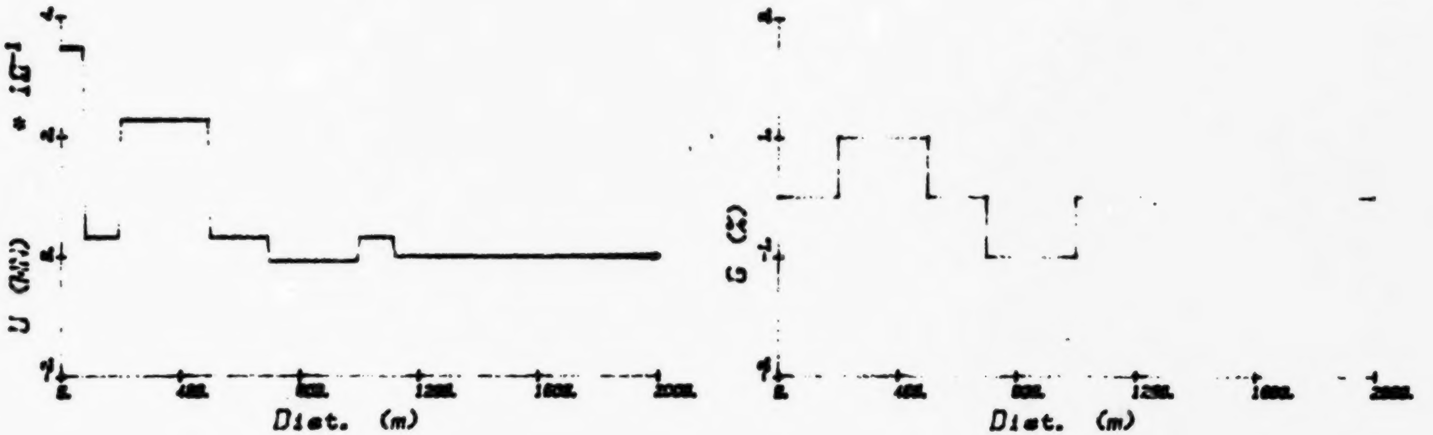
	Section 1	Section 2	Section 3	Section 4	Section 5
Distance	0.2 km	0.3 km	0.2 km	0.3 km	1 km
Grade	0%	-1%	0%	1%	0%
Nominal Speed	5 m/sec	5 m/sec	5 m/sec	5 m/sec	5 m/sec

The optimal switching strategy is the same as in Example 4 and the simulation results are presented in Figure 6.



(a): Velocity

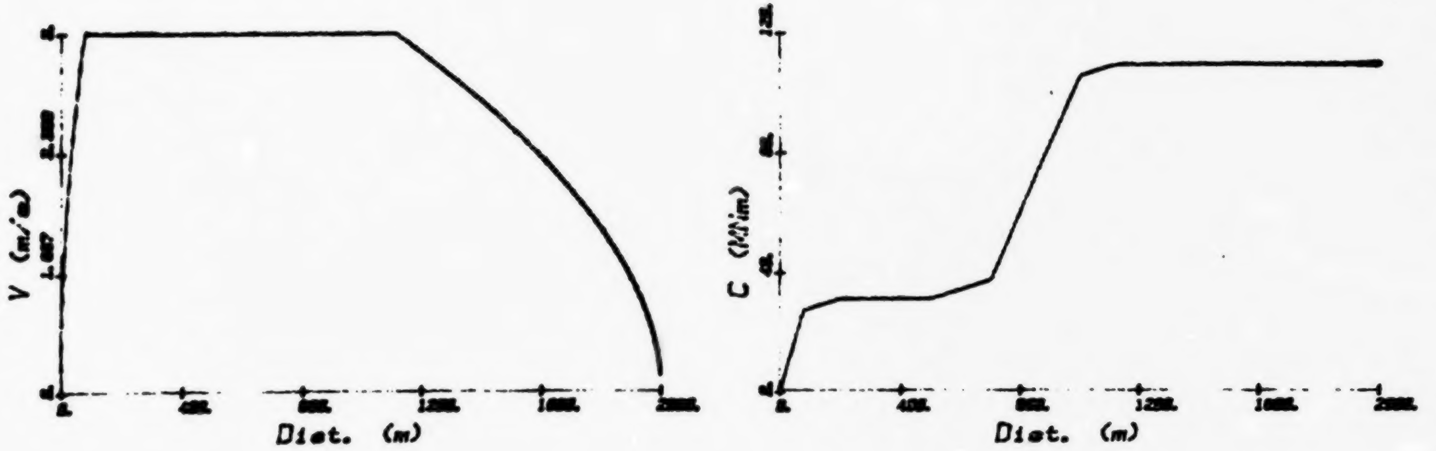
(c): Energy consumption



(b): Input force

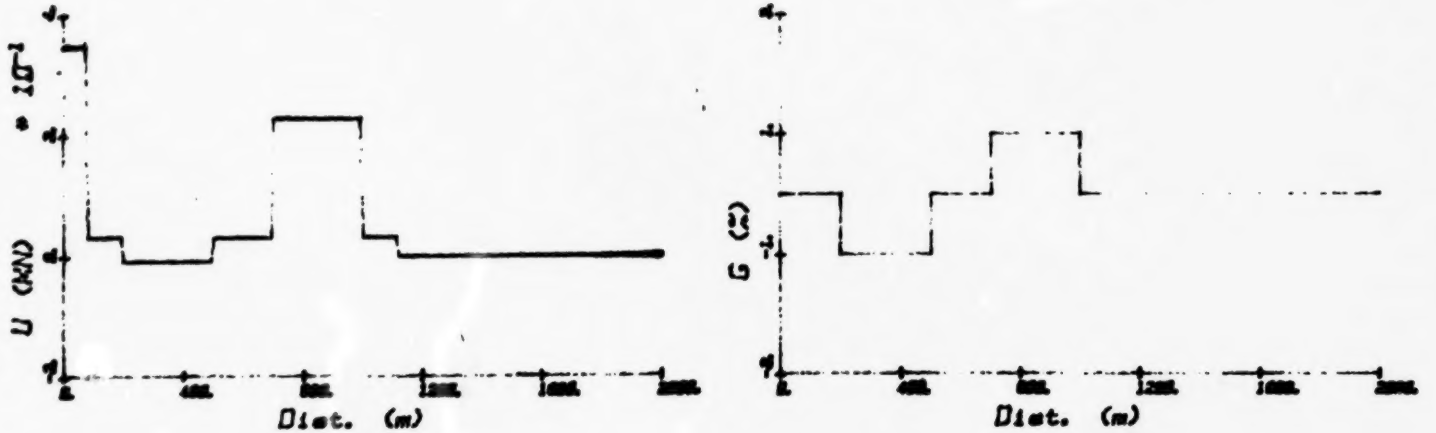
(d): Grade of sections

Figure 5 Train Simulation Results -- Travelling Over a Mountain



(a): Velocity

(c): Energy consumption



(b): Input force

(d): Grade of sections

Figure 6 Train Simulation Results -- Travelling Across a Valley

4. CONSTANT ACCELERATION (CA) CONTROL

4.1 Constant Acceleration (CA) Control Scheme

A simple constant acceleration (CA) control scheme is introduced in this section for the purpose of comparison.

This control scheme maintains constant acceleration (or deceleration) to bring the train to the required nominal speed. The input force for maintaining constant acceleration is:

$$F + U = R + G - m_T\ a \tag{39}$$

The constant acceleration control strategy includes the following schemes:

(1) constant deceleraton, $a < 0$ (instead of braking with maximum force and coasting deceleration)

(2) travelling with constant speed, $a = 0$

(3) constant acceleration, $a > 0$ (instead of driving with maximum force and coasting acceleration).

The complete solution can be obtained by solving the state equation in each section of the constant acceleration value. The solution in each section is given by:

$$v = v_0 + a\ t \tag{40}$$

$$s = v_0 t + \frac{1}{2}\ a\ t^2 \tag{41}$$

4.2 Switching Strategy

In order to make comparison with the ME control reasonable, a similar switching strategy as in Section 3.3 is given here:

(a) If $\theta_1 \geq \theta_2$, begin accelerating just past the grade or velocity change point. After reaching the new nominal velocity, v_2, maintain constant speed, v_2.

(b) If $\theta_1 \leq \theta_2$, begin decelerating at the retardation distance before the velocity change point. Upon reaching the velocity change point, the velocity has been reduced to the new nominal velocity, v_2, so that the train can continue on at this speed, v_2, on the next section, where θ_1, θ_2, v_1 and v_2 are the grade angles and nominal speeds of the sections.

The acceleration or retardation distance can be estimated easily by the formula:

$$s = \frac{v_1^2 - v_0^2}{2\,a} \tag{42}$$

which can be obtained by substituting equation (40) into equation (41).

4.3 Overall Control Scheme

The overall (CA) control strategy is:

(1) fill in the grade information and the nominal velocity for each section

(2) compute the retardation and acceleration distances for each section

(3) fill in the constant acceleration control scheme based on the switching strategy.

4.4 Illustrative Examples

A train with 2 M kg mass was simulated over a distance of 20 km with various grades and nominal velocities using the constant acceleration control strategy. For simplicity, the constant retardation value has the same magnitude as the acceleration value. This simulation is also performed by the simulation package LFTSIM.

Example 1: Travelling on Level Ground

The grade and nominal velocity data for the simulation are:

Distance	10 km
Grade	0%
Nominal Speed	9 m/sec

The acceleration constant for the CA control is $a = 0.04$ m/sec^2. The switching strategy and corresponding energy consumption are:

Distance (km)	Control Scheme
0 to 1.013	$a = 0.04$ m/sec
1.013 to 8.988	constant $v = 0$ m/sec
8.988 to 10.0	$a = -0.04$ m/sec

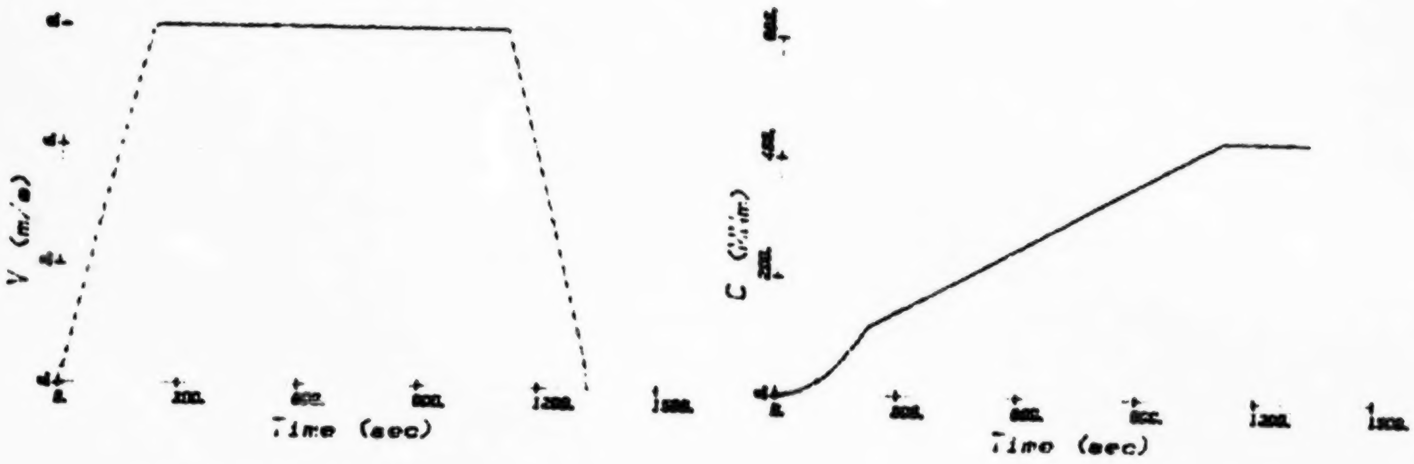
The time required for the train operations is $T = 1{,}327$ seconds. The simulation results are presented in Figure 7.

5. OPTIMALITY OF THE ME CONTROL SCHEME

5.1 Comparison of Optimal (ME) and Non-Optimal (CA) Controls

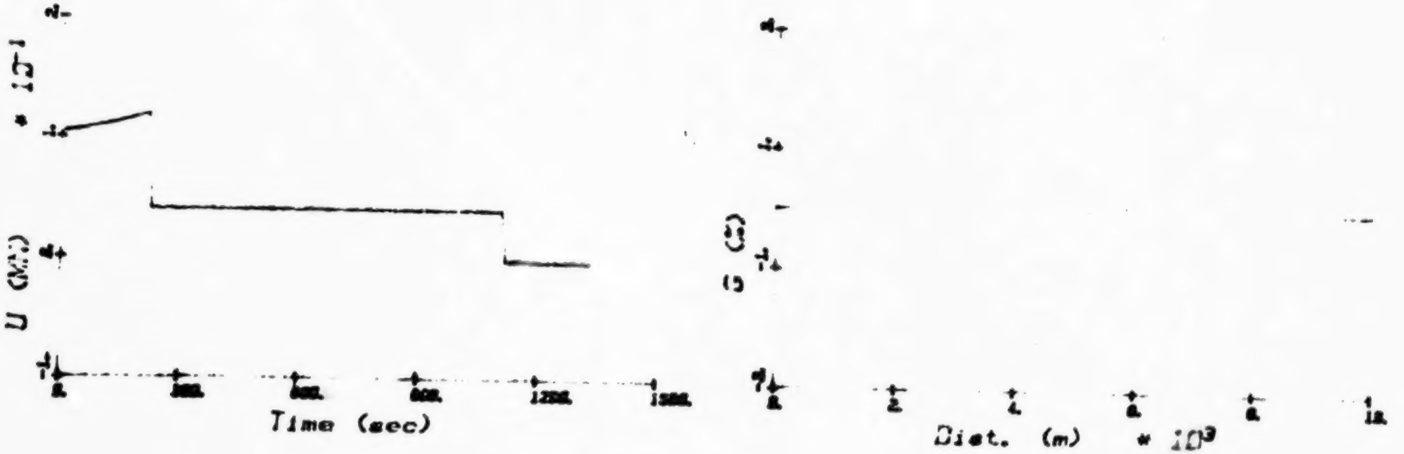
The minimum energy (ME)control strategy is introduced to reduce the energy consumption of a train travelling on grade sections with specified nominal speed. How optimal is the ME control strategy? The constant acceleration control strategy is introduced for comparative purposes. The only difference between the CA and ME controls is where acceleration or deceleration is required to attain the given nominal speed. Note that the nominal speed on the grade sections could be adjusted to let the train finish the journey in a definite time. However, if the time required for the whole journey is not fixed, lowering the nominal speed would reduce the energy consumption because the resistance force for low-speed train operation is small. Hence, it would only be fair to compare the energy consumptions of trains which reach their destinations in a given fixed time.

Suppose that a train is travelling at nominal speed, v_0, for a distance, S, using the minimum energy control strategy, and the time required for the whole journey is T seconds. We want to find the



(a): Velocity

(c): Energy consumption



(b): Input force

(d): Grade of sections

Figure 7 Train Simulation Results -- Travelling on Level Ground

corresponding destination with the same amount of time. Let $a > 0$ be the value of the acceleration constant and for simplicity the retardation constant is set to $-a$. If the train takes t_0 seconds to build up the velocity to v_0, it will take the same time, t_0 seconds, to bring the train to a full stop. Hence, the v-t curve is a piecewise linear curve, as shown in Figure 8.

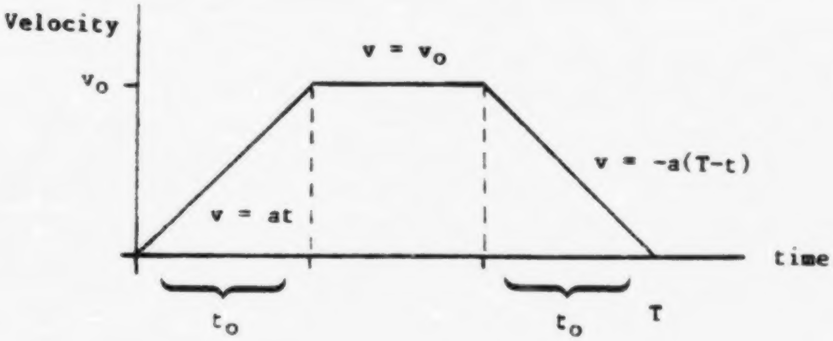


Figure 8 Velocity versus Time for Train Operations Using CA Control

The distance travelled by the train using the CA control strategy satisfies:

$$S = \frac{1}{2} a\, t_0^{\;2} + v_0\,(T-2t_0) + \frac{1}{2} a\, t_0^{\;2} = a\, t_0^{\;2} + v_0\,(T-2t_0) \qquad (43)$$

Substituting $t_0 = v_0/a$ into equation (43) yields:

$$a = \frac{v_0^{\;2}}{v_0 T - S} \qquad (44)$$

which is the appropriate acceleration constant for the CA control strategy to bring the train to the destination with time T seconds.

5.2 Illustrative Examples

A train with 2 M kg mass was simulated over a distance of 10 km of level ground with a nominal velocity 9 m/sec. Comparison is made of the ME and CA control strategies.

Example 1: Travelling with Optimal (ME) Control

The distance required to accelerate the train from start to stop is found to be 0.343 km by solving the BVP for the maximum throttling scheme. The stopping distance for coasting the train from 9 m/sec to a full stop is 2.45 km. The optimal switching strategy and the corresponding energy consumption are:

Distance (km)	Control Scheme	Energy Consumption (MJ)
0 to 0.343	maximum throttle	93.13
0.343 to 7.55	constant speed 9/m sec	288.87
7.55 to 10	coasting (dec.)	0.00
Total consumption		382.00

The total time required for the train movement is T = 1,419 seconds. The simulation results are shown in Figure 9.

Example 2: Travelling with Non-Optimal (CA) Control

The appropriate acceleration constant for the CA control strategy to bring the train to its destination in 1,419 seconds is computed using equation (44):

$$a = \frac{9^2}{9(1,419) - 10,000} = 0.02923 \text{ m/sec}^2$$

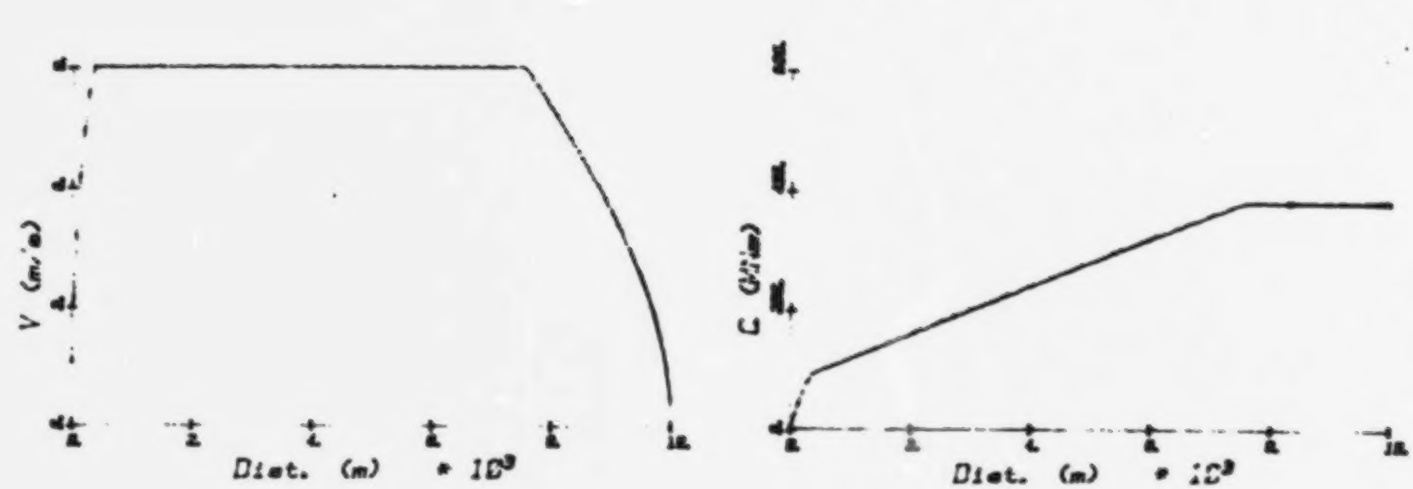
The time required to build up/down the velocity to 9 m/sec is:

$$t_0 = v_0/a = 9/0.02923 = 307.9 \text{ sec}$$

The CA switching strategy and corresponding energy consumption are:

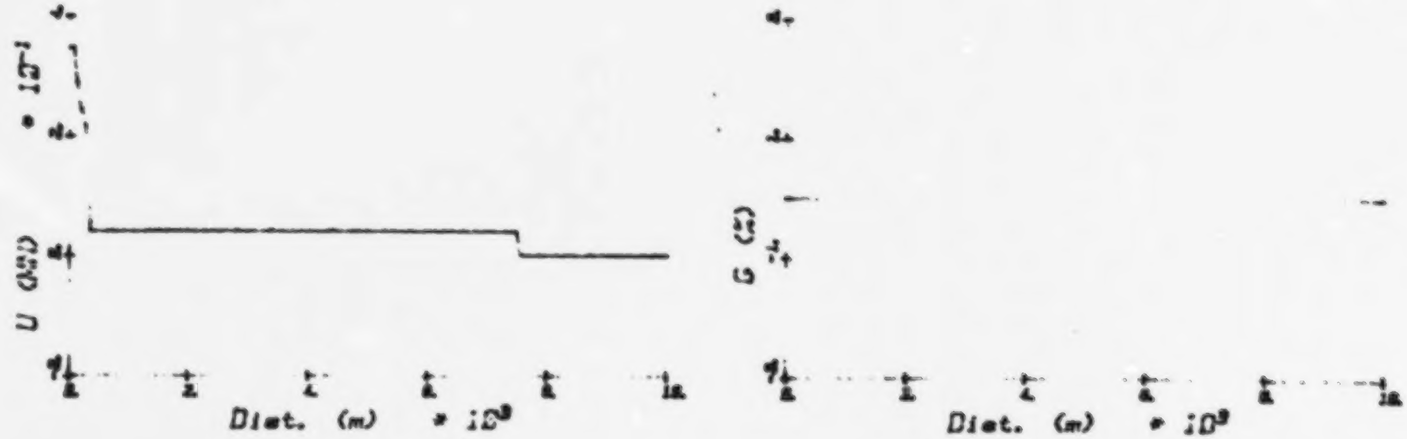
Distance (km)	Control Scheme	Energy Consumption (MJ)
0 to 1.351	constant acceleration	126.3
1.351 to 8.650	constant speed 9 m/sec	282.5
8.650 to 10	constant retardation	0.0
Total consumption		418.8

The simulation results are shown in Figure 10.



(a): Velocity

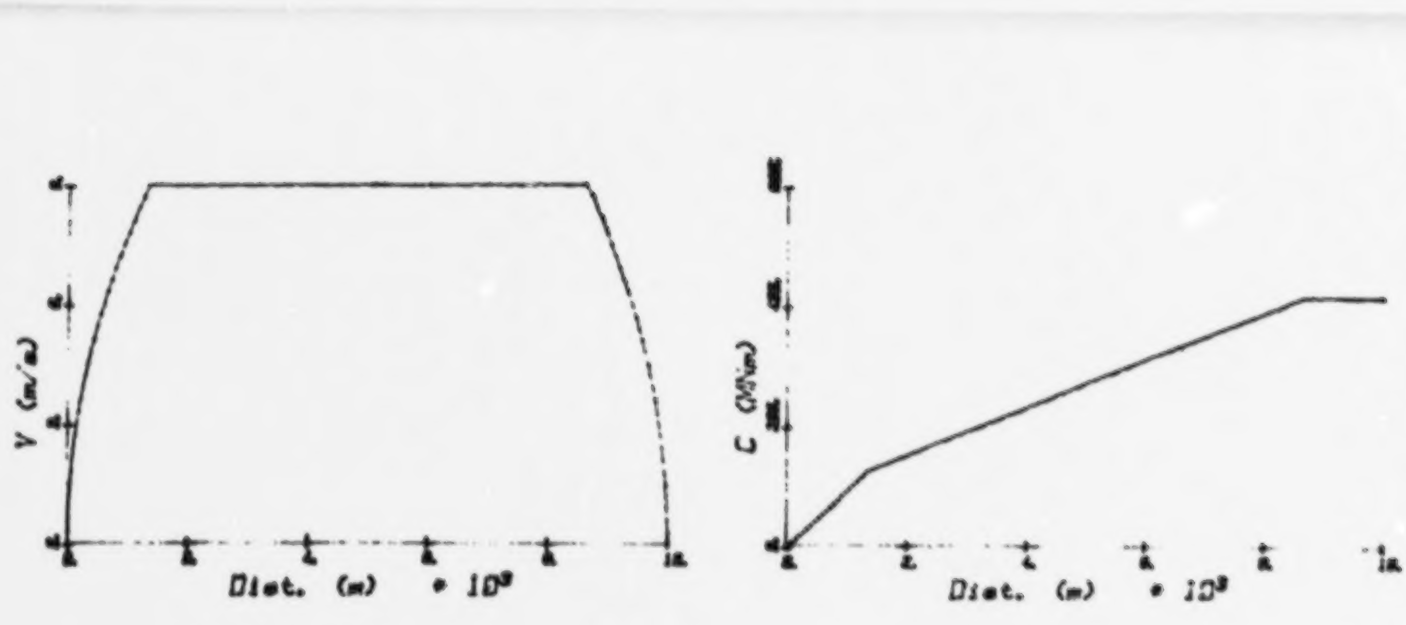
(c): Energy consumption



(b): Input force

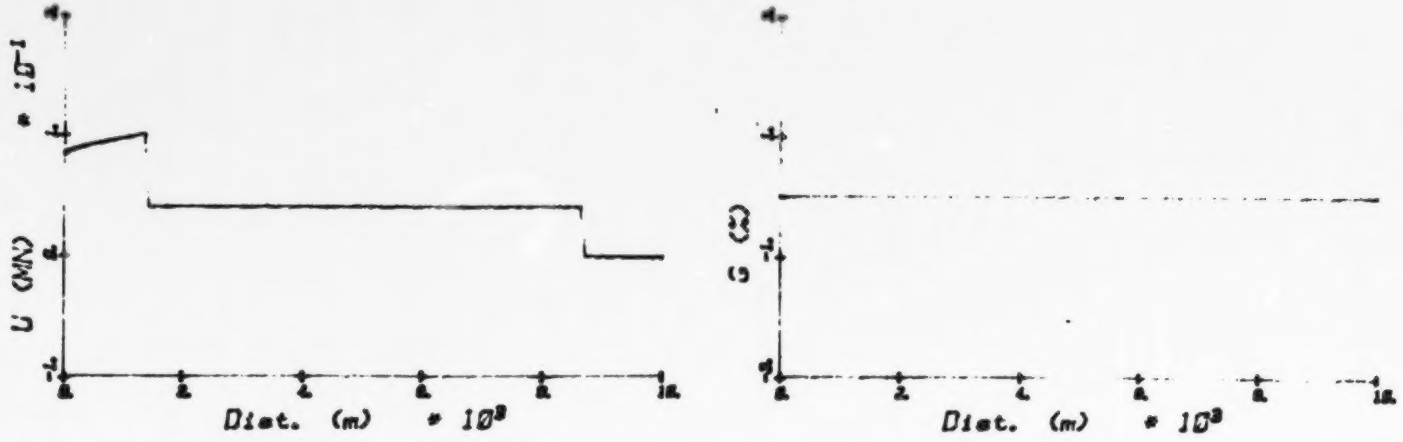
(d): Grade of sections

Figure 9 Train Simulation Results -- Travelling with Optimal (ME) Control



(a): Velocity

(c): Energy consumption



(b): Input force

(d): Grade of sections

Figure 10 Train Simulation Results -- Travelling with Non-Optimal (CA) Control

Comparison

Since both control strategies have a constant speed of 9 m/sec. in common, we can eliminate energy consumption of 181.5 MJ from each case. Hence, we get:

	Optimal (ME)	Non-Optimal (CA)
Total consumption	382.0	418.8
Common consumption	282.5	282.5
Net consumption	99.5	126.3

Thus, the energy saving using the optimal (ME) control is:

$$\frac{126.3 - 99.5}{99.5} = 26.9\%$$

6. MINIMUM FUEL CONSUMPTION (MF) CONTROL

6.1 Minimum Fuel Consumption Control Criterion

The rate of fuel consumption of a train at time t is:

$$C = C_1 P + C_2 = C_1 (P+Q) \tag{45}$$

where: $P = F^+ v/\eta$ = the power consumption of the train

C_1 and C_2 = constants

$Q = C_2/C_1$ = the penalty on idling.

The total fuel consumption of the train is:

$$\begin{aligned} J &= \int_0^T C(t)\,dt = \int_0^T (C_1 P + C_2)\,dt \\ &= \frac{C_1}{\eta} \int_0^T F^+(t)\,v(t)\,dt + C_2 T = \frac{C_1}{\eta} \int_0^T (F^+ v + \eta Q)\,dt \end{aligned} \tag{46}$$

The objective of the minimum fuel control problem is to minimize J, subject to train dynamics, constraints and boundary conditions.

Constants C_1, C_2 and Q can be found by the line fitting of real data (see Annex A).

6.2 Optimal Solution

The optimal solution can be obtained acording to Pontrjagin's maximum principle by maximizing the following Hamiltonian function with respect to F and U:

$$H\ (F,U,s,v,\lambda_1,\lambda_2) = -\frac{1}{2}\ (F^{+}|F|)\ v - \eta\, Q + \lambda_1\ v + \lambda_2\ (F+U-R-G) \tag{47}$$

$$= \begin{cases} \lambda_2\ (F+U) + \lambda_1 v - \lambda_2(R+G) - \eta Q & \text{for } F < 0 \\ (\lambda_2 - v)\ F + \lambda_2\ U + \lambda_1\ v - \lambda_2\ (R+G) - \eta Q & \text{for } F > 0 \end{cases}$$

subject to the state equations (equations (33) and (34)), the adjoint equations:

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial s} = 0 \tag{48}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial v} = \frac{1}{2}\ (F+|F|) - \lambda_1 + \frac{\partial R}{\partial v} \tag{49}$$

the boundary condition (equation (37)), the transversality condition (equation (38)) and all the contraints (equations (19) through (25)).

Since the Hamiltonian function (equation 47) differs from ME's equation (32) by only a constant term, $-\eta Q$, the maximum of H w.r.t. F and U is attended depends on λ_2, identical to the ME solution, by the following cases:

Case	Condition	Optimal Input
1	${}_{1}\quad 0$	F + U = braking with maximum force
2	${}_{2} = 0$	F + U = any braking value
3	$0 \quad {}_{2} \quad v$	U = 0, F = 0
4	${}_{2} = v$	U = 0, F = any throttling force
5	${}_{2} \quad v$	U = 0, F = driving with maximum throttling

The optimal control strategy also includes the following schemes:

(1) braking with maximum force
(2) coasting
(3) travelling at constant speed
(4) driving with maximum force.

The optimal solution can then be obtained by solving the boundary value problems of each case and the link of the solution of sub-intervals is governed by the same switching strategy described in Section 3.3.

6.3 Minimum Fuel Per Unit Distance

Consider a train travelling a distance, S, in time, T, with S as a function of time. To simplify the control criterion, let us define the MF criterion in terms of per-unit distance as follows:

$$J_F = \frac{\text{TOTAL FUEL CONSUMPTION}}{\text{TOTAL DISTANCE TRAVELLED}} = \frac{J}{S}$$

$$= \frac{\int_0^T C(t)\,dt}{\int_0^T v(t)\,dt} = \frac{C_1}{\eta}\,\frac{\int_0^T (F^+ v + \eta\, Q)\,dt}{\int_0^T v(t)\,dt} \qquad (50)$$

Since the train travels at constant speed for most of the journey, we can restrict our study to the special case where the train travels a distance, S, in time, T., with constant nominal velocity, v. The aim then is to try to find the best nominal speed, v, to minimize the MF criterion per unit distance. Assuming that the grade angle is fixed within the whole section, F+U should be constant to maintain constant velocity. The value of F+U can be obtained from the state equation (34).

$$F + U = R + G = R_{0T} + R_{1T}\, v + R_{2T}\, v^2 + m_T\, g \sin \theta \qquad (51)$$

where: R_{0T}, R_{1T} and R_{2T} = constants.

$$R_{iT} = \sum_{j=1}^{n} R_{ij} \qquad (i = 0,1,2)$$

The MF criterion (equation 50) to be minimized is:

$$J_F = \frac{C_1}{\eta} \frac{(F^{+}v + \eta Q)\ T}{v\ T} = \frac{C_1}{\eta} (F^{+} + \frac{\eta Q}{v}) \qquad (52)$$

6.4 Analytical Method for Best Nominal Velocities

The best nominal velocity can be attained by equating the derivative of equation (52) w.r.t. v to zero. This can be divided into two cases:

$F + U > 0$, i.e. $F > 0$ and $U = 0$

$$J_F = \frac{C_1}{\eta} (R_{0T} + R_{1T}v + R_{2T}v^2 + m_T\ g\ \sin 0 + \frac{\eta Q}{v}) \qquad (53)$$

$$\partial J_F/\partial v = \frac{C_1}{\eta} (R_{1T} + 2\ R_{2T}v - \frac{\eta Q}{v^2}) = 0 \qquad (54)$$

implies that the best nominal speed is the positive real root of the cubic polynomial equation:

$$2\ R_{2T}\ v^3 + R_{1T}\ v^2 - \eta\ Q = 0 \qquad (55)$$

$F + U \leq 0$, i.e. $F^{+} = 0$ and $V \leq 0$

$$J_F = \frac{C_1\ Q}{v} \qquad (54)$$

Hence, the best nominal speed is the maximum allowable speed that the train can run without positive throttle. Note that this case only occurs when the train is travelling on a severely downhill section such that the train is travelling with gravitational force.

For the case of ME, $Q = 0$, the best nominal speed for minimizing the energy consumption per unit distance is zero for the first case. This means that the train should not move at all to minimize energy consumption. Hence, the energy consumption criterion can only be considered when travelling time, T, is fixed, but optimal (MF) control can be obtained for variable travelling times. This is the major difference between the minimizations of the ME and MF criteria.

Note that the best nominal velocity is independent of the slope of grade for the case $F + U > 0$.

6.5 Overall Control Scheme

The overall MF control strategy is:

(1) compute the best nominal velocity of each section based on the given grade information and train data

(2) compute the retardation and acceleration distances for each section

(3) fill in the optimal control scheme based on the switching strategy.

6.6 Illustrative Examples

For a 3,000 hp (2.205 MW) train of 3,000 tons (mass = 2.722 M kg) with the following train data:

$$\eta = 0.82$$

$$Q = 0.08112 \text{ MW}$$

$$R_{1T} = 0.0013607 \text{ MW sec}^2/\text{m}^2$$

$$R_{2T} = 0.0001183 \text{ MW sec}^3/\text{m}^3$$

the roots of $2 R_{2T}v^3 + R_{1T}v^2 - \eta Q = 0$ can be obtained by the OPTV program (see Annex C):

$$5.0918, \; - 5.422 \pm 5.081 j \text{ (m/sec)}$$

Hence, the best nominal velocity for such a train is 5.0918 m/sec. (18.3312 km/h).

Furthermore, if R_{0T} = 0.03266 MN and C_1 = 0.07793 ℓ/MN sec., the minimum fuel consumption with the best nominal velocity travelling on level ground is:

$$J_F = \frac{C_1}{\eta} (R_{0T} + R_{1T}\, v_0 + R_{2T}\, v_0^{\,2} + \frac{\eta \ell}{v_0} = 5.2887\ \ell/\text{km}$$

7. OPTIMALITY OF THE MF CONTROL SCHEME

7.1 Comparison of Optimal (MF) and Non-Optimal Controls

Minimum fuel/energy control is introduced to reduce the energy consumption of a train travelling on grade sections. To assess its optimality, one may compare the MF control to either the CA or ME controls with a non-optimal nominal speed.

Note that there is no restriction on the travelling time requirement for comparison with the MF control.

7.2 Illustrative Examples

A train with 2.7 M kg mass was simulated over a 10 km distance on level ground. Comparison is made to the MF and non-optimal control strategies.

Optimal (MF) Control Versus Non-Optimal (ME) Control

The best nominal speed was found to be 5.092 m/sec., the optimal (MF) switching strategy and corresponding fuel consumption are:

Distance (km)	Control Scheme	Fuel (litres)
0 to 0.1148	maximum throttle	4.06
0.1148 to 9.088	constant speed	47.49 (5.293 L/km)
9.088 to 10.0	coasting (dec.)	2.23
Total consumption		53.78 (5.378 L/km)

Consider the ME control with nominal speeds of 4.0 and 6.0 m/sec., respectively. Their corresponding strategies and fuel consumptions are:

Distance (km)	Control Scheme	Fuel (litres)
0 to 0.07065	maximum throttle	2.55
0.07065 to 9.414	v_0 = 4 m/sec	50.11 (5.363 L/km)
9.414 to 10.0	coasting (dec.)	1.78
Total consumption		54.54 (5.454 L/km)

Distance (km)	Control Scheme	Fuel (litres)
0 to 0.1638	maximum throttle	5.62
0.1638 to 8.776	v_0 = 6 m/sec	45.92 (5.333 L/km)
8.776 to 10.0	coasting (dec.)	2.57
Total consumption		54.12 (5.412 L/km)

It is shown that the minimum fuel consumption control strategy requires the least fuel consumption among the ME control strategies.

Optimal (MF) Control Versus Non-Optimal (CA) Control

The optimal MF control is now compared with the CA control with various acceleration values and the best nominal speed. The results of these control strategies are listed below:

Control	Time Required (sec)	Fuel Used (litres)
Optimal (MF) Control	2,158.1	53.78 (5.378 L/km)
CA Control $a=0.05\ m/sec^2$	2,044.1	55.65 (5.565 L/km)
CA Control $a=0.04\ m/sec^2$	2,091.0	55.54 (5.554 L/km)
CA Control $a=0.03\ m/sec^2$	2,133.9	55.34 (5.534 L/km)
CA Control $a=0.02\ m/sec^2$	2,214.2	54.95 (5.495 L/km)

Thus, the MF control strategy also requires the least amount of fuel when it is compared with the CA control schemes.

8. MF CONTROL WITH QUANIZATION AND POWER CONSTRAINTS

8.1 Quantization and Contraint

Current train operation employs eight throttle positions. Each setting corresponds to a fixed ratio of the maximum power. Typical power ratios of the throttle settings are (Gruber and Bayoumi[3]):

Setting	Power Ratio
0	0.0
1	0.066
2	0.13
3	0.237
4	0.363
5	0.474
6	0.612
7	0.793
8	1.0

Note that the power ratios are not uniformly spaced.

For slow-speed train operation, the throttling force is bounded by the adhesion limit constraint (equations (9) and (20)). Therefore, the effective throttling force is:

$$F^{+} = \min \xi \; \eta P/v, \mu W$$

8.2 Method of Exhaustion for Best Nominal Velocity

Since the number of possible throttling positions is finite (=9), one can tableau the fuel consumption per unit distance for a train running with each of the fixed throttling positions along a grade section with constant grade angle.

For each throttling position, L = 0 to 8, we can obtain the power consumption for the train by:

$$p = r(L) \; P_{Tmax} \tag{55}$$

where r(L) is the power ratio corresponding to throttle position L.

The second step is to compute the constant velocity of a train travelling with the specified throttling position. For steady-state train operation without any brake,

$$F = F^{+} = R + G = R_{0T} + R_{1T}v + R_{2T}v^{2} + m_{T}\ g \sin 0 \qquad (56)$$

Hence, the constant speed, v, can be computed by solving the equation:

$$p = \frac{F^{+}\ v}{\eta} = \frac{1}{\eta}\left[\ (R_{0T} + m_{T}\ g \sin 0)\ v + R_{1T}\ v^{2} + R_{2T}\ v^{3}\ \right] \qquad (57)$$

i.e. v is the real non-negative root of the polynomial equation:

$$-\ \eta\ P + (R_{0} + mg \sin 0)\ v + R_{1}\ v^{2} + R_{2}\ v^{3} = 0 \qquad (58)$$

The effective throttling force can be calculated and limited by the constraint (equation 20):

$$F^{+} = \min \xi\ \eta\left[\ P/v;\ \mu W\ \right] \qquad (59)$$

The fuel consumption per unit distance for train operation can be computed as follows:

$$J_{F0} = C_{1}\ Q/v \qquad (60)$$

$$J_{F1} = C_{1}\ P/v \qquad (61)$$

$$J_{F} = \frac{C_{1}\ (P+Q)}{v} = J_{F0} + J_{F1} \qquad (62)$$

where: J_{F0} = fuel spend on idling

J_{F1} = fuel spent on thrust.

After we have computed the fuel consumption per unit distance for all possible throtting settings, we can easily pick up the optimal throttling positions and the corresponding best nominal velocity for minimum fuel consumption train operation along a given grade section.

8.3 Illustrative Examples

A program package called FUELTAB was created to generate a table listing of the throtting position, power available, nominal velocity, throttling force, and fuel consumption for each throttling position, L = 0 to 8, for various trains of different mass travelling on various grades. A listing of the program FUELTAB is shown in Annex D.

Consider a train with P_{max}/m_T = 1 hp/ton (0.822 W/kg) for various cases: P_{max} = 1000, 1500, 2000, 2500, 3000 hp, travelling on grades with grade angle -1 per cent, -0.5 per cent, 0 per cent, 0.5 per cnt and 1 per cent. Table listings of the FUELTAB results are shown in Annex E. A summary of the optimal setting and corresponding nominal velocities and fuel consumptions is presented in Table 1.

TABLE 1

OPTIMAL (MF) THROTTLE SETTING FOR VARIOUS TRAINS AND GRADE ANGLES

	Grade Angle (per cent)				
Horsepower	-1	-0.5	0	0.5	1
1,000	0	0	3	7	8
1,500	0	0	2	6	8
2,000	0	0	2	6	8
2,500	0	0	2	5	7
3,000	0	0	2	5	7

TABLE 1

BEST (MF) NOMINAL VELOCITIES (M/SEC) FOR
VARIOUS TRAINS AND GRADE ANGLES

	Grade Angle (per cent)				
Horsepower	-1	-0.5	0	0.5	1
1,000	39.11	24.00	8.265	7.810	5.816
1,500	39.11	24.00	5.415	6.188	5.816
2,000	39.11	24.00	5.415	6.188	5.816
2,500	39.11	24.00	5.415	4.893	5.816
3,000	39.11	24.00	5.415	4.893	4.657

TABLE 2

MF CONSUMPTION (L/KM) FOR VARIOUS TRAINS AND GRADE ANGLES

	Grade Angle (per cent)				
Horsepower	-1	-0.5	0	0.5	1
1,000	0.1614	0.2631	0.409	6.630	10.943
1,500	0.1614	0.2631	3.231	9.521	15.872
2,000	0.1614	0.2631	3.919	12.354	20.801
2,500	0.1614	0.2631	4.607	15.184	25.730
3,000	0.1614	0.2631	5.295	17.963	30.643

As the mass increases, the optimal throtte setting and the best nominal velocities decrease. The reason is that as the mass increases the amount of rolling resistance is larger, hence lowering the nominal speed will cut down fuel consumption.

We may also compare this result with the one obtained from the analytical method (see Section 6.6).

	With Quantization (exhaustion method)	Without Quantization (analytical method
Best (MF) nominal velocity (m/sec)	5.415	5.0918
MF consumption (L/km)	5.195	5.2887

9. CONCLUSION

The optimal control strategy has been designed to minimize energy and fuel consumption, and has been tested for trains travelling over various grades with known data.

A non-optimal constant acceleration control strategy has also been presented. The optimal control strategy for ME an MF consumption was compared with non-optimal control, providing a measure of the optimality for the ME and MF controls based on real simulation results.

The power constraints as well as the quantization of the throttling input is considered in the design. The proposed control strategy has very good potential for attaining minimum consumption for train operations.

Although the solution is worked out for trains, the derivation is general enough to be applied to any other moving vehicle because the whole moving body has been considered as a unit mass in the report.

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ANNEX A

ESTIMATION OF THE WEIGHTING OF THE MF CRITERION

The following is real data about the fuel consumption of a 3,000 hp train:

Throtting Setting	Fuel Consumption (gallons/hour)	P/m_L (ft^2/sec^2)
0	5.0	0.0
1	6.0	9.0
2	14.0	17.7
3	28.0	32.3
4	42.0	49.4
5	59.0	64.6
6	81.0	83.3
7	111.0	108.0
8	141.0	136.2

Consider $C = C_1 P + C_2$, best estimates of C_1 and C_2 can be obtained by minimizing w.r.t. C_1 and C_2 the sum of squares of errors:

$$SSE = \sum_{i=0}^{8} [\ C(i) - C_1\ P(i) - C_2\ ]^2$$

Equating the partial derivations of SSE to zero, one may get the system of linear equations:

$$C_1\ (\ \sum_{i=0}^{8} P(i)^2\ ) + C_2 \sum_{T=0}^{8} P(i) = \sum_{i=0}^{8} C(i)\ P(i)$$

$$C_1\ (\ \sum_{i=0}^{8} P(i)\ ) + 8\ C_2 = \sum_{i=0}^{8} C(i)$$

Hence:

$$C_1 = \frac{8 \sum_{i=0}^{8} C(i)\, P(i) - \sum_{i=0}^{8} C(i) \sum_{i=0}^{8} P(i)}{8 \sum_{i=0}^{8} P(i)^2 - \left(\sum_{i=0}^{8} P(i) \right)^2}$$

$$C_2 = \frac{8 \sum_{i=0}^{8} P(i)^2 \sum_{i=0}^{8} C(i) - \sum_{i=0}^{8} P(i) \sum_{i=0}^{8} C(i)\, P(i)}{8 \sum_{i=0}^{8} P(i)^2 - \left(\sum_{i=0}^{8} P(i) \right)^2}$$

Thus, optimal values for C_1 and C_2 can be computed using the real data. They are found to be:

C_1 = 0.0469 gallons/hp-hr = 0.0794 L/MW sec.

C_2 = 4.714 gallons/hr = 5.95×10^{-3} L/sec.

$Q = C_2/C_1$ = 0.0749 MW

A quicker way to obtain C_1 and C_2 is to use only the two end points data. Their values can be computed as follows:

C_1 = (141-5)/3,000 = 0.04533 (gallons/hp-hr) = 0.07677 L/MW sec.

C_2 = 5 gallons/hr = 6.3140×10^{-3} L/sec

$Q = C_2/C_1$ = 0.082246

ANNEX B

LFTSIM PROGRAM LISTING

```
C       PROGRAM         LFTSIM
C       PURPOSE         Long freight train minimum fuel control
C                       simulator
C       AUTHOR          K.Y. Wong
C
        PROGRAM LFTSIM
        DIMENSION DIST(0:10),GRADES(10),VMAXS(0:10)
        DIMENSION S1(10),S2(10),ICOAST(10)
        DIMENSION X(10),DX(10)
        DIMENSION TSAV(602),SSAV(602),VSAV(602),ESAV(602),
     *          USAV(602),FSAV(602)
        REAL N1,N2
        CHARACTER*1 ANS
        LOGICAL*1 TB(40),TX(10),TY(10)
        CHARACTER*40 TBB
        CHARACTER*10 TXX,TYY
        EQUIVALENCE (TB(1),TBB(1:1))
        EQUIVALENCE (TX(1),TXX(1:1))
        EQUIVALENCE (TY(1),TYY(1:1))
C
        COMMON/RES/RO,ROI,R1,R2,TNC
        COMMON/LOC/CMASS,TMASS,UW,PMAX,N1,N2,E1,Z
        COMMON/AB/A,B
        COMMON/VS/V1,V2,V3
        COMMON/ABS/A1,A2,A3,B1,B2,B3
C
      DATA IDEV/-1/
      DATA XLEN,YLEN/400.0,280.0/
      DATA SIZTIT,NTIT/4.0,40/
        DATA NTB,NTX,NTY/40,2*10/
      DATA XL,YL,FAC/125.0,75.0,0.8/
      DATA DVX,DVY/2*4.0/
        DATA DISPX,DISPY/120.,100./
C*
        CALL ASSIGN(1,'LFTSIM.INP',0)
        CALL ASSIGN(2,'LFTSIM.OUT',0)
        READ(1,1000)
        READ(1,1000)
 1000   FORMAT()
        READ(1,1001) CMASS
 1001   FORMAT(8X,G12.4)
        READ(1,1001) RO
        READ(1,1001) R1
        READ(1,1001) R2
        READ(1,1001) UW
        READ(1,1001) PMAX
        READ(1,1001) N1
        READ(1,1001) N2
        READ(1,1001) E1
        READ(1,1001) Z
        WRITE(6,*) 'NUMBER OF CARS ?'
        READ(5,*) TNC
      TMASS=CMASS*FNC
      CWEIGH=CMASS*9.81
      TWEIGH=TMASS*9.81
      V1=Z/E1
      V2=N2*FMAX/Z
      V3=N1*FMAX/UW
      WRITE(2,*) ' DATA OF TRAIN'
      WRITE(2,*) ' --------------'
      WRITE(2,2001) ' CMASS =',CMASS,'kg'
      WRITE(2,2001) ' NCAR  =',FNC
      WRITE(2,2001) ' TMASS =',TMASS,'kg'
      WRITE(2,2001) ' V1    =',V1,'m/s'
      WRITE(2,2001) ' V2    =',V2,'m/s'
      WRITE(2,2001) ' V3    =',V3,'m/s'
 2001 FORMAT(A,G12.4,1X,A)
      WRITE(2,1000)
C
      WRITE(6,*) 'NUMBER OF GRADES ?'
      READ(5,*) NG
      WRITE(6,*) 'ENTER DISTANCE OF SECTIONS:'
      READ(5,*) (DIST(I),I=1,NG)
      WRITE(6,*) 'ENTER GRADE OF SECTIONS:'
      READ(5,*) (GRADES(I),I=1,NG)
      WRITE(6,*) 'ENTER VELOCITY OF SECTIONS:'
      READ(5,*) (VMAXS(I),I=1,NG)
      WRITE(2,1000)
      WRITE(2,*) ' DATA OF GRADES'
      WRITE(2,*) ' ---------------'
      WRITE(2,2003) ' DIST  =',(DIST(I),'m   ',I=1,NG)
      WRITE(2,2003) ' GRADE =',(GRADES(I),'%   ',I=1,NG)
      WRITE(2,2003) ' VMAX  =',(VMAXS(I),'m/s',I=1,NG)
 2003 FORMAT(A,5(G12.4,1X,A))
C
      WRITE(2,1000)
      WRITE(2,*) ' DATA OF CONTROL
      WRITE(2,*) ' ---------------
      READ(1,1000)
      READ(1,1000)
      READ(1,1000)
      READ(1,1001) C1
      READ(1,1001) Q
      WRITE(6,*) 'CONTROL LAW (0=OFFLINE, 1=OPTIMAL):'
      READ(5,*) LAW
      IF (LAW-1) 110,111,111
  110 WRITE(2,*) ' OFFLINE SCHEDULE'
      CALL OFFIN(ACC,VMAXS,DIST,S1,S2,NG)
      GOTO 45
C
  111 WRITE(2,*) ' OPTIMAL SCHEDULE'
      CALL OPTIN(VMAXS,DIST,GRADES,S1,S2,ICOAST,NG)
   45 WRITE(2,2003) ' S1    =',(S1(I),'m   ',I=1,NG)
      WRITE(2,2003) ' S2    =',(S2(I),'m   ',I=1,NG)
      WRITE(2,2004) ' IC     =',(ICOAST(I),I=1,NG)
 2004 FORMAT(A,5(I5,12X))
      J=0
      I=0
      K=0
      WRITE(6,*) ' WATCH EVERY ? SEC'
      READ(5,*) ISHOW
      ISHOW=10*ISHOW
      ISCH=0
      SNEXT=0.
      SMAX=DIST(NG)
      WRITE(2,1000)
      WRITE(2,*) ' RESULTS'
      WRITE(2,*) ' -------'
      WRITE(2,*) '          time(sec)   dist(m)    vel(m/s)   ',
     *          '  force(MN)     fuel(l)   scheme'
C
      DO WHILE (S.LT.SMAX)              !start simulation
      IF (S.LT.SNEXT) GOTO 50           !
      I=I+1                             !
      ROI=RO+0.0981*GRADES(I)           !
      VMAX=VMAXS(I)                     !
      SNEXT=DIST(I)                     !
   50 CONTINUE                          !
      IF (LAW-1) 120,121,121
  120 CALL OFFSCH(S,I,V,ACC,S1,S2,U,ISCH)
      GOTO 100
  121 CALL OPTSCH(S,I,V,S1,S2,ICOAST,U,ISCH)
  100 CONTINUE
      IF (U) 51,51,52                   !
   51 IF (U.GE.(BKMAX(E1,Z,V)/CMASS)) GOTO 53
      WRITE(6,*) 'EXCEED MAX. BRAKE'    !
      GOTO 999                          !
   52 IF (U.LE.(THMAX(UW,V)/CMASS)) GOTO 53
      WRITE (6,*) 'EXCEED MAX. THROTTLE'
      GOTO 999                          !
   53 UU=U*CMASS/1.E+6                  !control (MN)
      IF (ISCH.EQ.ISCHO) GOTO 300       !
      WRITE(2,2005) ' CHANGE:',T,S,V,UU,F,ISCH
      ISCHO=ISCH                        !
  300 CONTINUE                          !
      CALL RKG(0.1,X,U,DX,4)            !train movement O.D.E.
      T=X(1)                            !time (sec)
      S=X(2)                            !distance (m)
      V=X(3)                            !speed  (m/s)
      E=X(4)                            !energy (Nm/kg)
      EE=E*CMASS/1.E+6                  !energy (MNm)
      F=C1*(EE+N1*Q*T)/N1               !fuel consumption (l)
      K=K+1                             !
      IF (K.LT.ISHOW) GOTO 200          !
      K=0                               !
      WRITE(6,*) T,S,V,UU,F,ISCH        !
```

```
        J=J+1                            !
        TSAV(J)=T                        !
        SSAV(J)=S                        !
        VSAV(J)=V                        !
        USAV(J)=UU                       !
        ESAV(J)=EE                       !
        FSAV(J)=F
  200   END DO                           !
        WRITE(6,*) T,S,V,UU,F,ISCH       !
        J=J+1                            !
        TSAV(J)=T                        !
        SSAV(J)=S                        !
        VSAV(J)=0.                       !
        USAV(J)=UU                       !
        ESAV(J)=EE                       !
        FSAV(J)=F
        WRITE(2,2005) ' FINAL :',T,S,V,UU,F,ISCH
 2005   FORMAT(A,5G12.4,I5)
        WRITE(2,2003) 'OFUEL    =',F/S,'l/m'
        WRITE(6,2003) ' FUEL    =',F/S,'l/m'
C
        WRITE(6,*) 'DO YOU WANT TO PLOT ?'
        READ(5,2006) ANS
 2006   FORMAT(A1)
        IF (ANS.NE.'Y') GOTO 999
  990   WRITE(6,*) 'X-VARIABLE = ? (0=EXIT,1=DIST.,2=TIME)'
        READ(5,*) IP
        IF (IP-1) 999,991,992
  991   TXX='Dist. (m)'
        GOTO 993
  992   TXX='Time (sec)'
        CALL BOUNDS(TSAV,J)
        WRITE(6,*) 'ENTER TMAX:'
        READ(5,*) TMAX
C
  993   CALL PLOTS('LFTME.PLT',IDEV)
        CALL PAPER(XLEN,YLEN)
C*
      CALL PLOT(0.0,0.0,-3)
      CALL SYMBOL(50.,10.,SIZTIT,
     *  'Fig.  : Train simulation results',0.0,NTIT)
        CALL ITALIC(10.)
C*    P.1
        CALL PLOT(0.0,DISPY,-3)
        TBB='Fig.  (a): Velocity'
        TYY='V (m/s)'
        VMAX=0.
        DO 600 I=1,NG
  600   IF (VMAXS(I).GT.VMAX) VMAX=VMAXS(I)
      IF (IP.EQ.1) CALL MPLT(SSAV,VSAV,J,1,0.,SMAX,0.,VMAX,
     *          DVX,DVY,TB,TX,TY,NTB,NTX,NTY,XL,YL,FAC,1)
      IF (IP.GT.1) CALL MPLT(TSAV,VSAV,J,1,0.,TMAX,0.,VMAX,
     *          DVX,DVY,TB,TX,TY,NTB,NTX,NTY,XL,YL,FAC,1)
```

```
C*    P.2
        DISPX1=DISPX/FAC
        DISPY1=DISPY/FAC
        CALL PLOT(0.0,-DISPY1,-3)
        TBB='Fig.  (b): Input force'
        TYY='U (MN)'
        CALL BOUNDS(USAV,J)
        WRITE(6,*) ' ENTER UMIN,UMAX:'
        READ(5,*) UMIN,UMAX
      IF (IP.EQ.1) CALL MPLT(SSAV,USAV,J,1,0.,SMAX,UMIN,UMAX,
     *           DVX,DVY,TB,TX,TY,NTB,NTX,NTY,XL,YL,FAC,1)
      IF (IP.GT.1) CALL MPLT(TSAV,USAV,J,1,0.,TMAX,UMIN,UMAX,
     *           DVX,DVY,TB,TX,TY,NTB,NTX,NTY,XL,YL,FAC,1)
C*    P.3
        CALL PLOT(DISPX1,DISPY1,-3)
        TBB='Fig.  (c): Fuel consumption'
        TYY='F (1)'
        CALL BOUNDS(FSAV,J)
        WRITE(6,*) ' ENTER FMAX:'
        READ(5,*) FMAX
      IF (IP.EQ.1) CALL MPLT(SSAV,FSAV,J,1,0.,SMAX,0.,FMAX,
     *           DVX,DVY,TB,TX,TY,NTB,NTX,NTY,XL,YL,FAC,1)
      IF (IP.GT.1) CALL MPLT(TSAV,FSAV,J,1,0.,TMAX,0.,FMAX,
     *           DVX,DVY,TB,TX,TY,NTB,NTX,NTY,XL,YL,FAC,1)
C*    P.4
        CALL PLOT(0.0,-DISPY1,-3)
        TBB='Fig.  (d): Grade of sections'
        TXX='Dist. (m)'
        TYY='G (%)'
      CALL MPLT(DIST,GRADES,NG+1,1,0.,SMAX,-3.,3.,DVX,DVY,
     *           TB,TX,TY,NTB,NTX,NTY,XL,YL,FAC,2)
C*
        CALL PLOT(0.0,0.0,999)
        GOTO 990
  999   STOP
        END
```

```
C***********************************************************************C
C*                                                                     *C
C*    SUBROUTINE      RKG                                              *C
C*    PURPOSE         TO SOLVE A SYSTEM OF FIRST ORDER O.D.E. WITH     *C
C*                    GIVEN INITIAL VALUE.                             *C
C*    METHOD          FORTH ORDER RUNGE-KUTTA FORMULAE MODIFIED BY     *C
C*                    GILL.                                            *C
C*    INPUT           STEP   - STEP LENGTH FOR INTEGRATION             *C
C*                    X      - INITIAL STATES                          *C
C*                    DX     - DERIVATIVES OF STATES                   *C
C*                    NX     - SYSTEM ORDER                            *C
C*    OUTPUT          X      - NEXT STATES                             *C
C*    ROUTINE NEEDED  FCT                                              *C
C*    WRITTEN BY      WONG, K.Y.                                       *C
C*    DATE            20-SEP-81                                        *C
C*    MODIFIED        12-OCT-82 FOR SINGLE INPUT CASE                  *C
C*                                                                     *C
C***********************************************************************C
      SUBROUTINE RKG(STEP,X,U,DX,NX)
      DIMENSION X(1),DX(1)
      DIMENSION A(4),B(4),C(4),Q(10)
      DATA A/0.5,0.2928932,1.707107,0.1666667/,
     *     B/2. ,1.       ,1.      ,2.       /,
     *     C/0.5,0.2928932,1.707107,0.5      /
      DO 10 J=1,4
      CALL DER(X,U,DX,NX)
      AA=A(J)
      BB=B(J)
      CC=C(J)
      DO 10 I=1,NX
      ZZ=AA*(DX(I)-BB*Q(I))
      X(I)=X(I)+STEP*ZZ
   10 Q(I)=Q(I)+3.*ZZ-CC*DX(I)
      RETURN
      END
```

```
C*********************************************************C
C*                                                       *C
C*    SUBROUTINE       DER                               *C
C*    PURPOSE          TO COMPUTE Y = THE RIGHT HAND SIDE OF THE *C
C*                     TRAIN MOVEMENT O.D.E.             *C
C*    WRITTEN BY       WONG, K.Y.                        *C
C*                                                       *C
C*********************************************************C
      SUBROUTINE DER(X,U,Y,NX)
      DIMENSION X(1),Y(1)
        COMMON/RES/R0,ROI,R1,R2,TNC
        Y(1)=1.
        Y(2)=X(3)
        R=ROI+(R1+R2*X(3))*X(3)
        IF (U) 10,10,20
   10   Y(3)=U-R
        Y(4)=0.
        GOTO 30
   20   Y(3)=U/TNC-R
        Y(4)=U*X(3)
   30   RETURN
      END
```

```
C
      FUNCTION SCOAST(V)
      COMMON/AB/A,B
      XX=A*V/B+1.
      IF (XX.LT.0) XX=-XX
      SCOAST=(V-B*ALOG(XX))/A)/A        !coasting distance
      RETURN
      END
```

```
C
      FUNCTION SSTOF(V)
      COMMON/VS/V1,V2,V3
      COMMON/ABS/A1,A2,A3,B1,B2,B3
      DV=V-V1
      IF (DV) 10,10,20
10    XX=A1*V/B1+1.
      IF (XX.LT.0) XX=-XX
      SSTOF=(V-B1*ALOG(XX)/A1)/A1
      GOTO 50
20    XX=A1*V1/B1+1.
      IF (XX.LT.0) XX=-XX
      SSTOF=(V1-B1*ALOG(XX)/A1)/A1
      DV=V-V2
      IF (DV) 30,30,40
30    XX=(A2*V+B2)/(A2*V1+B2)
      IF (XX.LT.0) XX=-XX
      SSTOF=SSTOF+(V-V1-B2*ALOG(XX)/A2)/A2
      GOTO 50
40    XX=(A2*V2+B2)/(A2*V1+B2)
      IF (XX.LT.0) XX=-XX
      SSTOF=SSTOF+(V2-V1-B2*ALOG(XX)/A2)/A2
      XX=(A3*V+B3)/(A3*V2+B3)
      IF (XX.LT.0) XX=-XX
      SSTOF=SSTOF+(V -V2-B3*ALOG(XX)/A3)/A3
50    RETURN
      END
```

```
C
      FUNCTION SACC(V)
      COMMON/VS/V1,V2,V3
      COMMON/ABS/A1,A2,A3,B1,B2,B3
      DV=V-V3
      IF (DV) 10,10,20
10    XX=(A1*V+B1)/B1
      IF (XX.LT.0) XX=-XX
      SACC=(B1*ALOG(XX)/A1-V)/A1
      GOTO 30
20    XX=(A1*V3+B1)/B1
      IF (XX.LT.0) XX=-XX
      S5=(B1*ALOG(XX)/A1-V3)/A1
      XX=(A2*V3+B2)/(A2*V+B2)
      IF (XX.LT.0) XX=-XX
      SACC=S5+(-V+V3-B2*ALOG(XX)/A2)/A2
30    RETURN
      END
      FUNCTION BKMAX(E1,Z,V)
      COMMON/VS/V1,V2,V3
      IF (V-V1) 10,10,20
10    BKMAX=-E1*V
      GOTO 50
20    IF (V-V2) 30,30,40
30    BKMAX=-Z
      GOTO 50
40    BKMAX=-Z*V2/V
50    RETURN
      END
```

```
      FUNCTION THMAX(UW,V)
      COMMON/VS/V1,V2,V3
      IF (V-V3) 10,10,20
10    THMAX=UW
      GOTO 30
20    THMAX=UW*V3/V
30    RETURN
      END
```

```
C
C     SUBROUTINE      BOUNDS
C     PURPOSE         PRINT OUT THE BOUNDS OF A VECTOR
C
      SUBROUTINE BOUNDS(X,NX)
      DIMENSION X(1)
      REAL XMIN,XMAX
      XMIN=X(1)
      XMAX=X(1)
      DO 10 I=1,NX
      IF (X(I).LT.XMIN) XMIN=X(I)
   10 IF (X(I).GT.XMAX) XMAX=X(I)
      WRITE(6,*) 'BOUNDS:'
      WRITE(6,601) XMIN,XMAX
  601 FORMAT(2G12.4)
      RETURN
      END
      SUBROUTINE OFFSCH(S,I,V,ACC,S1,S2,U,ISCH)
      DIMENSION S1(1),S2(1)
      COMMON/RES/R0,R0I,R1,R2,TNC
      R=R0I+(R1+R2*V)*V
      IF (S-S2(I)) 234,12,12              !
234   IF (S-S1(I)) 24,3,3                 !
 12   U=R-ACC                             !scheme 12 -- deceleration
      ISCH=12                             !
      GOTO 100                            !
 24   U=R+ACC                             !scheme 24 -- acceleration
      ISCH=24                             !
      GOTO 100                            !
  3   U=R                                 !scheme 3 -- constant velocity
      ISCH=3
100   IF (U.GT.0.) U=U*TNC                !throttle
      RETURN
      END



      SUBROUTINE OFFIN(ACC,VMAXS,DIST,S1,S2,NG)
      DIMENSION VMAXS(0:10),DIST(0:10),S1(1),S2(1)
      WRITE(6,*) 'ACCELERATION = ? (m/s**2)'
      READ(5,*) ACC
      WRITE(2,2003) ' ACC   =',ACC,'m/s**2'
2003  FORMAT(A,5(G12.4,1X,A))
      DO 30 I=NG,1,-1
      SA1=VMAXS(I)**2/(2.*ACC)
      SA0=VMAXS(I-1)**2/(2.*ACC)
      SA2=VMAXS(I+1)**2/(2.*ACC)
      S1(I)=DIST(I-1)
      S2(I)=DIST(I)
      IF (VMAXS(I).GT.VMAXS(I-1)) S1(I)-DIST(I-1)+SA1-SA0
 30   IF (VMAXS(I).GT.VMAXS(I+1)) S2(I)=DIST(I)+SA2-SA1
      RETURN
      END
      SUBROUTINE OPTI (VMAXS,DIST,GRADES,S1,S2,ICOAST,NG)
      DIMENSION VMAXS(0:10),DIST(0:10),GRADES(1),S1(1),S2(1),
     *        ICOAST(1)
      COMMON/RES/R0,R0I,R1,R2,TNC
      COMMON/LOC/CMASS,TMASS,UW,PMAX,N1,N2,E1,Z
      COMMON/AB/A,B
      COMMON/VS/V1,V2,V3
      COMMON/ABS/A1,A2,A3,B1,B2,B3
      DO 40 I=1,NG                                !fill in the opt. scheme
      VM0=VMAXS(I-1)                              !
      VM1=VMAXS(I)                                !
      VM2=VMAXS(I+1)                              !
      S1(I)=DIST(I-1)
      S2(I)=DIST(I)
      R0I=R0+0.0981*GRADES(I)                     !
      WRITE(2,*) ' GRADE',I
      A=R1+R2*VM1                             !
      B=ROI-R2*VM1*VM1/6.0                    !
      WRITE(2,2002) ' A,B   =',A,B
2002  FORMAT(A,5(G12.4,4X))
      IF (B) 21,21,22                         !
  21  ICOAST(I)=1                             !coasting do not slow down
      A1=R1+E1/CMASS+R2*V1                    !
      B1=ROI-R2*V1*V1/6.0                     !
      A2=R1+R2*(V1+V2)                        !
      B2=ROI+Z/CMASS-R2*(V2*V2+4.*V1*V2+V1*V1)/6.0
      VV=Z*V2/CMASS                           !
      WW=VM1-V2                               !
      XX=WW**-2                               !
      YY=ALOG(VM1/V2)/WW                      !
      ZZ=VM1+V2                               !
      A3=12.*VV*XX*(1.-0.5*ZZ*YY)+R1+R2*ZZ    !
      UU=ZZ*ZZ/4.-WW*WW/12.                   !
      B3=VV*XX*(-6.*ZZ+(3.*ZZ*ZZ+WW*WW)*YY)+ROI-R2*UU
      WRITE(2,2002) ' A1,B1 =',A1,B1          !
      WRITE(2,2002) ' A2,B2 =',A2,B2          !
      WRITE(2,2002) ' A3,B3 =',A3,B3          !
      SS1=SSTOP(VM1)                          !brake stopping dist.
      SS2=SSTOP(VM2)                          !
      WRITE(2,2003) ' SS1,2 =',SS1,'m',SS2,'m'!
2003  FORMAT(A,5(G12.4,1X,A))
      IF (VM1.GT.VM2) S2(I)=DIST(I)-SS1+SS2   !
      SC1=-SCOAST(VM1)                        !coasting acc. dist.
      SC0=-SCOAST(VM0)                        !
      WRITE(2,2003) ' SC0,1 =',SC0,'m',SC1,'m'!
      IF (VM0.LT.VM1) S1(I)=DIST(I-1)+SC1-SC0 !
      GOTO 40                                 !
  22  ICOAST(I)=-1.                           !coasting do slow down
      SC1=SCOAST(VM1)                         !
      SC2=SCOAST(VM2)                         !coasting dist.
      WRITE(2,2003) ' SC1,2 =',SC1,'m',SC2,'m'!
      IF (VM1.GT.VM2) S2(I)=DIST(I)-SC1+SC2   !
C     IF (S2(I).GE.DIST(I-1))                 !
      A1=R1+R2*V3                             !
      B1=ROI-UW/TMASS-R2*V3*V3/6.0            !
      VV=UW*V3/TMASS                          !
      WW=VM1-V3                               !
      XX=WW**-2                               !
      YY=ALOG(VM1/V3)/WW                      !
      ZZ=VM1+V3                               !
      A2=12.*VV*XX*(0.5*ZZ*YY-1.)+R1+R2*ZZ    !
      UU=ZZ*ZZ/4.-WW*WW/12.                   !
      B2=VV*XX*(6.*ZZ-(3.*ZZ*ZZ+WW*WW)*YY)+ROI-R2*UU
      WRITE(2,2002) ' A1,B1 =',A1,B1          !
      WRITE(2,2002) ' A2,B2 =',A2,B2          !
      SA0=SACC(VM0)                           !full power acc. dist.
      SA1=SACC(VM1)                           !
      WRITE(2,2003) ' SAO,1 =',SAO,'m',SA1,'m' !
      IF (VMO.LT.VM1) S1(I)=DIST(I-1)+SA1-SAO !
 40   CONTINUE                                !
      RETURN
      END




      SUBROUTINE OPTSCH(S,I,V,S1,S2,ICOAST,U,ISCH)
      DIMENSION S1(1),S2(1),ICOAST(1)
      COMMON/RES/RO,ROI,R1,R2,TNC
      COMMON/LOC/CMASS,TMASS,UW,PMAX,N1,N2,E1,Z
      COMMON/VS/V1,V2,V3
      IF (S-S2(1)) 234,12,12              !
 12   IF (ICOAST(I)) 2,1,1                !schemes for slow down
234   IF (S-S1(I)) 24,3,3                 !
 24   IF (ICOAST(I)) 4,4,2                !schemes for accelerating
  1   U=BKMAX(E1,Z,V)/CMASS               !scheme 1 -- max. brake
      ISCH=1                              !
      GOTO 100                            !
  2   U=0.0                               !scheme 2 -- coasting
      ISCH=2                              !
      GOTO 100                            !
  3   U=ROI+(R1+R2*V)*V                   !scheme 3 -- constant speed
      IF (U.GT.0.) U=U*TNC                !throttle
      ISCH=3                              !
      GOTO 100                            !
  4   U=THMAX(UW,V)/CMASS                 !scheme 4 -- max. throttle
      ISCH=4                              !
100   CONTINUE                            !
      RETURN
      END
```

ANNEX C

OPTV PROGRAM LISTING

```
C        PROGRAM             OPTV
C        PURPOSE             OBTAIN THE BEST NOMINAL VELOCITY BY COMPUTING
C                            THE REAL AND COMPLEX ROOTS OF A REAL POLYNOMIAL
C        AUTHOR              WONG, K.Y.
C
         PROGRAM OPTV
         DIMENSION A(37),W(37),ROOTR(37),ROOTI(37)
         REAL N,MM
         MM=1.0E+6
         CALL ASSIGN(2,'OPTV.OUT',0)
         IORD=3
         J=IORD+1
         Q=8.1121215E+4  !W
         N=0.82
         WRITE(6,*) 'ENTER TMASSO,TMASSF,TMASSI (kg*10**6):'
         READ(5,*) TMO,TMF,TMI
         K=IFIX((TMF-TMO)/TMI)
         TMASS=TMO*MM
         A(1)=-N*Q/MM
         A(2)=0.0
         DO 5 II=0,K
         TMASS=TMASS+TMI*MM
         R1=0.0005*TMASS
         R2=4.35E-5*TMASS
         WRITE(2,2000) II
 2000    FORMAT(//' ROOTS OF CUBIC EQUATION',I3,' FOR NOMINAL VEL',
     *          /' --------------------------------------'/)
         WRITE(2,2001) ' TMASS =',TMASS,'(kg)'
         WRITE(2,2001) ' R1    =',R1,'(Ns/m)'
         WRITE(2,2001) ' R2    =',R2,'(Ns**2/m**2)'
 2001    FORMAT(A,G16.6,1X,A)
         A(3)=R1/MM
         A(4)=(R2+R2)/MM
         WRITE(2,40)
   40    FORMAT(/' THE COEFFICIENTS ARE:'/)
         WRITE(2,50) (A(I),I=1,J)
   50    FORMAT(6G12.4)
         CALL POLRT(A,W,IORD,ROOTR,ROOTI,IER)
         IF (IER-1) 90,60,70
   60    WRITE(6,65)
   65    FORMAT(//' ORDER OF POLYNOMIAL LESS THAN ONE')
         GOTO 5
   70    IF (IER-3) 75,80,78
   75    WRITE(6,77)
   77    FORMAT(//' ORDER OF POLYNOMIAL GREATER THAN 36')
         GOTO 5
   78    WRITE(6,79)
   79    FORMAT(//' HIGH ORDER COEFFICIENT IS ZERO')
         GOTO 5
   80    WRITE(6,85)
   85    FORMAT(//' UNABLE TO DETERMINE ROOT. THOSE ALREADY FOUND ARE')
   90    WRITE(2,95)
 96    FORMAT(//4X,'REAL ROOT',6X,'COMPLEX ROOT'//)
       WRITE(2,97) (ROOTR(I),ROOTI(I),I=1,IORD)
 97    FORMAT(1X,2G16.7)
  5    CONTINUE
100    STOP
       END
```

```
C      SUBROUTINE        POLRT
C      PURPOSE           COMPUTE THE REAL AND COMPLEX ROOTS
C                        OF A REAL POLYNOMIAL
       SUBROUTINE POLRT(XCOF,COF,M,ROOTR,ROOTI,IER)
       DIMENSION XCOF(1),COF(1),ROOTR(1),ROOTI(1)
       DOUBLE PRECISION X0,Y0,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,
     1 SUMSQ,DX,DY,TEMP,ALPHA
       IFIT=0
       N=M
       IER=0
       IF (XCOF(N+1)) 10,25,10
 10    IF (N) 15,15,32
 15    IER=1
 20    RETURN
 25    IER=4
       GOTO 20
 30    IER=2
       GOTO 20
 32    IF (N-36) 35,35,30
 35    NX=N
       NXX=N+1
       N2=1
       KJ1=N+1
       DO 40 L=1,KJ1
       MT=KJ1-L+1
 40    COF(MT)=XCOF(L)
 45    X0=0.00500101
       Y0=0.01000101
       IN=0
 50    X=X0
       X0=-10.0*Y0
       Y0=-10.0*X
       X=X0
       Y=Y0
       IN=IN+1
       GOTO 59
 55    IFIT=1
       XPR=X
       YPR=Y
 59    ICT=0
```

```
60    UX=0.0
      UY=0.0
      V=0.0
      YT=0.0
      XT=1.0
      U=COF(N+1)
      IF (U) 65,130,65
65    DO 70 I=1,N
      L=N-I+1
      TEMP=COF(L)
      XT2=X*XT-Y*YT
      YT2=X*YT+Y*XT
      U=U+TEMP*XT2
      V=V+TEMP*YT2
      FI=I
      UX=UX+FI*XT*TEMP
      UY=UY-FI*YT*TEMP
      XT=XT2
70    YT=YT2
      SUMSQ=UX*UX+UY*UY
      IF (SUMSQ) 75,110,75
75    DX=(V*UY-U*UX)/SUMSQ
      X=X+DX
      DY=-(U*UY+V*UX)/SUMSQ
      Y=Y+DY
78    IF (DABS(DY)+DABS(DX)-1.0D-05) 100,80,80
80    ICT=ICT+1
      IF (ICT-500) 60,85,85
85    IF (IFIT) 100,90,100
90    IF (IN-5) 50,95,95
95    IER=3
      GOTO 20
100   DO 105 L=1,NXX
      MT=KJ1-L+1
      TEMP=XCOF(MT)
      XCOF(MT)=COF(L)
105   COF(L)=TEMP
      ITEMP=N
      N=NX
      NX=ITEMP
      IF (IFIT) 120,55,120
110   IF (IFIT) 115,50,115
115   X=XPR
      Y=YPR
120   IFIT=0
122   IF(DABS(Y)-1.0D-4*DABS(X)) 135,125,125
125   ALPHA=X+X
      SUMSQ=X*X+Y*Y
      N=N-2
      GOTO 140
130   X=0.0
      NX=NX-1
      NXX=NXX-1
135   Y=0.0
      SUMSQ=0.0
      ALPHA=X
      N=N-1
140   COF(2)=COF(2)+ALPHA*COF(1)
145   DO 150 L=2,N
150   COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
155   ROOTI(N2)=Y
      ROOTR(N2)=X
      N2=N2+1
      IF (SUMSQ) 160,165,160
160   Y=-Y
      SUMSQ=0.0
      GOTO 155
165   IF (N) 20,20,45
      END
```

ANNEX D

FUELTAB PROGRAM LISTING

```
C       PROGRAM             FUELTAB
C       PURPOSE             GENERATE A TABLE LISTING THE FOLLOWINGS:
C                           L -- THROTTLING POSITION
C                           P -- POWER AVAILIABLE
C                           V -- VELOCITY
C                           U -- THRUST
C                           FO-- FUEL SPEND ON IDLING
C                           F1-- FUEL SPEND ON THRUST
C                           F -- TOTAL FUEL CONSUMPTION
C
C       AUTHOR              WONG, K.Y.
C
        PROGRAM FUELTAB
        DIMENSION A(37),W(37),ROOTR(37),ROOTI(37)
        DIMENSION PS(9)
        REAL N,MM
        DATA PS/0.,0.1457,0.2866,0.5229,0.7998,
     *  1.0458,1.3486,1.7485,2.205/
        TT=1000.          !ONE THOUSAND
        MM=1.0E+6         !ONE MILLION
        CALL ASSIGN(2,'FUELTAB.OUT',0)
        IORD=3
        NA=IORD+1
        TON3K=2.715542E+6            !kg
        C1=0.0778342E-6              !litre/Ws
        Q=0.0811212E+6               !W
        N=0.82                       !
        UW=0.347                     !MN (u=0.2, W=195 tons)
C
        WRITE(6,*) 'ENTER TMASSO,TMASSF,TMASSI (kg*10**6):
        READ(5,*) TMO,TMF,TMI
        IF (TMO.LE.0.) TMO=TON3K
        IF (TMF.LE.0.) TME=TON3K
        IF (TMI.LE.0.) TMI=1.0
        K=IFIX((TMF-TMO)/TMI)+1
        TMASS=(TMO-TMI)*MM
C
        WRITE(6,*) 'ENTER GRADE ANGLE: GO,GF,GI (%):'
        READ(5,*) GO,GF,GI
        IF (GI.LE.0.0) GI=1.0
        NG=IFIX((GF-GO)/GI)+1
C
        DO 5 II=1,K                  !FOR VARIOUS TRAIN MASS
        TMASS=TMASS+TMI*MM
        RO=0.012*TMASS               !N
        R1=0.0005*TMASS              !Ns/m
        R2=4.35E-5*TMASS             !Ns**2/m**2
        G=GO-GI
        DO 5 JJ=1,NG                 !FOR VARIOUS GRADE ANGLES
        G=G+GI
        WRITE(2,2001) '1DATA OF TRAIN & GRADE'
        WRITE(2,2001) ' ------------------------'
```

```
      WRITE(2,2001) ' TMASS =',TMASS,'kg'
      WRITE(2,2001) ' Grade =',G,'%'
      WRITE(2,2001) ' R0    =',R0,'N'
      WRITE(2,2001) ' R1    =',R1,'Ns/m'
      WRITE(2,2001) ' R2    =',R2,'Ns**2/m**2'
      WRITE(2,2001) ' C1    =',C1*MM,' litre/MWs'
      WRITE(2,2001) ' Q     =',Q/MM,'MW'
      WRITE(2,2001) ' n     =',N,' '
 2001 FORMAT(A,G16.6,1X,A)
!
      WRITE(2,2001) '0TABLE OF FUEL CONSUMPTION'
      WRITE(2,2001) ' --------------------------'
      WRITE(2,*) '|  L  |  P(MW)  |  V(m/s) |  U(MN)  | F0(l/km)| ',
     *           F1(l/km)| F(l/km) |'
      DO 10 L=0,8                    !FOR VARIOUS THROTTLE SETTINGS
      P=PS(L+1)*TMASS/TON3K
      A(1)=-N*P                      !
      A(2)=(R0+TMASS*0.0981*G)/MM    !R0+mgsin0 (MW)
      A(3)=R1/MM
      A(4)=R2/MM
      PRINT*, 'A=',(A(I),I=1,NA)!!!!!
      CALL POLRT(A,W,IORD,ROOTR,ROOTI,IER)
      IF (IER-1) 90,60,70
   60 WRITE(6,65)
   65 FORMAT(//' ORDER OF POLYNOMIAL LESS THAN ONE')
      GOTO 5
   70 IF (IER-3) 75,80,78
   75 WRITE(6,77)
   77 FORMAT(//' ORDER OF POLYNOMIAL GREATER THAN 36')
      GOTO 5
   78 WRITE(6,79)
   79 FORMAT(//' HIGH ORDER COEFFICIENT IS ZERO')
      GOTO 5
   80 WRITE(6,85)
   85 FORMAT(//' UNABLE TO DETERMINE ROOT. THOSE ALREADY FOUND ARE')
   90 V=0.
      DO 95 I=IORD,1,-1
      IF (ABS(ROOTI(I)).GT.1.E-6) GOTO 95
      IF (ROOTR(I).LT.0.) GOTO 95
      V=ROOTR(I)
      GOTO 97
   95 CONTINUE
   97 CONTINUE
      IF (V.EQ.0.0) V=1.E-6
      U=N*P/V                        !MN
      IF (U.GT.UW) U=UW
      F1=C1*P*MM/V
      F0=C1*Q/V
      F=F1+F0
   10 WRITE(2,2002) L,P,V,U,F0*TT,F1*TT,F*TT
 2002 FORMAT(I5,1X,7F10.1)
      WRITE(2,2001) '0L -- THROTTLING POSITION'
      WRITE(2,2001) ' P -- POWER AVAILIABLE (MW)'
      WRITE(2,2001) ' V -- VELOCITY (m/s)'
      WRITE(2,2001) ' U -- THRUST (MN)'
      WRITE(2,2001) ' F0-- FUEL SPEND ON IDLING (l/km)'
      WRITE(2,2001) ' F1-- FUEL SPEND ON THRUST (l/km)'
      WRITE(2,2001) ' F -- TOTAL FUEL CONSUMPTION (l/km)'
5     CONTINUE
      STOP
      END
```

ANNEX E

TABLE LISTING OF THE FUELTAB RESULTS

The following is a listing of throttle positions and the corresponding fuel consumptions for the steady-state operation of a freight train moving along a fixed grade. Various train mass and grade angles are considered. The maximum power of the trains is chosen to keep the power-to-mass ratio at one hp/ton (0.822 W/kg).

DESCRIPTION OF SYMBOLS

L: throttle position, $L=0$ to 8

P: power available (MW), $P=0$ to P_{max}

V: velocity (m/s), V = positive real root of the polynomial equation $- P + (R_0 + m\ g \sin 0)\ v + R_1\ v^2 + R_2\ v^3 = 0$

U: thrust (MN), $U = \text{sat}\ (\ P/v,\ W)$

F_0: fuel spent on idling (L/km), $F_0 = C_1\ Q/v$

F_1: fuel spent on thrust (L/km), $F_1 = C_1\ P/v$

F: total fuel consumption (L/km), $F = F_0 + F_1$

SUMMARY OF OPTIMAL SETTINGS FOR EACH CASE

Train Mass (tons)	Train Mass (M kg)	Grade Angle -1%	-0.5%	0%	0.5%	1%
1,000	0.907	0	0	3	7	8
1,500	1.361	0	0	2	6	8
2,000	1.814	0	0	2	6	8
2,500	2.268	0	0	2	5	8
3,000	2.722	0	0	2	5	7

```
DATA OF TRAIN & GRADE
---------------------
TMASS =       907185.      kg
Grade =     -1.00000       %
R0    =       10886.2      N
R1    =       453.592      Ns/m
R2    =       39.4625      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L  |   P(MW)   |   V(m/s)  |   U(MN)    | F0(1/km)| F1(1/km)| F(1/km)
   0       0.0000    39.1120      0.0000      0.1614     0.0000     0.1614
   1       0.0487    39.3972      0.0010      0.1603     0.0962     0.2564
   2       0.0957    39.6676      0.0020      0.1592     0.1879     0.3470
   3       0.1747    40.1096      0.0036      0.1574     0.3390     0.4964
   4       0.2672    40.6107      0.0054      0.1555     0.5121     0.6676
   5       0.3494    41.0420      0.0070      0.1538     0.6626     0.8164
   6       0.4505    41.5563      0.0089      0.1519     0.8438     0.9958
   7       0.5841    42.2101      0.0113      0.1496     1.0771     1.2267
   8       0.7366    42.9245      0.0141      0.1471     1.3357     1.4828
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =       907185.      kg
Grade =     -0.500000      %
R0    =       10886.2      N
R1    =       453.592      Ns/m
R2    =       39.4625      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L  |   P(MW)   |   V(m/s)  |   U(MN)    | F0(1/km)| F1(1/km)| F(1/km)
   0       0.0000    23.9977      0.0000      0.2631     0.0000     0.2631
   1       0.0487    24.6788      0.0016      0.2558     0.1535     0.4094
   2       0.0957    25.2918      0.0031      0.2496     0.2946     0.5443
   3       0.1747    26.2387      0.0055      0.2406     0.5182     0.7588
   4       0.2672    27.2457      0.0080      0.2317     0.7633     0.9950
   5       0.3494    28.0672      0.0102      0.2250     0.9689     1.1938
   6       0.4505    29.0025      0.0127      0.2177     1.2091     1.4268
   7       0.5841    30.1350      0.0159      0.2095     1.5087     1.7182
   8       0.7366    31.3144      0.0193      0.2016     1.8309     2.0326
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =      907185.      kg
Grade =    0.000000E+00 %
RO    =      10886.2      N
R1    =      453.592      Ns/m
R2    =      39.4625      Ns**2/m**2
C1    =    0.778342E-01 litre/MWs
Q     =    0.811212E-01 MW
n     =    0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L   |  P(MW)   |  V(m/s) |   U(MN)   | FO(1/km)| F1(1/km)| F(1/km)
   0      0.0000     0.0000     0.0000**********     0.0000**********
   1      0.0487     3.1424     0.0127     2.0093     1.2056     3.2149
   2      0.0957     5.4148     0.0145     1.1661     1.3763     2.5423
   3      0.1747     8.2651     0.0173     0.7639     1.6451     2.4090
   4      0.2672    10.7682     0.0203     0.5864     1.9313     2.5176
   5      0.3494    12.5602     0.0228     0.5027     2.1650     2.6677
   6      0.4505    14.4161     0.0256     0.4380     2.4325     2.8704
   7      0.5841    16.4758     0.0291     0.3832     2.7595     3.1427
   8      0.7366    18.4638     0.0327     0.3420     3.1053     3.4472


DATA OF TRAIN & GRADE
---------------------
TMASS =      907185.      kg
Grade =    0.500000       %
RO    =      10886.2      N
R1    =      453.592      Ns/m
R2    =      39.4625      Ns**2/m**2
C1    =    0.778342E-01 litre/MWs
Q     =    0.811212E-01 MW
n     =    0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L   |  P(MW)   |  V(m/s) |   U(MN)   | FO(1/km)| F1(1/km)| F(1/km)
   0      0.0000     0.0000     0.0000**********     0.0000**********
   1      0.0487     0.7162     0.0557     8.8160     5.2898    14.1058
   2      0.0957     1.3996     0.0561     4.5113     5.3246     9.8359
   3      0.1747     2.5228     0.0568     2.5028     5.3895     7.8922
   4      0.2672     3.7987     0.0577     1.6621     5.4746     7.1367
   5      0.3494     4.8932     0.0585     1.2904     5.5574     6.8477
   6      0.4505     6.1880     0.0597     1.0204     5.6669     6.6872
   7      0.5841     7.8096     0.0613     0.8085     5.8217     6.6302
   8      0.7366     9.5417     0.0633     0.6617     6.0089     6.6706
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =       907185.      kg
Grade =       1.00000      %
R0    =       10886.2      N
R1    =       453.592      Ns/m
R2    =       39.4625      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L   |  P(MW)   |  V(m/s) |   U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0      0.0000     0.0000     0.0000**********     0.0000*********
   1      0.0487     0.3989     0.1001    15.8303     9.4985   25.3287
   2      0.0957     0.7831     0.1003     8.0632     9.5167   17.5799
   3      0.1747     1.4238     0.1006     4.4347     9.5496   13.9842
   4      0.2672     2.1682     0.1011     2.9121     9.5916   12.5037
   5      0.3494     2.8232     0.1015     2.2365     9.6321   11.8686
   6      0.4505     3.6205     0.1020     1.7440     9.6857   11.4296
   7      0.5841     4.6571     0.1028     1.3558     9.7624   11.1182
   8      0.7366     5.8162     0.1039     1.0856     9.8578   10.9434


DATA OF TRAIN & GRADE
---------------------
TMASS =     0.136078E+07 ks
Grade =      -1.00000      %
R0    =       16329.3      N
R1    =       680.389      Ns/m
R2    =       59.1938      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L   |  P(MW)   |  V(m/s) |   U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0      0.0000    39.1120     0.0000     0.1614     0.0000    0.1614
   1      0.0730    39.3972     0.0015     0.1603     0.1442    0.3045
   2      0.1436    39.6676     0.0030     0.1592     0.2818    0.4410
   3      0.2620    40.1096     0.0054     0.1574     0.5085    0.6659
   4      0.4008    40.6107     0.0081     0.1555     0.7681    0.9236
   5      0.5241    41.0420     0.0105     0.1538     0.9939    1.1477
   6      0.6758    41.5563     0.0133     0.1519     1.2657    1.4177
   7      0.8762    42.2101     0.0170     0.1496     1.6157    1.7652
   8      1.1049    42.9245     0.0211     0.1471     2.0036    2.1507
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.136078E+07 kg
Grade =    -0.500000      %
R0    =      16329.3      N
R1    =      680.389      Ns/m
R2    =      59.1938      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L   |  P(MW)   |   V(m/s)  |    U(MN)    | F0(1/km)| F1(1/km)| F(1/km)
   0      0.0000    23.9977     0.0000       0.2631     0.0000     0.2631
   1      0.0730    24.6788     0.0024       0.2558     0.2303     0.4861
   2      0.1436    25.2918     0.0047       0.2496     0.4420     0.6916
   3      0.2620    26.2387     0.0082       0.2406     0.7773     1.0179
   4      0.4008    27.2457     0.0121       0.2317     1.1449     1.3767
   5      0.5241    28.0672     0.0153       0.2250     1.4533     1.6782
   6      0.6758    29.0025     0.0191       0.2177     1.8136     2.0313
   7      0.8762    30.1350     0.0238       0.2095     2.2631     2.4726
   8      1.1049    31.3144     0.0289       0.2016     2.7464     2.9480


DATA OF TRAIN & GRADE
---------------------
TMASS =     0.136078E+07 kg
Grade =     0.000000E+00 %
R0    =      16329.3     N
R1    =      680.389     Ns/m
R2    =      59.1938     Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L   |  P(MW)   |   V(m/s)  |    U(MN)    | F0(1/km)| F1(1/km)| F(1/km)
   0      0.0000     0.0000     0.0000**********     0.0000**********
   1      0.0730     3.1424     0.0191       2.0093     1.8084     3.8177
   2      0.1436     5.4148     0.0217       1.1661     2.0644     3.2305
   3      0.2620     8.2651     0.0260       0.7639     2.4676     3.2315
   4      0.4008    10.7682     0.0305       0.5864     2.8969     3.4833
   5      0.5241    12.5602     0.0342       0.5027     3.2475     3.7502
   6      0.6758    14.4161     0.0384       0.4380     3.6487     4.0867
   7      0.8762    16.4758     0.0436       0.3832     4.1392     4.5225
   8      1.1049    18.4638     0.0491       0.3420     4.6579     4.9998
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.136078E+07 kg
Grade =     0.500000      %
R0    =      16329.3      N
R1    =      680.389      Ns/m
R2    =      59.1938      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L  |   P(MW)   |   V(m/s)  |   U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0      0.0000      0.0000      0.0000**********      0.0000**********
   1      0.0730      0.7162      0.0836     8.8160      7.9346   16.7506
   2      0.1436      1.3996      0.0841     4.5113      7.9869   12.4982
   3      0.2620      2.5228      0.0852     2.5028      8.0842   10.5870
   4      0.4008      3.7987      0.0865     1.6621      8.2119    9.8740
   5      0.5241      4.8932      0.0878     1.2904      8.3360    9.6264
   6      0.6758      6.1880      0.0896     1.0204      8.5003    9.5206
   7      0.8762      7.8096      0.0920     0.8085      8.7325    9.5410
   8      1.1049      9.5417      0.0950     0.6617      9.0133    9.6750
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.136078E+07 kg
Grade =      1.00000      %
R0    =      16329.3      N
R1    =      680.389      Ns/m
R2    =      59.1938      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L  |   P(MW)   |   V(m/s)  |   U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0      0.0000      0.0000      0.0000**********      0.0000**********
   1      0.0730      0.3989      0.1501    15.8303     14.2477   30.0780
   2      0.1436      0.7831      0.1504     8.0632     14.2750   22.3382
   3      0.2620      1.4238      0.1509     4.4347     14.3244   18.7590
   4      0.4008      2.1682      0.1516     2.9121     14.3875   17.2996
   5      0.5241      2.8232      0.1522     2.2365     14.4481   16.6846
   6      0.6758      3.6205      0.1531     1.7440     14.5285   16.2725
   7      0.8762      4.6571      0.1543     1.3558     14.6437   15.9994
   8      1.1049      5.8162      0.1558     1.0856     14.7867   15.8723
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.181437E+07 kg
Grade =     -1.00000      %
R0    =      21772.4      N
R1    =      907.185      Ns/m
R2    =      78.9251      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L  |  P(MW)   |  V(m/s)  |   U(MN)    | F0(1/km)| F1(1/km)| F(1/km)
   0     0.0000    39.1120     0.0000       0.1614    0.0000    0.1614
   1     0.0973    39.3972     0.0020       0.1603    0.1923    0.3526
   2     0.1915    39.6676     0.0040       0.1592    0.3757    0.5349
   3     0.3494    40.1096     0.0071       0.1574    0.6780    0.8354
   4     0.5344    40.6107     0.0108       0.1555    1.0242    1.1797
   5     0.6987    41.0420     0.0140       0.1538    1.3251    1.4790
   6     0.9011    41.5563     0.0178       0.1519    1.6877    1.8396
   7     1.1682    42.2101     0.0227       0.1496    2.1542    2.3038
   8     1.4733    42.9245     0.0281       0.1471    2.6714    2.8185
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.181437E+07 kg
Grade =    -0.500000      %
R0    =      21772.4      N
R1    =      907.185      Ns/m
R2    =      78.9251      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L  |  P(MW)   |  V(m/s)  |   U(MN)    | F0(1/km)| F1(1/km)| F(1/km)
   0     0.0000    23.9977     0.0000       0.2631    0.0000    0.2631
   1     0.0973    24.6788     0.0032       0.2558    0.3070    0.5629
   2     0.1915    25.2918     0.0062       0.2496    0.5893    0.8389
   3     0.3494    26.2387     0.0109       0.2406    1.0364    1.2770
   4     0.5344    27.2457     0.0161       0.2317    1.5266    1.7583
   5     0.6987    28.0672     0.0204       0.2250    1.9377    2.1627
   6     0.9011    29.0025     0.0255       0.2177    2.4182    2.6359
   7     1.1682    30.1350     0.0318       0.2095    3.0174    3.2269
   8     1.4733    31.3144     0.0386       0.2016    3.6619    3.8635
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.181437E+07 kg
Grade =     0.000000E+00 %
RO    =      21772.4      N
R1    =      907.185      Ns/m
R2    =      78.9251      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------

|  L  |  P(MW)   |  V(m/s)  |   U(MN)   | FO(l/km)| F1(l/km)| F(l/km)
   0      0.0000     0.0000     0.0000**********      0.0000**********
   1      0.0973     3.1424     0.0254      2.0093      2.4112      4.4205
   2      0.1915     5.4148     0.0290      1.1661      2.7526      3.9186
   3      0.3494     8.2651     0.0347      0.7639      3.2901      4.0540
   4      0.5344    10.7682     0.0407      0.5864      3.8626      4.4489
   5      0.6987    12.5602     0.0456      0.5027      4.3300      4.8327
   6      0.9011    14.4161     0.0513      0.4380      4.8649      5.3029
   7      1.1682    16.4758     0.0581      0.3832      5.5190      5.9022
   8      1.4733    18.4638     0.0654      0.3420      6.2105      6.5525


DATA OF TRAIN & GRADE
---------------------
TMASS =     0.181437E+07 kg
Grade =     0.500000     %
RO    =      21772.4      N
R1    =      907.185      Ns/m
R2    =      78.9251      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------

|  L  |  P(MW)   |  V(m/s)  |   U(MN)   | FO(l/km)| F1(l/km)| F(l/km)
   0      0.0000     0.0000     0.0000**********      0.0000**********
   1      0.0973     0.7162     0.1115      8.8160     10.5795     19.3955
   2      0.1915     1.3996     0.1122      4.5113     10.6492     15.1605
   3      0.3494     2.5228     0.1136      2.5028     10.7789     13.2817
   4      0.5344     3.7987     0.1154      1.6621     10.9492     12.6114
   5      0.6987     4.8932     0.1171      1.2904     11.1147     12.4051
   6      0.9011     6.1880     0.1194      1.0204     11.3337     12.3541
   7      1.1682     7.8096     0.1227      0.8085     11.6434     12.4519
   8      1.4733     9.5417     0.1266      0.6617     12.0177     12.6794
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =      0.181437E+07 kg
Grade =        1.00000     %
RO    =        21772.4     N
R1    =        907.185     Ns/m
R2    =        78.9251     Ns**2/m**2
C1    =      0.778342E-01 litre/MWs
Q     =      0.811212E-01 MW
n     =      0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L  |  P(MW)   |   V(m/s) |   U(MN)   | FO(1/km)| F1(1/km)| F(1/km)
    0      0.0000     0.0000     0.0000**********     0.0000**********
    1      0.0973     0.3989     0.2001    15.8303    18.9969    34.8272
    2      0.1915     0.7831     0.2005     8.0632    19.0334    27.0965
    3      0.3494     1.4238     0.2012     4.4347    19.0992    23.5338
    4      0.5344     2.1682     0.2021     2.9121    19.1833    22.0954
    5      0.6987     2.8232     0.2030     2.2365    19.2642    21.5007
    6      0.9011     3.6205     0.2041     1.7440    19.3713    21.1153
    7      1.1682     4.6571     0.2057     1.3558    19.5249    20.8806
    8      1.4733     5.8162     0.2077     1.0856    19.7156    20.8012


DATA OF TRAIN & GRADE
---------------------
TMASS =      0.226796E+07 kg
Grade =       -1.00000     %
RO    =        27215.5     N
R1    =        1133.98     Ns/m
R2    =        98.6563     Ns**2/m**2
C1    =      0.778342E-01 litre/MWs
Q     =      0.811212E-01 MW
n     =      0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L  |  P(MW)   |   V(m/s) |   U(MN)   | FO(1/km)| F1(1/km)| F(1/km)
    0      0.0000    39.1120     0.0000     0.1614     0.0000     0.1614
    1      0.1217    39.3972     0.0025     0.1603     0.2404     0.4007
    2      0.2394    39.6676     0.0049     0.1592     0.4697     0.6288
    3      0.4367    40.1096     0.0089     0.1574     0.8475     1.0049
    4      0.6680    40.6107     0.0135     0.1555     1.2802     1.4357
    5      0.8734    41.0420     0.0175     0.1538     1.6564     1.8103
    6      1.1263    41.5563     0.0222     0.1519     2.1096     2.2615
    7      1.4603    42.2101     0.0284     0.1496     2.6928     2.8424
    8      1.8416    42.9245     0.0352     0.1471     3.3393     3.4864
```

```
DATA OF TRAIN & GRADE
---------------------

TMASS =      0.226796E+07 kg
Grade =     -0.500000     %
R0    =       27215.5     N
R1    =       1133.98     Ns/m
R2    =       98.6563     Ns**2/m**2
C1    =      0.778342E-01 litre/MWs
Q     =      0.811212E-01 MW
n     =      0.820000

TABLE OF FUEL CONSUMPTION
-------------------------

|  L  |   P(MW)   |  V(m/s)  |   U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0      0.0000     23.9977     0.0000      0.2631     0.0000     0.2631
   1      0.1217     24.6788     0.0040      0.2558     0.3838     0.6396
   2      0.2394     25.2918     0.0078      0.2496     0.7366     0.9863
   3      0.4367     26.2387     0.0136      0.2406     1.2955     1.5361
   4      0.6680     27.2457     0.0201      0.2317     1.9082     2.1400
   5      0.8734     28.0672     0.0255      0.2250     2.4221     2.6471
   6      1.1263     29.0025     0.0318      0.2177     3.0227     3.2404
   7      1.4603     30.1350     0.0397      0.2095     3.7718     3.9813
   8      1.8416     31.3144     0.0482      0.2016     4.5774     4.7790


DATA OF TRAIN & GRADE
---------------------

TMASS =      0.226796E+07 kg
Grade =      0.000000E+00 %
R0    =       27215.5     N
R1    =       1133.98     Ns/m
R2    =       98.6563     Ns**2/m**2
C1    =      0.778342E-01 litre/MWs
Q     =      0.811212E-01 MW
n     =      0.820000

TABLE OF FUEL CONSUMPTION
-------------------------

|  L  |   P(MW)   |  V(m/s)  |   U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0      0.0000      0.0000     0.0000**********     0.0000**********
   1      0.1217      3.1424     0.0318      2.0093     3.0140     5.0233
   2      0.2394      5.4148     0.0362      1.1661     3.4407     4.6068
   3      0.4367      8.2651     0.0433      0.7639     4.1126     4.8766
   4      0.6680     10.7682     0.0509      0.5864     4.8282     5.4146
   5      0.8734     12.5602     0.0570      0.5027     5.4126     5.9153
   6      1.1263     14.4161     0.0641      0.4380     6.0811     6.5191
   7      1.4603     16.4758     0.0727      0.3832     6.8987     7.2819
   8      1.8416     18.4638     0.0818      0.3420     7.7631     8.1051
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.226796E+07 kg
Grade =     0.500000     %
R0    =      27215.5     N
R1    =      1133.98     Ns/m
R2    =      98.6563     Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
| L  |  P(MW)   |  V(m/s) |   U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0      0.0000     0.0000     0.0000**********    0.0000**********
   1      0.1217     0.7162     0.1393     8.8160    13.2244    22.0404
   2      0.2394     1.3996     0.1402     4.5113    13.3115    17.8228
   3      0.4367     2.5228     0.1419     2.5028    13.4736    15.9764
   4      0.6680     3.7987     0.1442     1.6621    13.6865    15.3487
   5      0.8734     4.8932     0.1464     1.2904    13.8934    15.1838
   6      1.1263     6.1880     0.1493     1.0204    14.1671    15.1875
   7      1.4603     7.8096     0.1533     0.8085    14.5542    15.3627
   8      1.8416     9.5417     0.1583     0.6617    15.0221    15.6838
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.226796E+07 kg
Grade =      1.00000     %
R0    =      27215.5     N
R1    =      1133.98     Ns/m
R2    =      98.6563     Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
| L  |  P(MW)   |  V(m/s) |   U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0      0.0000     0.0000     0.0000**********    0.0000**********
   1      0.1217     0.3989     0.2502    15.8303    23.7461    39.5764
   2      0.2394     0.7831     0.2507     8.0632    23.7917    31.8549
   3      0.4367     1.4238     0.2515     4.4347    23.8739    28.3086
   4      0.6680     2.1682     0.2526     2.9121    23.9791    26.8912
   5      0.8734     2.8232     0.2537     2.2365    24.0802    26.3167
   6      1.1263     3.6205     0.2551     1.7440    24.2142    25.9581
   7      1.4603     4.6571     0.2571     1.3558    24.4061    25.7619
   8      1.8416     5.8162     0.2596     1.0856    24.6445    25.7301
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.272155E+07 kg
Grade =     -1.00000     %
R0    =      32658.7     N
R1    =      1360.78     Ns/m
R2    =      118.388     Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L  |   P(MW)   |  V(m/s) |    U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0       0.0000    39.1120     0.0000      0.1614    0.0000    0.1614
   1       0.1460    39.3972     0.0030      0.1603    0.2885    0.4488
   2       0.2872    39.6676     0.0059      0.1592    0.5636    0.7228
   3       0.5241    40.1096     0.0107      0.1574    1.0170    1.1744
   4       0.8016    40.6107     0.0162      0.1555    1.5363    1.6918
   5       1.0481    41.0420     0.0209      0.1538    1.9877    2.1415
   6       1.3516    41.5563     0.0267      0.1519    2.5315    2.6834
   7       1.7524    42.2101     0.0340      0.1496    3.2313    3.3809
   8       2.2099    42.9245     0.0422      0.1471    4.0071    4.1542
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.272155E+07 kg
Grade =    -0.500000     %
R0    =      32658.7     N
R1    =      1360.78     Ns/m
R2    =      118.388     Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L  |   P(MW)   |  V(m/s) |    U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0       0.0000    23.9977     0.0000      0.2631    0.0000    0.2631
   1       0.1460    24.6788     0.0049      0.2558    0.4605    0.7164
   2       0.2872    25.2918     0.0093      0.2496    0.8839    1.1336
   3       0.5241    26.2387     0.0164      0.2406    1.5546    1.7952
   4       0.8016    27.2457     0.0241      0.2317    2.2899    2.5216
   5       1.0481    28.0672     0.0306      0.2250    2.9066    3.1315
   6       1.3516    29.0025     0.0382      0.2177    3.6273    3.8450
   7       1.7524    30.1350     0.0477      0.2095    4.5261    4.7356
   8       2.2099    31.3144     0.0579      0.2016    5.4928    5.6945
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =     0.272155E+07 kg
Grade =     0.000000E+00 %
R0    =      32658.7      N
R1    =      1360.78      Ns/m
R2    =      118.388      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L   |   P(MW)   |   V(m/s)  |   U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0       0.0000      0.0000      0.0000**********      0.0000**********
   1       0.1460      3.1424      0.0381      2.0093      3.6168      5.6261
   2       0.2872      5.4148      0.0435      1.1661      4.1288      5.2949
   3       0.5241      8.2651      0.0520      0.7639      4.9352      5.6991
   4       0.8016     10.7682      0.0610      0.5864      5.7939      6.3802
   5       1.0481     12.5602      0.0684      0.5027      6.4951      6.9978
   6       1.3516     14.4161      0.0769      0.4380      7.2974      7.7354
   7       1.7524     16.4758      0.0872      0.3832      8.2785      8.6617
   8       2.2099     18.4638      0.0981      0.3420      9.3158      9.6577


DATA OF TRAIN & GRADE
---------------------
TMASS =     0.272155E+07 kg
Grade =     0.500000     %
R0    =      32658.7      N
R1    =      1360.78      Ns/m
R2    =      118.388      Ns**2/m**2
C1    =     0.778342E-01 litre/MWs
Q     =     0.811212E-01 MW
n     =     0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
|  L   |   P(MW)   |   V(m/s)  |   U(MN)   | F0(1/km)| F1(1/km)| F(1/km)
   0       0.0000      0.0000      0.0000**********      0.0000**********
   1       0.1460      0.7162      0.1672      8.8160     15.8693     24.6853
   2       0.2872      1.3996      0.1683      4.5113     15.9738     20.4851
   3       0.5241      2.5228      0.1703      2.5028     16.1684     18.6712
   4       0.8016      3.7987      0.1730      1.6621     16.4238     18.0860
   5       1.0481      4.8932      0.1756      1.2904     16.6721     17.9625
   6       1.3516      6.1880      0.1791      1.0204     17.0006     18.0209
   7       1.7524      7.8096      0.1840      0.8085     17.4651     18.2736
   8       2.2099      9.5417      0.1899      0.6617     18.0266     18.6883
```

```
DATA OF TRAIN & GRADE
---------------------
TMASS =    0.272155E+07 kg
Grade =     1.00000     %
R0    =     32658.7     N
R1    =     1360.78     Ns/m
R2    =     118.388     Ns**2/m**2
C1    =    0.778342E-01 litre/MWs
Q     =    0.811212E-01 MW
n     =    0.820000

TABLE OF FUEL CONSUMPTION
-------------------------
| L |  P(MW)  |  V(m/s) |  U(MN)  | F0(l/km)| F1(l/km)| F(l/km)
  0    0.0000    0.0000    0.0000**********  0.0000**********
  1    0.1460    0.3989    0.3002  15.8303   28.4954   44.3256
  2    0.2872    0.7831    0.3008   8.0632   28.5501   36.6132
  3    0.5241    1.4238    0.3018   4.4347   28.6487   33.0834
  4    0.8016    2.1682    0.3032   2.9121   28.7749   31.6870
  5    1.0481    2.8232    0.3044   2.2365   28.8963   31.1328
  6    1.3516    3.6205    0.3061   1.7440   29.0570   30.8010
  7    1.7524    4.6571    0.3085   1.3558   29.2873   30.6431
  8    2.2099    5.8162    0.3116   1.0856   29.5734   30.6590
```